Splitting, Squeezing and Diluting: Policy Moderation when Candidacy is Endogenous and Voting is Sincere

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Plurality Voting is the electoral rule in which every citizen is given one indivisible vote to cast and the election winner is the candidate who receives the most votes. Several countries, such as the U.S. and Canada, elect their policy-makers by means of Plurality Voting.

Some people have claimed that giving citizens multiple votes to cast would elect more centrist candidates and lead to more moderate policies as compared to Plurality Voting. A policy is said to be moderate compared to another policy if it is preferred by the median citizen.
Two arguments have been put forward to justify the claim that giving citizens multiple votes to cast would result in more moderate policies as compared to Plurality Voting.

- One argument is related to the *wasted vote phenomenon*. In this argument, voting is assumed to be strategic.

- The other argument is related to the *squeezing phenomenon*. In this argument, voting is assumed to be sincere.
Consider the following 3-candidate election under Plurality Voting.

\[ \text{L} \quad \text{C} \quad \text{R} \]
Consider the same 3-candidate election now under Dual Voting.

\[
\begin{align*}
&L & X_L & C & X_C & R \\
&L & \cdots & \cdots & \cdots & \cdots \\
&C & \cdots & \cdots & \cdots & \cdots \\
&R & \cdots & \cdots & \cdots & \cdots 
\end{align*}
\]
Both arguments have been studied in settings where the set of candidates is exogenously given. However, different electoral rules provide different incentives for candidates to stand for election.

- Dellis (JET 2009) endogenizes candidacy and revisits the argument related to the wasted vote phenomenon.

- The present paper adopts a similar framework and revisits the argument related to the phenomenon of squeezing.
Model

Consider a community that must elect a representative to choose and implement a policy.

- Set of alternatives: $X$, a non-empty interval on $\mathbb{R}$.
- Set of citizens: $\mathcal{N}$.
  - Citizen $\ell$’s unique ideal policy: $x_\ell \in X$.
  - Distribution of ideal policies over $X$: $F$, strictly increasing, continuous and symmetric about the median $m$.
  - Citizen $\ell$’s utility: $u_\ell (x) = u (|x - x_\ell|)$ for all $x \in X$, with $u (\cdot)$ concave.
- Set of potential candidates: $\mathcal{P} \subset \mathcal{N}$.
Policy-making process

The policy-making process is modeled following the citizen-candidate approach to electoral competition.

1. **Candidacy stage.** Each potential candidate chooses whether or not to stand for election at a utility cost $\delta > 0$. Candidacy decisions are made simultaneously and non-cooperatively. Candidates cannot credibly commit to the policy they will implement if elected.

2. **Election stage.** An election is held to select the policy-maker. In order to capture the squeezing phenomenon, voting is assumed to be sincere.

3. **Policy selection stage.** The elected policy-maker chooses and implements policy. In case nobody stands for election, the status-quo policy $x_0 \in X$ is kept in place.
Multiple Voting rules

Under a Multiple Voting rule, every citizen has $q$ votes to cast (or $c - 1$ votes if the number of candidates $c \leq q$). The election winner is the candidate who receives the most votes.

Examples of Multiple Voting rules:

- Plurality Voting ($q = 1$).
- Dual Voting ($q = 2$).
- Approval Voting ($q = +\infty$).
Multiple Voting rules

A Multiple Voting rule always yields policy moderation compared to Plurality Voting if

1. The Multiple Voting rule is Approval Voting, i.e., every citizen can vote for as many candidates as she wishes.
2. Voting is restricted to be relatively sincere, i.e., every citizen votes for all the candidates she prefers to the winning lottery.
3. Attention is restricted to serious equilibria, i.e., equilibria in which every candidate is elected with a strictly positive probability.

This result provides a (conditional) theoretical underpinning for the claim that giving citizens multiple votes to cast would lead to more moderate policies as compared to Plurality Voting.
However, if any of the three conditions is not satisfied, then the Multiple Voting rule sometimes leads to more *extreme* policies as compared to Plurality Voting.

This happens because:

- Multiple Voting rules that admit only completely-filled ballots induce multiple similar candidacies, which triggers a dilution effect whereby the votes received by a centrist candidate are diluted in a greater mass of votes. This dilution effect strengthens the squeezing phenomenon.

- Multiple Voting rules that admit truncated ballots are characterized by a multiplicity of permissible voting profiles, which triggers an expansion of the set of equilibrium outcomes.

- The presence of spoilers renders the elimination of the squeezing phenomenon irrelevant.
Conclusion

When candidacy is endogenous,

1. Holding the election under a Multiple Voting rule does not always yield policy moderation compared to Plurality Voting.

2. Holding the election under an Ordinal Voting rule always yields policy moderation compared to Plurality Voting if the Ordinal Voting rule is Coombs Voting. Under Coombs Voting, voters must rank-order the different candidates. A candidate is elected if he is ranked first by a majority of voters. Otherwise, the candidate who is ranked last by the largest number of voters is eliminated and, on every voter’s ballot, the candidates who are ranked below the eliminated candidate are moved up by one rank. This process is repeated until a candidate is ranked first by a majority of voters.
Policy-making process

This modelization of the policy-making process includes two key features of elections:

1. Endogenous candidacy.
   Theoretical support: Dutta et al. (Econometrica 2001).

2. Policy-motivation on the part of candidates.
   Empirical support: Levitt (AER 1996), Lee et al. (QJE 2004).
   Theoretical support: Alesina (AER 1988).
Policy selection stage

Given that:

1. it is the last stage of the game
2. candidates cannot pre-commit to the policy they will implement if elected

⇒ the policy-maker chooses to implement his ideal policy.
Election stage

\( \mathcal{C} \subseteq \mathcal{P} \): Non-empty set of \( c \) candidates.

\( \alpha_\ell (C) = (\alpha_\ell^r)_{r \in \{1, \ldots, c\}} \): Citizen \( \ell \)'s voting strategy, with \( \alpha_\ell^r \) the set of candidates that citizen \( \ell \) ranks at the \( r^{th} \) position with a positive probability.

\( \alpha (C) \): Profile of voting strategies.

**Definition.** For a given non-empty set of candidates \( C \), \( \alpha^* (C) \) is a *profile of admissible voting strategies* if for every citizen \( \ell \):

1. \( \alpha_\ell^* (C) \) is sincere, i.e., if citizen \( \ell \) votes for a candidate \( i \), then she also votes for all the candidates she prefers to candidate \( i \) and she ranks above candidate \( i \) all the candidates she strictly prefers to candidate \( i \); and

2. \( \alpha_\ell^* (C) \) is weakly undominated.
Candidacy stage

$\delta$: Utility cost of running for election.

ei ∈ {0, 1}: Candidacy strategy of potential candidate $i$, with $e_i = 1$ if potential candidate $i$ enters the race.

e = (ei)i∈P: Profile of candidacy strategies.

$\mathcal{C}(e) \equiv \{i \in P : e_i = 1\}$: Set of candidates.

**Definition.** A profile of candidacy strategies $e^*$ is a *candidacy equilibrium* if the candidacy strategy of every potential candidate is a best response to the candidacy strategies of the other potential candidates, given the voting function $\alpha(\cdot)$. ||
Political equilibrium

**Definition.** A *political equilibrium* consists of a pair \((e^*, \alpha^*(.))\) such that:

1. \(\alpha^*(C)\) is a profile of admissible voting strategies for any non-empty set of candidates \(C \subseteq \mathcal{P}\); and
2. \(e^*\) is a candidacy equilibrium given the voting function \(\alpha^*(.))\).

Two types of equilibria:

- **Serious equilibria.** In equilibrium all candidates are elected with a positive probability.
- **Spoiler equilibria.** In equilibrium some of the candidates are elected with probability zero.
Policy moderation

**Definition.** Let $e$ and $\tilde{e}$ be any two equilibria. Denote the probability that alternative $x \in X$ is implemented in $e$ ($\tilde{e}$, resp.) by $p_x$ ($\tilde{p}_x$, resp.). For all alternative $x$, write $x \in e$ ($\tilde{e}$, resp.) if and only if $p_x > 0$ ($\tilde{p}_x > 0$, resp.). Then,

1. $e$ and $\tilde{e}$ are equivalent if and only if $p_x = \tilde{p}_x$ for all alternative $x$.

2. $e$ is moderate compared to $\tilde{e}$ ($\neq e$) if and only if $u_m(x) \geq u_m(y) \geq u_m(z)$ for all $x \in (e \setminus \tilde{e})$, $y \in (e \cap \tilde{e})$ and $z \in (\tilde{e} \setminus e)$, with at least one inequality strict.

3. $e$ is extreme compared to $\tilde{e}$ ($\neq e$) if and only if $\tilde{e}$ is moderate compared to $e$.||
Policy moderation

**Definition.** A voting procedure $V$ yields *policy moderation* compared to another voting procedure $\tilde{V}$ if and only if:

1. every equilibrium under $V$ either has an equivalent equilibrium under $\tilde{V}$ or is moderate compared to all equilibria under $\tilde{V}$; and
2. every equilibrium under $\tilde{V}$ either has an equivalent equilibrium under $V$ or is extreme compared to all equilibria under $V$.

A voting procedure $V$ yields *policy extremism* compared to another voting procedure $\tilde{V}$ if and only if $\tilde{V}$ yields policy moderation compared to $V$.||
Under Plurality Voting, two candidates standing on the same platform would split their votes, thereby helping the election of rival candidates ⇒ Only one left and one right candidates are standing for election.

There is no such risk of splitting votes under a Multiple Voting rule ⇒ Triggers entry at the left and right positions until candidates at those positions capture all the leftist and rightist votes ⇒ A centrist candidate still receives only centrist votes as under Plurality Voting, whereas a left (right) candidate receives centrist votes in addition to the leftist (rightist) votes he receives under Plurality Voting ⇒ The votes received by a centrist candidate are diluted in a greater mass of votes. This *dilution effect* reinforces the *squeezing effect*, which triggers policy extremism.
Consider a community that must elect a representative to choose a tax rate.

- Set of possible tax rates: $X = [0, 1]$.
- Status-quo tax rate: $x_0 = 0$.
- Citizen $\ell$'s utility: $u_\ell(x) = -|x - x_\ell|$.
- Citizens' ideal tax rates are uniformly distributed over $X$.
- Candidacy cost: $\delta = \frac{1}{5}$. 
The (serious) equilibrium set under Plurality Voting:

- One-position serious equilibria: \( \{x\} \) for all \( x \in \left[ \frac{2}{5}, \frac{3}{5} \right] \).
- Two-position serious equilibria: \( \{x, 1 - x\} \) for all \( x \in \left( \frac{1}{6}, \frac{3}{10} \right) \).

The serious equilibrium set under Dual Voting with only completely-filled ballots admissible:

- One-position serious equilibria: \( \{x\} \) for all \( x \in \left[ \frac{2}{5}, \frac{3}{5} \right] \).
- Two-position serious equilibria: \( \{x, 1 - x\} \) for all \( x \in \left[ \frac{1}{10}, \frac{3}{10} \right] \).
The set of equilibrium outcomes under Plurality Voting:

The set of equilibrium outcomes under Dual Voting:
Consider \( \left\{ \frac{1}{8}, \frac{7}{8} \right\} \). Suppose that the election is held under Plurality Voting.

- There is one candidate at each position and the two candidates tie for the first place.
- Suppose a potential candidate at \( \frac{1}{2} \) enters the race \( \Rightarrow \)
  - The candidate at \( \frac{1}{8} \) receives the votes of all the citizens with ideal tax rate \( x_\ell < \frac{5}{16} \).
  - The candidate at \( \frac{1}{2} \) receives the votes of all the citizens with ideal tax rate \( x_\ell \in \left( \frac{5}{16}, \frac{11}{16} \right) \).
  - The candidate at \( \frac{7}{8} \) receives the votes of all the citizens with ideal tax rate \( x_\ell > \frac{11}{16} \).

\( \Rightarrow \) A potential candidate at \( \frac{1}{2} \) wants to enter the race since he would win the election outright.

- \( \left\{ \frac{1}{8}, \frac{7}{8} \right\} \) cannot be supported as an equilibrium.
Suppose that the election is held under Dual Voting.

- There are two candidates at each position and all four candidates tie for the first place.
- Suppose a potential candidate at $\frac{1}{2}$ enters the race ⇒
  - A candidate at $\frac{1}{8}$ receives a vote from all the citizens with ideal tax rate $x_\ell < \frac{5}{16}$ and from half the citizens with ideal tax rate $x_\ell \in \left( \frac{5}{16}, \frac{1}{2} \right)$.
  - The candidate at $\frac{1}{2}$ receives a vote from all the citizens with ideal tax rate $x_\ell \in \left( \frac{5}{16}, \frac{11}{16} \right)$.
  - A candidate at $\frac{7}{8}$ receives a vote from all the citizens with ideal tax rate $x_\ell > \frac{11}{16}$ and from half the citizens with ideal tax rate $x_\ell \in \left( \frac{1}{2}, \frac{11}{16} \right)$.

⇒ A potential candidate at $\frac{1}{2}$ does not want to enter the race since the candidates at $\frac{1}{8}$ and $\frac{7}{8}$ would still be tying for the first place.
Admissibility of truncated ballots

That even more extreme policies can be implemented when truncated ballots are admissible follows because:

- Voters are no longer forced to cast all their votes ⇒
  - Weakens the incentives for multiple similar candidacies.
  - Extremists can choose not to vote for a centrist candidate while centrist voters can choose to vote for all the left or right candidates.
  ⇒ Reinforces the dilution effect.
- The votes of a defecting candidate need not be transferred to his closest neighbor ⇒ Deter incumbent candidates from defecting ⇒ Multiposition serious equilibria can exist.

Notice that admitting truncated ballots can also lead to the adoption of more moderate policies compared to when only completely-filled ballots are admissible.
Consider a community that must elect a representative to choose a tax rate.

- Set of possible tax rates: $X = [0, 1]$.
- Citizen $\ell$’s utility: $u_{\ell}(x) = -|x - x_\ell|$.
- Citizens’ ideal tax rates are uniformly distributed over $X$.
- Candidacy cost: $\delta = \frac{1}{5}$.
- The election is held under Dual Voting.

The set of two-position serious equilibria is given by:

- $\{x, 1 - x\}$ for all $x \in \left[\frac{1}{10}, \frac{3}{10}\right]$: Completely-filled ballots.
- $\{x, 1 - x\}$ for all $x \in \left(0, \frac{3}{10}\right]$: Truncated ballots.
Consider $\left\{ \frac{1}{20}, \frac{19}{20} \right\}$. Suppose that only completely-filled ballots are admissible.

- There are two candidates at each position and all four candidates tie for the first place.
- Suppose a potential candidate at $\frac{1}{2}$ enters the race $\Rightarrow$
  - A candidate at $\frac{1}{20}$ receives a vote from all the citizens with ideal tax rate $x_\ell < \frac{11}{40}$ and from half the citizens with ideal tax rate $x_\ell \in \left( \frac{11}{40}, \frac{1}{2} \right)$.
  - The candidate at $\frac{1}{2}$ receives a vote from all the citizens with ideal tax rate $x_\ell \in \left( \frac{11}{40}, \frac{29}{40} \right)$.
  - A candidate at $\frac{19}{20}$ receives a vote from all the citizens with ideal tax rate $x_\ell > \frac{29}{40}$ and from half the citizens with ideal tax rate $x_\ell \in \left( \frac{1}{2}, \frac{29}{40} \right)$.

$\Rightarrow$ A potential candidate at $\frac{1}{2}$ wants to enter the race since he would win the election outright.

- $\left\{ \frac{1}{20}, \frac{19}{20} \right\}$ cannot be supported as an equilibrium.
Suppose that truncated ballots are admissible.

- There is only one candidate at each position and the two candidates tie for the first place.
- Suppose a potential candidate at \( \frac{1}{2} \) enters the race \( \Rightarrow \) There exists a voting profile such that
  - The candidate at \( \frac{1}{20} \) receives a vote from all the citizens with ideal tax rate \( x_\ell < \frac{1}{2} \).
  - The candidate at \( \frac{1}{2} \) receives a vote from all the citizens with ideal tax rate \( x_\ell \in \left( \frac{11}{40}, \frac{29}{40} \right) \).
  - The candidate at \( \frac{19}{20} \) receives a vote from all the citizens with ideal tax rate \( x_\ell > \frac{1}{2} \).

\( \Rightarrow \) A potential candidate at \( \frac{1}{2} \) does not want to enter the race since the candidates at \( \frac{1}{20} \) and \( \frac{19}{20} \) would still be tying for the first place.
Spoiler equilibria

Under Plurality Voting, the votes of a defecting candidate are transferred to his closest neighbor $\Rightarrow$ No spoiler equilibrium exists under Plurality Voting.

Under a Multiple Voting rule, the votes of a defecting candidate need not be transferred to a close neighbor $\Rightarrow$ Spoiler equilibria can exist under a Multiple Voting rule.

- The presence of spoiler candidates can trigger a multiplicity of candidates’ positions, thereby helping support self-fulfilling prophecies that deter entry by new candidates and exit by incumbent candidates $\Rightarrow$ The presence of spoilers renders the elimination of the squeezing effect irrelevant $\Rightarrow$ Policies can be adopted that are more extreme than under Plurality Voting.
Consider a community that must elect a representative to choose a tax rate.

- Set of possible tax rates: \( X = [0, 1] \).
- Citizen \( \ell \)'s utility: \( u_\ell (x) = -|x - x_\ell| \).
- Citizens’ ideal tax rates are uniformly distributed over \( X \).
- Candidacy cost: \( \delta = \frac{1}{5} \).

Under Plurality Voting, all equilibrium tax rates lie in \( (\frac{1}{6}, \frac{5}{6}) \).

Under Approval Voting, there exists a spoiler equilibrium with two serious contenders standing at \( \left\{ \frac{1}{10}, \frac{9}{10} \right\} \) and two spoiler candidates standing at \( \left\{ \frac{1}{20}, \frac{19}{20} \right\} \).
Relative Sincerity

**Definition.** For a given non-empty set of candidates $C$, a voting strategy for citizen $\ell$, $\alpha_\ell (C)$, is *relatively sincere* if

1. the voting strategy is sincere; and

2. citizen $\ell$ does not truncate her ballot without casting a vote for all the candidates she prefers to the winning lottery.

**Proposition.** Consider the set of serious equilibria in relatively sincere voting strategies. Then, a Multiple Voting rule always yields policy moderation compared to Plurality Voting if and only if the Multiple Voting rule is Approval Voting.

However, extreme spoiler equilibria in relatively sincere voting strategies can exist, even under Approval Voting.
Intuition:

- Approval Voting is the only Multiple Voting rule under which truncated ballots are admissible and no $q$ candidates are ever standing at the same position.

- With $q$ left candidates and $q$ right candidates standing for election, the Relative Sincerity refinement has no bite. With less than $q$ left (right) candidates standing for election, Relative Sincerity implies that the leftist (rightist) voters must be voting for a centrist candidate they prefer to the winning lottery $\Rightarrow$ The squeezing effect is eliminated $\Rightarrow$ If the left and right candidates’ positions are sufficiently polarized, a potential candidate in-between would win the election outright $\Rightarrow$ Policy moderation.

- The presence of spoilers renders the elimination of the squeezing effect irrelevant.