Fundamentals Based Exchange Rate Prediction Revisited

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Predicting FX movements: where do we stand

- Meese and Rogoff (*JIE*, 1983): out-of-sample forecast performance monetary exchange rate models inferior to random walk forecasts (horizon maximal 1 year)

- Mark (*AER*, 1995):
  Present-day monetary model-based error correction terms beat random walk at horizons of 3 to 4 years.

\[ \Delta s_{t+k} = \delta + \beta (s_t - f_t) + \varepsilon_{t+k,t} \]
• HOWEVER: No robust evidence for long-horizon predictability


• Only robust evidence for long-horizon predictability within multi-country panel data: Mark and Sul (*JIE*, 2001) and Groen (*JMCB*, 2005).

• Reasons:
  - short span post-Bretton Woods sample,
  - and persistence of disequilibria.

**This paper:** Does imperfect measurement of macroeconomic fundamentals cause persistence in disequilibria?
Overview of results

- Canadian, UK and US economies: fundamental dynamics driven by 2 dynamic factors
  
  Factor #1 → fundamental nominal dynamics  
  Factor #2 → fundamental real dynamics

- Exchange rate is cointegrated with corresponding domestic and foreign factors

- Factors-based equilibrium error outperform RW/AR forecasts from approx. 1-year ahead onwards.
Outline

Fundamentals based exchange rate model

Estimating dynamic factors

Estimating factors-based ‘fundamental’ levels

Exchange rate forecasting

Concluding remarks
Fundamentals based exchange rate model

- Fundamentals PV exchange rate model:

\[ s_t = \mu + \frac{1}{1 + \omega} \sum_{j=0}^{\infty} \left( \frac{\omega}{1 + \omega} \right)^j E_t(f_{t+j} - f^*_{t+j}) \]

- OFTEN with

\[ f_t = \eta + m_t - \delta y_t \text{ and } f^*_t = \eta^* + m^*_t - \delta y^*_t: \text{ observed} \]

- Motivates the predictive regression

\[ \Delta s_{t+k} = \delta + \beta [s_t - (f_t - f^*_t)] + \varepsilon_{t+k,t}. \]

- BUT: are \( f_t \) and \( f^*_t \) truly observable?
Fundamentals based exchange rate model

- Fundamentals PV exchange rate model:

\[ s_t = \mu + \frac{1}{1 + \omega} \sum_{j=0}^{\infty} \left( \frac{\omega}{1 + \omega} \right)^j E_t(f_{t+j} - f_{t+j}^*) \]

- OFTEN with

\[ f_t = \eta + m_t - \delta y_t \text{ and } f_t^* = \eta^* + m_t^* - \delta y_t^* : \text{ observed} \]

- Motivates the predictive regression

\[ \Delta s_{t+k} = \delta + \beta [s_t - (f_t - f_t^*)] + \varepsilon_{t+k,t}. \]

- BUT: are \( f_t \) and \( f_t^* \) truly observable?
Engel and West (JPE, 2005): some imperfectly measured fundamentals with unobserved measurement error $z_t$.

This paper: all fundamentals are imperfectly measured

Home:

$$\frac{1}{1+\omega} \sum_{j=0}^{\infty} \left( \frac{\omega}{1+\omega} \right)^j E_t(f_{t+j} + z_{t+j}) \approx H \left( \begin{array}{c} \hat{F}_{1t} \\ \hat{F}_{2t} \end{array} \right)$$

Abroad

$$\frac{1}{1+\omega} \sum_{j=0}^{\infty} \left( \frac{\omega}{1+\omega} \right)^j E_t(f_{t+j}^* + z_{t+j}^*) \approx H^* \left( \begin{array}{c} \hat{F}_{1t}^* \\ \hat{F}_{2t}^* \end{array} \right)$$

$(\hat{F}_{1t} \hat{F}_{2t})'$ and $(\hat{F}_{1t}^* \hat{F}_{2t}^*)'$: dynamic factors of the economies
Estimating dynamic factors

- We have $N$ data series: $X_{it}; i = 1, \ldots, N, t = 1, \ldots, T$.
- The economy is driven by $r$ dynamic factors $F_t = (F_{1t} \cdots F_{rt})'$:
  \[
  \tilde{X}_{it} = \lambda_{i0}' F_t + \lambda_{i1}' F_{t-1} + \cdots + \lambda_{ip}' F_{t-p} + e_{it}
  \]
  \[
  e_{it} \sim I(0), \ E(e_{it}) = 0
  \]
  \[
  F_t = F_{t-1} + u_t
  \]
  \[
  u_t \sim I(0), \ E(u_t) = 0.
  \]
- How to estimate $F_t$?
  Rewrite in error correction form:
  \[
  \tilde{X}_{it} = \gamma_{i0}' F_t - \gamma_{i1}' \Delta F_{t-1} - \cdots - \gamma_{ip}' \Delta F_{t-p} + e_{it},
  \]
  where $\gamma_{ik} = \lambda_{ik} + \lambda_{i,k+1} + \cdots + \lambda_{ip}$.
Estimating dynamic factors

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- The economy is driven by $r$ dynamic factors $F_t = (F_{1t} \cdots F_{rt})'$:

  $$\tilde{X}_{it} = \lambda'_{i0} F_t + \lambda'_{i1} F_{t-1} + \cdots + \lambda'_{ip} F_{t-p} + e_{it}$$

  $$e_{it} \sim I(0), \ E(e_{it}) = 0$$

  $$F_t = F_{t-1} + u_t$$

  $$u_t \sim I(0), \ E(u_t) = 0.$$ 

- How to estimate $F_t$?
  Rewrite in error correction form:

  $$\tilde{X}_{it} = \gamma'_{i0} F_t - \gamma'_{i1} \Delta F_{t-1} - \cdots - \gamma'_{ip} \Delta F_{t-p} + e_{it},$$

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Estimating dynamic factors

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\tilde{X}_{it} = \lambda_{i0}'F_t + \lambda_{i1}'F_{t-1} + \cdots + \lambda_{ip}'F_{t-p} + e_{it}
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\[
F_t = F_{t-1} + u_t
\]

$u_t \sim I(0), \ E(u_t) = 0.$

- How to estimate $F_t$?
  Bai (*JofE*, 2004): $F_t \propto T \mathcal{S}_r$;
  $\mathcal{S}_r$: eigenvectors for 1 to $r$ largest eigenvalues of

\[
\frac{\tilde{X}\tilde{X}'}{T^2N}; \quad \tilde{X} = (\tilde{X}_1 \cdots \tilde{X}_N), \quad \tilde{X}_i = (\tilde{X}_{i1} \cdots \tilde{X}_{iT})'
\]
How to determine $r$?

- FIRST: demeaned, standardised first differences. Determines $r + rp$, use upper bound equal to 12 for Bai and Ng (Ectrica, 2004):

  \[
  PC_1 = \ln(V(k)) + 1k \left( \left( \frac{N + T}{NT} \right) \ln \left( \frac{NT}{N + T} \right) \right);
  \]

  \[
  PC_2 = \ln(V(k)) + 1k \left( \left( \frac{N + T}{NT} \right) \ln C_{NT}^2 \right);
  \]

  \[
  PC_3 = \ln(V(k)) + 1k \left( \frac{\ln(C_{NT}^2)}{C_{NT}^2} \right);
  \]

  where $C_{NT}^2 = \min(N, T)$, $V(k) = (\sum_{i=1}^{N} \sum_{t=1}^{T} \hat{e}_{it})/NT$. 
How to determine $r$?

- SECOND: detrended, standardised logs of levels. Determines $r$, use upper bound equal to $r + rp$ for Bai (*JofE*, 2004).

\[
\begin{align*}
PC_1 &= \ln(V(k)) + \frac{T}{4 \ln \ln(T)} k \left( \left( \frac{N + T}{NT} \right) \ln \left( \frac{NT}{N + T} \right) \right); \\
PC_2 &= \ln(V(k)) + \frac{T}{4 \ln \ln(T)} k \left( \left( \frac{N + T}{NT} \right) \ln C_{NT}^2 \right); \\
PC_3 &= \ln(V(k)) + \frac{T}{4 \ln \ln(T)} k \left( \frac{\ln(C_{NT}^2)}{C_{NT}^2} \right);
\end{align*}
\]

where $C_{NT}^2 = \min(N, T)$, $V(k) = (\sum_{i=1}^{N} \sum_{t=1}^{T} \hat{e}_{it})/NT$. 
The data and results


- UK: 86 Macroeconomic time series. Source: Kapetanios, Labhard and Price (2005), IFS, GFD.

  US: 91 Macroeconomic time series. Source: FRED® St. Louis Federal Reserve Database, IFS, GFD.

  Canada: 96 Macroeconomic time series. Source: Galbraith and Tkacz (2007), IFS, GFD.
Results:

- What is $r + rp$?
  - UK: $PC_1 = PC_2 = 6, PC_3 = 5$.
  - US: $PC_1 = PC_2 = 6, PC_3 = 8$.
  - Canada: $PC_1 = PC_2 = 4, PC_3 = 9$.

- What is $r$?
  - UK: $IPC_1 = IPC_2 = 2, IPC_3 = 1$.
  - US: $IPC_1 = IPC_2 = IPC_3 = 2$.
  - Canada: $IPC_1 = IPC_2 = 2, IPC_3 = 1$.

Fundamental dynamics depends on 2 dynamic factors

Giannone et al. *NBER Macroeconomic Annual, 2005*.
Results:

- What is $r + rp$?
  - UK: $PC_1 = PC_2 = 6, PC_3 = 5$.
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- What is $r$?
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  - Canada: $IPC_1 = IPC_2 = 2, IPC_3 = 1$.

Fundamental dynamics depends on 2 dynamic factors

Giannone *et al.* (*NBER Macroeconomic Annual, 2005*).
What do the dynamic factors describe?

- Factors are purely statistical → interpretation?
- For each economy: categorise the individual series
  A: real/GDP components
  B: labour market
  C: international (import, export, terms-of-trade etc.)
  D: money and credit
  E: interest rates and stock prices
  F: prices
- For each economy:
  compute Newey and West (1987, 1994) covariance matrix between each $\Delta X_{it}$ and $(\Delta F_{1t} \quad \Delta F_{2t})$

$\Rightarrow$ compute squared long-run correlation for $\Delta X_{it}, \Delta F_{1t}$
and $\Delta X_{it}, \Delta F_{2t}$. 
Squared long-run correlations: UK
Squared long-run correlations: US
Squared long-run correlations: Canada
Estimating factors-based ‘fundamental’ levels

‘Fundamental’ exchange rate level: Rotate \( \hat{F}_t = (\hat{F}_{1t} \hat{F}_{2t})' \) and \( \hat{F}^*_t = (\hat{F}^*_1 \hat{F}^*_2)' \) to the log exchange rate \( s_t \):

\[
s_t = \alpha_0 + \alpha_1 t + \delta' \begin{pmatrix} \hat{F}_{1t} \\ \hat{F}_{2t} \\ \hat{F}^*_1 \\ \hat{F}^*_2 \end{pmatrix} + \text{error}.
\]

This suggests a ‘fundamental’ exchange rate level:

\[
s^C_t = \hat{\alpha}_0 + \hat{\alpha}_1 t + \hat{\delta'} \begin{pmatrix} \hat{F}_t \\ \hat{F}^*_t \end{pmatrix}.
\]
Cointegration test exchange rate and factors

|   |   | LR \textsubscript{UK/US} (q|5) | LR \textsubscript{Can/US} (q|5) | 95\%  | 99\%  |
|---|---|-----------------|-----------------|-------|-------|
| 3 | 0 | 99.08***        | 119.26***       | 86.96 | 95.38 |
| 1 |   | 51.89           | 61.66           | 62.61 | 70.22 |
| 2 |   | 33.36           | 33.83           | 42.20 | 48.59 |
| 3 |   | 18.63           | 15.15           | 25.47 | 30.65 |
| 4 |   | 8.71            | 5.63            | 12.39 | 16.39 |

\[
(\hat{\beta}', \hat{\beta}_0) = \begin{pmatrix}
  1 & -0.020^{**} & -0.004 & 0.016^{**} & -0.003 & -0.620^{***} & 0.002^{**} \\
  [0.007] & [0.003] & [0.007] & [0.003] & [0.047] & [0.001]
\end{pmatrix}

\textit{US dollar/UK pound sterling rate}

\[
(\hat{\beta}', \hat{\beta}_0) = \begin{pmatrix}
  1 & -0.013^{**} & -0.001 & 0.013^{***} & 0.002 & -0.133^{***} & 0.002^{***} \\
  [0.006] & [0.002] & [0.005] & [0.002] & [0.019] & [0.003]
\end{pmatrix}

\textit{US dollar/Canadian dollar rate}
Exchange rate forecasting

- Our predictive regression:

$$\Delta s_{t+h,t} = \alpha^h + \beta^h (s^c_t - s_t) + \sum_{i=1}^{p} \tilde{\rho}_i \Delta s_{t-i+1,t-i} + \epsilon_{t+h,t}$$

or

$$\Delta s_{t+h,t} = \alpha^h + \beta^h (s^c_t - s_t) + \epsilon_{t+h,t}$$

With:
- $h = 1, 2, 3, 4, 8, 12, 16$
- $s^c_t$: ‘fundamental’ exchange rate level $\rightarrow$ rotation of 2 domestic dynamic factors and 2 US dynamic factors towards $s_t$.
Recursive updating:

2. Extract 2 domestic dynamic factors $F_t$ and 2 US dynamic factors $F_t^*$ from the domestic and US panels over the sample $t = 1, \ldots, t_0 - h$ to construct $s_t^c$ for each $h$.
3. Estimate
\[
\Delta s_{t+h} = \alpha^h + \beta^h (s_t^c - s_t) ( + \sum_{i=1}^{p} \tilde{\gamma}_i \Delta s_{t-i+1,t-i} ) + \epsilon_{t+h,t}
\]
over the sample $t = 1, \ldots, t_0 - h$ for each $h$.
4. Extract 2 domestic dynamic factors $F_t$ and 2 US dynamic factors $F_t^*$ from the domestic and US panels over the sample $t = 1, \ldots, t_0$ to construct $s_t^c$.
5. Generate for $h$ the forecast
\[
\Delta \hat{s}_{t_0+h, t_0} = \hat{\alpha}_{t_0-h}^h + \hat{\beta}_{t_0-h}^h (s_{t_0}^c - s_{t_0}) ( + \sum_{i=1}^{p} \hat{\gamma}_i \Delta s_{t-i+1,t-i} )
\]
6. Repeat for $t_0 + 1, \ldots, T - h$ for each $h$. 
Recursive updating:

2. Extract 2 domestic dynamic factors $F_t$ and 2 US dynamic factors $F^*_t$ from the domestic and US panels over the sample $t = 1, \ldots, t_0 - h$ to construct $s^c_t$ for each $h$.
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   \[
   \Delta \hat{s}_{t_0+h,t_0} = \hat{\alpha}^h_{t_0-h} + \hat{\beta}^h_{t_0-h}(s^c_{t_0} - s_{t_0}) + \sum_{i=1}^{p} \hat{\rho}_i \Delta s_{t-i+1,t-i}
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Recursive updating:


2. Extract 2 domestic dynamic factors \( F_t \) and 2 US dynamic factors \( F^*_t \) from the domestic and US panels over the sample \( t = 1, \ldots, t_0 - h \) to construct \( s^c_t \) for each \( h \).

3. Estimate
   \[
   \Delta s_{t+h,t} = \alpha^h + \beta^h(s^c_t - s_t) + \sum_{i=1}^{p} \hat{\varrho}_i \Delta s_{t-i+1,t-i} + \epsilon_{t+h,t}
   \]
   over the sample \( t = 1, \ldots, t_0 - h \) for each \( h \).

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   \[
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   \]

6. Repeat for \( t_0 + 1, \ldots, T - h \) for each \( h \).
Recursive updating:

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   \]
   over the sample $t = 1, \ldots, t_0 - h$ for each $h$.
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   \[
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   \]
6. Repeat for $t_0 + 1, \ldots, T - h$ for each $h$. 
• Compare against two non-fundamental benchmarks:
  • $AR(p)$ model:
    \[
    \Delta s_{t+h,t} = \alpha^h + \sum_{i=1}^{p} \varphi_i \Delta s_{t-i+1,t-i} + \epsilon_{t+h,t}
    \]
  • Random walk, no change forecast (see Meese and Rogoff (1983)).
  • Forecasts evaluation with the MSE criterion:
    \[
    MSE = \frac{1}{T - t_0 - h} \sum_{s=t_0}^{T-h} e_{s,s+h}^2
    \]
    relative to the MSE of the benchmark forecasts.
• Compare against two *non-fundamental* benchmarks:
  • $AR(p)$ model:
    \[
    \Delta s_{t+h, t} = \alpha^h + \sum_{i=1}^{p} \varrho_i \Delta s_{t-i+1, t-i} + \epsilon_{t+h, t}
    \]
  • Random walk, no change forecast (see Meese and Rogoff (1983)).
  • Forecasts evaluation with the MSE criterion:
    \[
    MSE = \frac{1}{T - t_0 - h} \sum_{s=t_0}^{T-h} \epsilon_{s, s+h}^2
    \]
    relative to the MSE of the benchmark forecasts.
HOWEREVER: finite-sample bias fundamentals-based MSE.

use ‘adjusted’ MSE for fundamentals forecasts (Clark and West (2006a, 2006b)):

\[
MSE_{F}^{adj} = MSE_{F} - \left( \frac{1}{T - t_0 - h} \sum_{s=t_0}^{T-h} \left( \Delta \hat{s}^B_{s,s+h} - \Delta \hat{s}^F_{s,s+h} \right)^2 \right);
\]

- Test for significance MSE difference

\[
z_{MSE} = \sqrt{T - t_0 - h} \left( \frac{MSE_B - MSE_{F}}{\sqrt{\text{Var}(u_{t+h} - (MSE_B - MSE_{F}))}} \right)
\]

\[
z_{MSE} = \sqrt{T - t_0 - h} \left( \frac{MSE_B - MSE_{F}^{adj}}{\sqrt{\text{Var}(u_{t+h}^{adj} - (MSE_B - MSE_{F}^{adj}))}} \right)
\]

based on one-sided bootstrap distributions.
Algorithm Bootstrap Distributions

- DGP Benchmark: $\Delta s_t = c + \sum_{j=1}^{p} \rho_j \Delta s_{t-j} + \epsilon_t$; $RW$: $\rho_1 = \cdots = \rho_p = 0$
- DGPs Macro data (analogous for the foreign variables):

$$
\Delta \hat{F}_t = \sum_{j=1}^{p} \chi_j \Delta \hat{F}_{t-j} + u_t
$$
$$
\hat{e}_{it} = \sum_{j=1}^{p} \zeta_j \hat{e}_{i,t-j} + u_{it}^\delta; i = 1, \ldots, N
$$
$$
\tilde{X}_{it} = \hat{\gamma}_{i0} \hat{F}_t - \hat{\gamma}_{i1} \Delta \hat{F}_{t-1} - \cdots - \hat{\gamma}_{ip} \Delta \hat{F}_{t-p} + \hat{e}_{it}
$$

- Resample $\epsilon_t, u_t$ and the $u_{it}^\delta$'s $\rightarrow$ artificial data: $\hat{s}_t, \hat{X}_{1t}, \ldots, \hat{X}_{Nt}, \hat{X}^*_1, \ldots, \hat{X}^*_N$.
- Extract home and foreign dynamic factors from $\hat{X}_{1t}, \ldots, \hat{X}_{Nt}, \hat{X}^*_1, \ldots, \hat{X}^*_N$ $\rightarrow$ artificial factors-based ‘fundamental’ exchange rate $\hat{s}_t^c$.
- Do forecast evaluation exercise and save $z_{MSE}$ test statistics.
- Repeat 10,000 times and compute $p$-value of empirical $z_{MSE}$ statistics.
Algorithm Bootstrap Distributions

- DGP Benchmark: $\Delta s_t = c + \sum_{j=1}^{p} \rho_j \Delta s_{t-j} + \epsilon_t$; $RW: \rho_1 = \cdots = \rho_p = 0$

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\[
\Delta \hat{F}_t = \sum_{j=1}^{p} \chi_j \Delta \hat{F}_{t-j} + u_t
\]

\[
\hat{e}_{it} = \sum_{j=1}^{p} \zeta_j \hat{e}_{i,t-j} + u_{it}^e; i = 1, \ldots, N
\]

\[
\tilde{X}_{it} = \hat{\gamma}_{i0} \hat{F}_t - \hat{\gamma}_{i1} \Delta \hat{F}_{t-1} - \cdots - \hat{\gamma}_{ip} \Delta \hat{F}_{t-p} + \hat{e}_{it}
\]

- Resample $\epsilon_t, u_t$ and the $u_{it}^e$'s $\rightarrow$ artificial data:

$\hat{s}_t, \hat{X}_{1t}, \ldots, \hat{X}_{Nt}, \hat{X}_{1t}^*, \ldots, \hat{X}_{Nt}^*$.

- Extract home and foreign dynamic factors from $\hat{X}_{1t}, \ldots, \hat{X}_{Nt}, \hat{X}_{1t}^*, \ldots, \hat{X}_{Nt}^*$ $\rightarrow$ artificial factors-based ‘fundamental’ exchange rate $\hat{s}_t^c$.

- Do forecast evaluation exercise and save $z_{MSE}$ test statistics.

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- DGPs Macro data (analogous for the foreign variables):

$$\Delta \hat{F}_t = \sum_{j=1}^{p} \chi_j \Delta \hat{F}_{t-j} + u_t$$

$$\hat{e}_{it} = \sum_{j=1}^{p} \zeta_j \hat{e}_{i,t-j} + u_{it}^e; i = 1, \ldots, N$$

$$\tilde{X}_{it} = \hat{\gamma}'_{i0} \hat{F}_t - \hat{\gamma}'_{i1} \Delta \hat{F}_{t-1} - \cdots - \hat{\gamma}'_{ip} \Delta \hat{F}_{t-p} + \hat{e}_{it}$$

- Resample $\epsilon_t, u_t$ and the $u_{it}^e$'s → artificial data: $\tilde{s}_t, \tilde{X}_{1t}, \ldots, \tilde{X}_{Nt}, \tilde{X}_{1t}^*, \ldots, \tilde{X}_{Nt}^*$.
- Extract home and foreign dynamic factors from $\tilde{X}_{1t}, \ldots, \tilde{X}_{Nt}, \tilde{X}_{1t}^*, \ldots, \tilde{X}_{Nt}^*$ → artificial factors-based ‘fundamental’ exchange rate $\tilde{s}_t^c$.
- Do forecast evaluation exercise and save $z_{MSE}$ test statistics.
- Repeat 10,000 times and compute $p$-value of empirical $z_{MSE}$ statistics.
Also: performance ‘traditional’ fundamental forecasts relative to the $AR(p)$ and RW benchmarks using

$$
\Delta s_{t+h,t} = \alpha^h + \beta^h (s^M_t - s_t) + \sum_{i=1}^{p} \tilde{\gamma}_i \Delta s_{t-i+1,t-i} + \epsilon_{t+h,t}
$$

with

$$
s^M_t = \hat{\alpha}_0 + \hat{\alpha}_1 t + (m_t - m^*_t) - (y_t - y^*_t)
$$

This is the specification used by Mark (1995).
Out-of-sample forecasts: 1-qtr ahead
Out-of-sample forecasts: 4-qtrs ahead
Out-of-sample forecasts: 16-qtrs ahead
## US dollar/pound sterling: AR benchmark

<table>
<thead>
<tr>
<th>$h$</th>
<th>$(\text{MSE}<em>{\text{AR}}-\text{MSE}</em>{\text{F}})$</th>
<th>$(\text{MSE}<em>{\text{AR}}-\text{MSE}</em>{\text{F}}^{adj})$</th>
<th>$(\text{MSE}<em>{\text{AR}}-\text{MSE}</em>{\text{M}})$</th>
<th>$(\text{MSE}<em>{\text{AR}}-\text{MSE}</em>{\text{M}}^{adj})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.27 (1.27)</td>
<td>10.32 (2.06)</td>
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### US dollar/Canada dollar: AR benchmark

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### US dollar/pound sterling: RW benchmark

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## US dollar/Canada dollar: RW benchmark

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**Concluding remarks**

- A better measurement of fundamental drivers
  → better measurement of ‘long-run’ exchange rate levels.

- ‘Fundamental factors’-actual exchange rate gap
  → good forecasts over long horizons (as in other studies)
  but also over the next year (or shorter).

- Work ahead:
  - Other exchange rates
  - Forecasting conventional (monetary) fundamentals?