Collateral Constraints and Macroeconomic Adjustment in an Open Economy

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This paper analyzes a small open economy Ramsey growth model with convex investment costs and a collateral constraint on borrowing. The economy’s adjustment speed depends on the fraction of the capital stock that can be used as collateral. In the presence of non-convexities, a higher loan-to-value ratio of the capital stock may produce a bifurcation in the dynamics by increasing the economy’s adjustment speed.
Contribution of the paper

To motivate, formulate and analytically solve the following problem:

\[ \text{Max} \int_{0}^{\infty} u(c) e^{-\rho t} \, dt \]

subject to

\[ \dot{b} = c + \psi(i, k) + rb - f(k) \]
\[ \dot{k} = i + \delta k \]
\[ b \leq aqk \quad 0 \leq a \leq 1 \]
\[ k_0, b_0 \text{ given} \]

where \( \psi(i, k) = i + \phi\left(\frac{i}{k} - \delta\right)k, \quad \phi' \geq 0, \phi'' > 0, \phi(0) = 0, \phi'(0) = 0 \)
Background

Economics profession: 1970s push to develop intertemporal open economy models

One research avenue: How to open the Ramsey model to international capital flows?

The problem: instantaneous adjustment of the capital stock ⇒ uninteresting dynamics
- Solution 1: Add a production function for nontraded capital.

- Solution 2: Add installation costs for investment

Lucas and Prescott (1973 *Econometrica*),

Hayashi (1982 *Econometrica*, received 8/79)

Applications: Abel-Blanchard (1983 *Econometrica*, received 2/80)

Lipton-Sachs (1983 *JJE*, 10/1980 NBER #572)


Canonical Open Economy Ramsey Model:

Blanchard-Fischer (1979), (Turnovsky 1994, 1997)

No-Ponzi-Game constraint:

\[ b_t \leq q_t k_t + \int_0^\infty w(k_t) e^{-r(s-t)} ds \]

Additional borrowing constraints:

\[ \rho = r \quad \text{Blanchard (1983)} \]
\[ \rho(c) = r \quad \text{Obstfeld (1981)} \]
\[ \rho = r(b) \quad \text{Uribe and others} \]
\[ b \leq \bar{b} \quad \text{Atkeson (1996)} \]
\[ b \leq V(k) \quad \text{Cohen and Sachs (1986)} \]
\[ b \leq qk \quad \text{Kiyotaki and Moore (1997)} \]
\[ b \leq aqk \quad \text{Mendoza and Smith (2006), Mendoza (2010)} \]
Pledgeable Income from Capital:

\[ b_t \leq a \int_{t}^{\infty} [f'(k_s)k_s - \psi(i_s, k_s)]e^{-r(s-t)} ds \Rightarrow b_t \leq a q_t k_t \quad 0 \leq a \leq 1 \]

i. a constraint on pledgeable net income from capital implies a collateral constraint on the value of the capital stock

ii. the upper limit on pledgeable capital income can be due to sovereign risk, inalienability of human capital, or other unmodeled financial frictions
Completely Pledgeable Capital Income  
(Kiyotaki-Moore Constraint)

\[
\begin{align*}
\text{Max} & \int_0^\infty u(c)e^{-\rho t} dt \\
\text{subject to} & \\
\dot{b} &= c + \psi(i, k) + rb - f(k) \\
\dot{k} &= i + \delta k \\
b &\leq qk \\
k_0, b_0 &\text{ given}
\end{align*}
\]

\[e^{\rho t} H = u(c) - \lambda [c + \psi(i, k) + rb - f(k)] + q^*(i - \delta k) + \gamma^*(qk - b)\]

First-order necessary conditions are:

\[u'(c) = \lambda\]
\[\psi_i(i, k) = q \quad \text{where} \quad q = q^*/\lambda \quad \Rightarrow \quad \dot{k} = i(q, k) - \delta k\]

The co-state equations are:

\[\dot{\lambda}/\lambda = \rho - r - \gamma\]
\[\dot{q} = -f'(k) + \psi_k (k, i) + (r + \delta)q\]
Binding collateral constraint \((b = qk)\):

\[
\dot{b} = qk + \dot{q}k = c + \psi(i, k) + rqk - f(k) \Rightarrow
\]

\[
c = f(k) - f'(k)k = w(k)
\]

Consumption is constrained to equal the real wage.

*Capital stock dynamics are the same as \(\rho = r\) assumption without explicit borrowing constraint:*

\[
\begin{pmatrix}
\dot{k} \\
\dot{q}
\end{pmatrix}
= 
\begin{pmatrix}
0 & 1/\psi_{ii} \\
-f''(k) & r
\end{pmatrix}
\begin{pmatrix}
k - \tilde{k} \\
q - \tilde{q}
\end{pmatrix}
\]

The speed of adjustment is \(\mu\):

\[
\mu = \frac{r}{2} - \sqrt{\left(\frac{r}{2}\right)^2 - \frac{f''(k)}{\psi_{ii}}}
\]

\[
k = \tilde{k} + (k_0 - \tilde{k})e^{\mu t}
\]
Closed Economy

\[
\begin{bmatrix}
\dot{c} \\
\dot{k}
\end{bmatrix} =
\begin{bmatrix}
0 & \frac{f''(k)}{\left(\frac{\sigma}{c} + \psi_{ii}\right)} \\
-1 & \rho
\end{bmatrix}
\]

with corresponding eigenvalues given by:

\[
\mu_{1,2} = \frac{\rho}{2} \pm \sqrt{\left(\frac{\rho}{2}\right)^2 - \frac{f''(k)}{\frac{\sigma}{c} + \psi_{ii}}}
\]
Figure 1. Growth with Collateralized Borrowing

\[ \dot{c} = 0 \]

\[ \dot{k} = 0, b = 0 \]

\[ r \bar{b} = [f'(k) - \delta]k \]
Adjustment speed with partially pledgeable capital income:

\[ b_t \leq a \int_{t}^{\infty} \left[ f'(k_s)k_s - \psi(i_s, k_s) \right] e^{-r(s-t)} ds \Rightarrow b_t \leq a \xi_t k_t \quad 0 \leq a \leq 1 \]

\[
\mu_{b=agk} = \frac{\rho}{2} - \sqrt{\left( \frac{\rho}{2} \right)^2 - \frac{f''(k)}{(1-a)^2 \sigma c} + \psi_{ii}}
\]

Special cases:

i. \( \psi_{ii} = 0, \ a = 0 \Rightarrow \) standard (closed economy) Ramsey model
ii. \( \psi_{ii} > 0, \ a = 0 \Rightarrow \) Abel-Blanchard model
iii. \( \psi_{ii} > 0, \ a = 1 \Rightarrow \) Canonical open economy Ramsey model, Kiyotaki-Moore borrowing constraint
iv. \( \psi_{ii} = 0, \ a = 1 \Rightarrow \) instantaneous capital stock adjustment
Non-convexities

Let \( \delta = \delta(k), \quad \delta'(k) \leq 0 \), where \( k \) is an external effect (knowledge).

Assume \( \delta(k) \) is concave-convex, resulting in three equilibria \( (\underline{k}, \hat{k}, \bar{k}) \) where \( f'(k) = \rho + \delta \)

*Figure 2*
Adjustment speed with the external effect:

\[ \mu_{1,2} = \frac{\rho - \delta'(k)k}{2} \pm \sqrt{\left( \frac{\rho - \delta'(k)k}{2} \right)^2 - \frac{f''(k) - \delta'(k)}{(1 - a)^2 \frac{\sigma}{c} + \psi_{ii}}} \]

At \( k \) and \( \bar{k} \) the equilibria are saddlepoints.

At \( \hat{k} \) the Hamiltonian is convex and the determinant of the Jacobian is positive, so the equilibrium is unstable:

- node if both eigenvalues are positive
- focus if eigenvalues are complex

As \( a \to 1 \), there will be a bifurcation in the dynamics for \( \psi_{ii} \) small enough.
Figure 3

\[ \dot{c} = 0 \]

\[ \dot{k} = 0 \quad (b = 0) \]

\[ r\tilde{b} = a\tilde{k} \]
Aghion, Banerjee, and Piketty (1999) and Aghion, Bacchetta, and Banerjee (2004) have also constructed models in which changes in the loan-to-value ratio can alter the adjustment dynamics. However, their models employ either linear (Ak) or Leontief production functions and rely on cobweb dynamics created by price movements arising from the interaction between savers and entrepreneurs. In addition, they characterize unstable equilibria as indicative of negative consequences of capital market liberalization (an increase in $a$) in developing countries.
Conclusion

\[ \max_{c,i} \int_0^\infty u(c) e^{-rt} dt \]

subject to

\[ \dot{b} = c + \psi(i,k) + rb - f(k) \]
\[ \dot{k} = i + \delta k \]
\[ b \leq aqk \quad 0 \leq a \leq 1 \]

\[ \begin{array}{c}
k_0, b_0 \text{ given} \\
npg : \lim_{t \to \infty} b_t e^{-rt} = 0
\end{array} \]