Inflation Bets or Deflation Hedges? The Changing Risks of Nominal Bonds

John Y. Campbell, Adi Sunderam, and Luis M. Viceira
Harvard University

Econometric Society Summer Meetings

Boston, June 6, 2009
CAPM Beta of Bonds
(Realized beta based on 3 months of daily stock and bond returns)
CAPM Beta of Deflation
(3-yr rolling window of Shocks to -Log(Inflation) and Stock Returns)
The Risk of Nominal Bonds

• This evidence is consistent with changes in conventional wisdom about the risk of nominal bonds.

• Early 1980’s:
  – Bonds are exposed to the risk of stagflation.
  – Avoid them unless the term premium is high.
  – Henry Kaufman, “Dr. Doom.”

• Early 2000’s:
  – Bonds are hedges against the risk of deflation.
  – “Anchor to windward.”
  – Hold them even at a low term premium.
A Suggested Interpretation

• Changes in measured bond risks appear to be related to the changing behavior of the Phillips Curve.

• When the Phillips Curve is stable (early 1960’s, 2000’s), inflation falls as unemployment rises.
  – Then bonds do well in bad times and hedge macroeconomic risk.
  – Stocks and bonds are negatively correlated.

• When the Phillips Curve is unstable (1970’s and 1980’s), inflation and unemployment move together (stagflation).
  – Then bonds do badly in bad times and are risky.
  – Stocks and bonds are positively correlated.
A Bond Pricing Model

• This paper studies how this time-varying inflation risk affects the pricing of nominal government bonds.

• We build a model that integrates a changing nominal-real covariance with other known influences on bond prices.
  – Real interest rate, risk aversion, expected inflation.

• Log inflation-indexed yields are affine, and log nominal yields are quadratic in the state variables.
Real Term Structure

- Real stochastic discount factor (SDF):

\[-m_{t+1} = x_t + \frac{\sigma^2}{2} z_t^2 + z_t \varepsilon_{m,t+1},\]

- $x_t$ is real rate

- $z_t$ drives time-variation in the conditional volatility of SDF ("risk aversion") as well as in the conditional covariance of the real SDF with realized returns.

- Both $x_t$ and $z_t$ follow homoskedastic AR(1) processes
Affine Real Term Structure

• Real term structure is exponential affine:

\[ P_{n,t} = \exp \left\{ A_n + B_{x,n}x_t + B_{z,n}z_t \right\} \]

• Yet both the risk premium on real bonds and the risk premium on equities vary over time with \( z_t \):

\[
\begin{align*}
E_t \left[ r_{e,t+1} - r_{1,t+1} \right] + \frac{1}{2} \operatorname{Var}_t \left( r_{e,t+1} - r_{1,t+1} \right) &= \left( B_{x,n-1} \sigma_{mx} + B_{z,n-1} \sigma_{mz} \right) z_t \\
E_t \left[ r_{e,t+1} - r_{1,t+1} \right] + \frac{1}{2} \operatorname{Var}_t \left( r_{e,t+1} - r_{1,t+1} \right) &= \left( \beta_{ex} \sigma_{xm} + \beta_{em} \sigma_m^2 \right) z_t.
\end{align*}
\]
Nominal Term Structure

• Log nominal SDF: $m_{t+1} - \pi_{t+1}$

• Log inflation: $\pi_{t+1} = \lambda_t + \xi_t + \frac{\sigma_t^2}{2} \psi_t^2 + \psi_t \varepsilon_{\pi,t+1}$

• Expected inflation:
  
  – Permanent component:
  
    $\lambda_{t+1} = \lambda_t + \varepsilon_{\Lambda,t+1} + \psi_t \varepsilon_{\lambda,t+1}$

  – Transitory component:
  
    $\xi_{t+1} = \phi_\xi \xi_t + \psi_t \varepsilon_{\xi,t+1}$. 
Nominal Term Structure

• State variable $\psi_t$ follows AR(1) process:

$$\psi_{t+1} = \mu_\psi (1 - \phi_\psi) + \phi_\psi \psi_t + \varepsilon_{\psi,t+1}.$$ 

• This variable is the main innovation of our model, and generates time-variation in:
  – The conditional volatility of realized and expected inflation $(\psi_t)^2$.
  – The covariance of the real SDF $m_t$ with inflation $(\psi_t)$.

• The nominal-real covariance (and thus bond risk premia) can switch sign as $\psi_t$ takes positive or negative values.
Nominal Term Structure

- Nominal term structure is linear-quadratic:

\[
P_{n,t}^s = \exp \left\{ A_n^s + B_{x,n}^s x_t + B_{z,n}^s z_t + B_{\lambda,n}^s \lambda_t + B_{\xi,n}^s \xi_t + B_{\psi,n}^s \psi_t \\
+ C_{z,n}^s z_t^2 + C_{\psi,n}^s \psi_t^2 + C_{z\psi,n}^s z_t \psi_t \right\}
\]

- Nominal bond risk premia:

\[
\log E_t \left[ \frac{P_{n-1,t+1}^s}{P_{n,t}^s} \right] - r_{1,t+1}^s = \gamma_{z,n} z_t + \beta_{z,n} z_t^2 + \beta_{z\psi,n} z_t \psi_t
\]

- As inflation behaves countercyclically or procyclically, nominal bonds command a positive risk premium or a negative risk premium.
Estimation

- State variables are unobserved and model is non-affine.


- Observed variables (1953:Q1-2005:Q4) / measurement equations:
  - Nominal yield curve (3 months, 1 year, 3 years, 10 years).
  - Realized inflation.
  - Yield on constant maturity 10-year inflation-indexed bond (UK and US).
  - Equity returns and dividend-price ratio (proxy for risk aversion).
  - Realized covariance of returns on nominal bonds and equities in daily data.
  - Realized volatility of nominal bond returns in daily data.
Parameter Estimates

• All state variables follow persistent processes.

• Correlation of almost 50% between shocks to the transitory component of expected inflation and shocks to \( m_t \):
  – Positive risk premium in the nominal term structure when \( \psi_t > 0 \).
  – Negative risk premium in the nominal term structure when \( \psi_t < 0 \).

• On average transitory expected inflation risk premium is positive. Has largest effects on intermediate maturities.

• Estimated long-run expected inflation risk premium is close to zero.
Constrained Models

• Our model nests prior models with
  – Constant real and nominal bond risk premia (Campbell and Viceira 2001, 2002).

• We have estimated models that constrain $z_t$, $\psi_t$ and $z_t\psi_t$ to be constant

• Our estimates strongly reject the restriction that $\psi_t$ is constant:
  – Thus nominal bond risk premia have a time-varying component which is different from the time-varying component of the risk premium on real assets
Table 2. Sample and implied moments for 3mo excess returns.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Sample and Implied Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual Data</td>
</tr>
<tr>
<td>3yr YS mean</td>
<td>.674</td>
</tr>
<tr>
<td>10yr YS mean</td>
<td>1.13</td>
</tr>
<tr>
<td>3yr YS std dev</td>
<td>.401</td>
</tr>
<tr>
<td></td>
<td>[.996]</td>
</tr>
<tr>
<td>10yr YS std dev</td>
<td>.642</td>
</tr>
<tr>
<td></td>
<td>[.998]</td>
</tr>
<tr>
<td>3yr RXR mean</td>
<td>1.06</td>
</tr>
<tr>
<td>10yr RXR mean</td>
<td>1.79</td>
</tr>
<tr>
<td>3yr RXR std dev</td>
<td>4.01</td>
</tr>
<tr>
<td></td>
<td>[.928]</td>
</tr>
<tr>
<td>10yr RXR std dev</td>
<td>10.00</td>
</tr>
<tr>
<td></td>
<td>[.011]</td>
</tr>
<tr>
<td>10yr TIPS yield mean</td>
<td>3.37‡</td>
</tr>
<tr>
<td></td>
<td>[.999]</td>
</tr>
<tr>
<td>10yr TIPS YS mean</td>
<td>.001</td>
</tr>
<tr>
<td>10yr TIPS RXR mean</td>
<td>.039</td>
</tr>
<tr>
<td>10yr TIPS RXR std dev</td>
<td>3.27</td>
</tr>
</tbody>
</table>
Table 2. Sample and implied moments for 3mo excess returns.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Actual Data</th>
<th>Full model</th>
<th>Constant $z$</th>
<th>Constant $\psi$</th>
<th>CS EXR</th>
<th>CP EXR</th>
</tr>
</thead>
<tbody>
<tr>
<td>3yr EXR stdev</td>
<td>.334</td>
<td>.339</td>
<td>.007</td>
<td></td>
<td>1.60</td>
<td>1.04</td>
</tr>
<tr>
<td>10yr EXR stdev</td>
<td>.408</td>
<td>.420</td>
<td>.010</td>
<td></td>
<td>3.81</td>
<td>1.51</td>
</tr>
<tr>
<td>10yr TIPS EXR stdev</td>
<td>.003</td>
<td>.000</td>
<td>.001</td>
<td></td>
<td>.001</td>
<td>.004</td>
</tr>
<tr>
<td>3yr RXR $\sigma(CS)$</td>
<td>.810</td>
<td>.300</td>
<td>.285</td>
<td></td>
<td>.629</td>
<td>.575</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[.037]</td>
<td>[.023]</td>
<td></td>
<td>[.259]</td>
<td>[.255]</td>
</tr>
<tr>
<td>10yr RXR $\sigma(CS)$</td>
<td>2.55$^+$</td>
<td>.488</td>
<td>.510</td>
<td></td>
<td>.843</td>
<td>.649</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[.000]</td>
<td>[.000]</td>
<td></td>
<td>[.003]</td>
<td>[.000]</td>
</tr>
<tr>
<td>10yr TIPS RXR $\sigma(CS)$</td>
<td></td>
<td>.179</td>
<td>.184</td>
<td></td>
<td>.188</td>
<td>.132</td>
</tr>
<tr>
<td>3yr RXR $\sigma(CP)$</td>
<td>1.00</td>
<td>.653</td>
<td>.647</td>
<td></td>
<td>1.29</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[.118]</td>
<td>[.093]</td>
<td></td>
<td>[.785]</td>
<td>[.760]</td>
</tr>
<tr>
<td>10yr RXR $\sigma(CP)$</td>
<td>2.23$^+$</td>
<td>1.10</td>
<td>1.27</td>
<td></td>
<td>3.11</td>
<td>2.02</td>
</tr>
</tbody>
</table>
Model Fit

• Model somewhat understates mean yield spreads and mean bond excess returns, but it has a harder time fitting their realized volatility.

• Model-implied risk premium on real bonds is very small and is almost constant in practice.

• Thus the model attributes average nominal bond risk premia and their time variation to the covariance of inflation with the real economy.

• Model generates economically meaningful variation of nominal bond risk premia (30bp-50bp p.a.).
This variation is small relative to the volatility of realized excess nominal bond returns.

It is also small relative to the in-sample variability in expected returns generated by Campbell-Shiller and Cochrane-Piazzesi predictive regressions.

Generate more variability in expected excess returns by replacing the measurement equation linking the dividend yield and $z_t$ with one linking bond excess returns to either

- the yield spread (as in CS regressions) or
- CP’s “tent” variable.
Implications for the Yield Curve

• We plot the yield curve at the sample mean of all the state variables.

• We then vary each state variable to its sample minimum and maximum, holding the other state variables at their sample mean.

• Our model identifies the contribution of each factor to the average level, slope, and curvature of the yield curve.
The real interest rate is both a “level” factor and a “slope” factor:

Figure 8

Responses of real yield curve and nominal yield curve to real interest rate
Risk aversion is a “slope” factor:

Figure 9

Responses of real yield curve and nominal yield curve to risk aversion
The permanent expected inflation is a “level” factor, while the transitory expected inflation is a “slope” factor:

**Figure 10**

Response of nominal yield curve to permanent (left) and transitory (right) expected inflation
The nominal-real covariance is mainly a “curvature” factor:

![Graph showing the response of nominal yield curve to nominal-real covariance.](image)

**Figure 11**

Response of nominal yield curve to nominal-real covariance
Impact of $\psi$ on the curvature of the nominal yield curve:

- The estimated price of risk is much higher for transitory expected inflation, steepening the yield curve at intermediate maturities.

- When $\psi$ changes sign, the difference in risk prices pulls intermediate-term yields down more strongly than long-term yields.

- As $\psi$ becomes large, bond volatility also rises. Convexity lowers the yield needed to deliver a given expected simple return. Effect is much more pronounced on long maturity bonds.
Implications for Risk Premia

• The curvature factor or nominal-real covariance $\psi$ is an important determinant of time-variation in expected excess nominal bond returns in our model.

• Nominal bond risk premia are determined by:
  – Price of risk $\times$ quantity of risk
  – Risk aversion $\times$ nominal-real covariance
  – $z_t \times \psi_t$

• Both matter, but the nominal-real covariance $\psi$ is more important.
  – $z$ moves long-term nominal bond yields only modestly.
Implications for Risk Premia

Response of nominal bond risk premia to nominal-real covariance
Time-Variation in Nominal Bond Risk Premia

History of nominal bond risk premium
Cochrane and Piazzesi (2005) find that a tent-shaped linear combination of forward rates predicts excess bond returns better than yield spreads.

Their tent variable is high when 3-year yield is high relative to the average of short-term and 10-year yields.

Thus, it captures the concavity (curvature) of the yield curve.

Our model interprets this evidence as the result of changes in the nominal-real covariance.
Our model generates the same type of predictability as the one observed in the data using CP Tent variable:
However, our model does not fully match the degree of predictability of bond returns generated by unrestricted CP regressions:

<table>
<thead>
<tr>
<th>3-month holding period</th>
<th>Moment</th>
<th>Unconstrained</th>
<th>Full model</th>
<th>Constant $z$</th>
<th>Constant $\psi$</th>
<th>CS EXR</th>
<th>CP EXR</th>
</tr>
</thead>
<tbody>
<tr>
<td>3yr RXR</td>
<td>.049</td>
<td>.030</td>
<td>.030</td>
<td>.029</td>
<td>.014</td>
<td>.024</td>
<td></td>
</tr>
<tr>
<td>10yr RXR</td>
<td>.050</td>
<td>.031</td>
<td>.031</td>
<td>.023</td>
<td>.003</td>
<td>.014</td>
<td></td>
</tr>
<tr>
<td>1-year holding period</td>
<td>Moment</td>
<td>Unconstrained</td>
<td>Full model</td>
<td>Constant $z$</td>
<td>Constant $\psi$</td>
<td>CS EXR</td>
<td>CP EXR</td>
</tr>
<tr>
<td>3yr RXR</td>
<td>.184</td>
<td>.146</td>
<td>.146</td>
<td>.134</td>
<td>.059</td>
<td>.127</td>
<td></td>
</tr>
<tr>
<td>10yr RXR</td>
<td>.194†</td>
<td>.172</td>
<td>.171</td>
<td>.129</td>
<td>.020</td>
<td>.113</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3-month holding period</th>
<th>Moment</th>
<th>Unconstrained</th>
<th>Full model</th>
<th>Constant $z$</th>
<th>Constant $\psi$</th>
<th>CS EXR</th>
<th>CP EXR</th>
</tr>
</thead>
<tbody>
<tr>
<td>3yr RXR</td>
<td>.049</td>
<td>.029</td>
<td>.029</td>
<td>.028</td>
<td>.002</td>
<td>.024</td>
<td></td>
</tr>
<tr>
<td>10yr RXR</td>
<td>.050</td>
<td>.017</td>
<td>.017</td>
<td>.017</td>
<td>.001</td>
<td>.012</td>
<td></td>
</tr>
<tr>
<td>1-year holding period</td>
<td>Moment</td>
<td>Unconstrained</td>
<td>Full model</td>
<td>Constant $z$</td>
<td>Constant $\psi$</td>
<td>CS EXR</td>
<td>CP EXR</td>
</tr>
<tr>
<td>3yr RXR</td>
<td>.180</td>
<td>.124</td>
<td>.124</td>
<td>.134</td>
<td>.002</td>
<td>.121</td>
<td></td>
</tr>
<tr>
<td>10yr RXR</td>
<td>.214†</td>
<td>.056</td>
<td>.055</td>
<td>.071</td>
<td>.014</td>
<td>.051</td>
<td></td>
</tr>
</tbody>
</table>

Table 4
Conclusions

• Often assume that broad asset classes have a stable risk structure over time. For nominal bonds, this is a mistake.

• Our model implies that the risk premia of nominal bonds have changed as a result of time variation in risk aversion and in the covariance between inflation and the real economy.

• Our results are qualitatively consistent with empirical findings of predictability in excess bond returns,
  – But it does not replicate the high explanatory power of Campbell-Shiller and Cochrane-Piazzesi bond return predictive regressions.

• Thanks!
CAPM beta of bonds (2002.06-2008.04)

3-month centered beta, 10-year Treasury on S&P500
Nominal Yields
Real Yields

![Graph showing Real Yields from 1980 to 2010]

- **10yr Gilts**
- **10yr TIPS**
Equity Dividend Yield
Real State Variables
Expected Inflation Components
Inflation-Recession Covariance

Time Series of $\psi_t$

Year


$\psi_t$
Stable Phillips Curve

Inflation (- $R_{bond}$) vs. Unemployment (- Output)

Good times

Bad times
Unstable Phillips Curve

Inflation (- R_{bond})

Unemployment (- Output)

Bad times

Good times
Figure 3
CAPM beta of bonds and the yield spread
(1962.07-2003.12)

Realized beta of bonds based on 3-months of daily returns on stocks and bonds (right axis), and annualized log yield spread (right axis).