Monetary Policy Regime Shifts and Inflation Persistence

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Introduction

- Bayesian estimation of a small-scale New Keynesian model with switching in:
  - Taylor rule coefficients
  - Shock volatilities

- Assess how regime changes impact inflation persistence
  - Connect structural parameters to inflation autocorrelation - composition effect
  - Evaluate the relative importance of shifts in policy coefficients vs. shifts in volatilities in explaining inflation persistence
Related Literature


- Literature on changes in inflation persistence
Markov-Switching New Keynesian (MSNK) Model: Households and Firms

- The representative household chooses \( \{C_t, H_t, B_t\}_{t=0}^{\infty} \) to maximize lifetime utility

\[
E_t \sum_{t=0}^{\infty} \beta_t \left( \frac{(C_t/A_t)^{1-\tau}}{1 - \tau} - H_t \right),
\]

subject to

\[
P_tC_t + Q_tB_t = B_{t-1} + W_tH_t + P_tD_t - P_tT_t,
\]

- Monopolistically competitive intermediate goods-producing firms pay a quadratic cost of price adjustment

- The representative final-goods producing firm combines intermediate goods using Dixit-Stiglitz CRS technology
The MSNK Model: Exogenous Shock Processes

- Aggregate productivity follows

\[ \ln A_t = \lambda + \ln A_{t-1} + \ln a_t, \]

where

\[ \ln a_t = \rho_a \ln a_{t-1} + \epsilon_{at} \]

- Productivity grows on average at \( \lambda \), but is subject to serially correlated shocks

- Shocks to the markup follow

\[ \ln u_t = (1 - \rho_u) \ln u + \rho_u \ln u_{t-1} + \epsilon_{ut}, \]
The monetary authority sets the short-term nominal rate using the following rule

\[ R_t = r \Pi \left( \frac{\Pi_t}{\Pi} \right)^{\alpha(s_t)} \left( \frac{Y_t}{A_t y^*} \right)^{\gamma(s_t)} \exp(e_t), \]

where

\[ \ln e_t = \rho_e \ln e_{t-1} + \epsilon_{et} \]

\( s_t \) evolves according to a Markov chain
Private sector relations and monetary policy

\[
\begin{align*}
    x_t &= E_t x_{t+1} - \tau^{-1} (R_t - E_t \pi_{t+1} - E_t a_{t+1}) \\
    \pi_t &= \beta E_t \pi_{t+1} + \kappa (x_t + u_t) \\
    R_t &= \alpha \pi_t + \gamma x_t + e_t
\end{align*}
\]

Exogenous shock processes

\[
\begin{align*}
    \hat{a}_t &= \rho_a \hat{a}_{t-1} + \varepsilon_{at} \\
    \hat{u}_t &= \rho_u \hat{u}_{t-1} + \varepsilon_{ut} \\
    \hat{e}_t &= \rho_e \hat{e}_{t-1} + \varepsilon_{et}
\end{align*}
\]
MSNK Model: Switching Monetary Policy Rule

- Private sector relations and monetary policy

\[
x_t = E_t x_{t+1} - \tau^{-1} (R_t - E_t \pi_{t+1} - E_t a_{t+1}) \\
\pi_t = \beta E_t \pi_{t+1} + \kappa (x_t + u_t) \\
R_t = \alpha (s_t) \pi_t + \gamma (s_t) x_t + e_t
\]

- Exogenous shock processes

\[
\hat{a}_t = \rho_a \hat{a}_{t-1} + \sigma_a \varepsilon_{at} \\
\hat{u}_t = \rho_u \hat{u}_{t-1} + \sigma_u \varepsilon_{ut} \\
\hat{e}_t = \rho_e \hat{e}_{t-1} + \sigma_e \varepsilon_{et}
\]
MSNK Model: Switching Shock Volatility

- Private sector relations and monetary policy

\[ x_t = E_t x_{t+1} - \tau^{-1} (R_t - E_{t+1} \pi_t + E_t \pi_{t+1} - E_t a_{t+1}) \]
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa (x_t + u_t) \]
\[ R_t = \alpha \pi_t + \gamma x_t + e_t \]

- Exogenous shock processes

\[ \hat{a}_t = \rho_a \hat{a}_{t-1} + \sigma_a (r_t) \varepsilon_{at} \]
\[ \hat{u}_t = \rho_u \hat{u}_{t-1} + \sigma_u (r_t) \varepsilon_{ut} \]
\[ \hat{e}_t = \rho_e \hat{e}_{t-1} + \sigma_e (r_t) \varepsilon_{et} \]
Private sector relations and monetary policy

\[ x_t = E_t x_{t+1} - \tau^{-1} (R_t - E_t \pi_{t+1} - E_t a_{t+1}) \]
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa (x_t + u_t) \]
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Exogenous shock processes

\[ \hat{a}_t = \rho_a \hat{a}_{t-1} + \sigma_a (r_t) \varepsilon_{at} \]
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\[ \hat{e}_t = \rho_e \hat{e}_{t-1} + \sigma_e (r_t) \varepsilon_{et} \]
The ‘Stacked’ System

- As in Gordon and St-Amour (2000) and Bansal and Zhou (2002), distribute probability over conditional expectations

\[ E_t[\pi_{t+1}|s_t = 1] = p_{11}E_t[\pi_{1t+1}] + (1 - p_{11})E_t[\pi_{2t+1}] \]
\[ E_t[\pi_{t+1}|s_t = 2] = (1 - p_{22})E_t[\pi_{1t+1}] + p_{22}E_t[\pi_{2t+1}] \]

- Substitute these expressions for conditional expectations

- Creates a ‘stacked’ system

\[ Ay_t = BE_t y_{t+1} + \Gamma \epsilon_t \]

where \( y_t = [\pi_{1t}, \pi_{2t}, x_{1t}, x_{2t}]' \)

- Unique, bounded solution of stacked system is the MSV solution (Davig and Leeper (2007))
The ‘Stacked’ System

- Monetary policy can be passive, yet not induce indeterminacy in the stacked system

- Restrict parameters to determinate region of the stacked system during estimation

- Next steps in estimation:
  - Need to consider non-MSV solutions (Farmer, Waggoner, and Zha (2009))
Channels of Changing Inflation Persistence

- The population moment for the serial correlation of inflation

\[ \rho_\pi(s_t) = w_a(s_t)\rho_a + w_u(s_t)\rho_u + (1 - w_a(s_t) - w_u(s_t))\rho_e \]

- Weights change when monetary policy regime changes

- A shift to more active policy shifts weight from more persistent to less persistent shocks

- A decline in the relative volatility of more persistent shocks can also transfer weight to less persistence shocks
Graphical Illustration of the Weight of the Most Persistent Shock
The solution of the MSNK maps into a regime-switching state-space representation

\[
Z_t = A_z + B_z(s_t)X_t + [1, 0, 0]' \ln A_t , \quad Z_t = [\ln Y_t, \pi_t, R_t]',
\]

\[
X_t = \rho X_{t-1} + \epsilon_t , \quad X_t = [\hat{a}_t, \hat{u}_t, \hat{e}_t]', \quad \epsilon_t = [\epsilon_a t, \epsilon_u t, \epsilon_e t]',
\]

\[
\ln A_t = \lambda + \ln A_{t-1} + \hat{a}_t.
\]

Bayesian MCMC as in Schorfheide (2005)

Three observables: per capita real GDP, inflation (log difference of GDP deflator), and 3 month Treasury bill rate

Data from 1953:Q1 to 2006:Q4
Switching Monetary Rule: Estimates

- Switching MP rule (90% posterior interval):
  - Active Regime:
    \[ \alpha_1 \in [1.81, 2.30] \]
    \[ \gamma_1 \in [.029, .161] \]
  - Passive (or Less-Active) Regime
    \[ \alpha_2 \in [.80, .98] \]
    \[ \gamma_2 \in [.048, .176] \]
- Passive monetary policy amplifies shocks, whereas active monetary policy dampens them
Switching Monetary Rule: Probability of Active Regime

- Passive policy in the 1970s captures higher volatility
Switching Monetary Rule: Implied Inflation Persistence

- Not distinguishing inflation vs. “inflation gap” persistence

- Measure of persistence

\[ \pi_t = \rho \pi(s_t) \pi_{t-1} + u_t \]

- Model-implied inflation persistence:
  - \( \rho_1 = [.77, .88] \)
  - \( \rho_2 = [.92, .97] \)

- Inflation persistence drops under active policy

- Caveat: Obtained by ignoring expectation formation effect of future regime changes (the next step: compute the model implied persistence by simulation)
MSNK Model:
Switching Shock Volatility: Estimates and Model-implied Persistence

▶ Volatility Regime

\[ \sigma_{a,1} \in [0.014, 0.018], \sigma_{f,1} \in [0.004, 0.006], \sigma_{i,1} \in [0.006, 0.008] \]
\[ \sigma_{a,2} \in [0.007, 0.009], \sigma_{f,2} \in [0.0014, 0.0018], \sigma_{i,2} \in [0.0030, 0.0038] \]

▶ Model-implied Persistence

\[ (r_t = 1) \in [0.8226, 0.8762] \]
\[ (r_t = 2) \in [0.7627, 0.8184] \]
Switching Volatility: Prob of High Volatility Regime

- Capturing the rise and fall (Great Moderation) of volatility
- Move to a more flexible framework: non-synchronous switching in volatility and monetary policy
MSNK Model: Switching Policy and Volatility: Estimates and Model-implied Persistence

- **Policy Regime:**
  \[
  \alpha_1 \in [1.53, 1.69], \quad \gamma_1 \in [0.041, 0.164] \\
  \alpha_2 \in [1.07, 1.21], \quad \gamma_2 \in [0.031, 0.134]
  \]

- **Volatility Regime**
  \[
  \sigma_{a,1} \in [0.013, 0.017], \quad \sigma_{f,1} \in [0.002, 0.004], \quad \sigma_{i,1} \in [0.005, 0.007] \\
  \sigma_{a,2} \in [0.006, 0.008], \quad \sigma_{f,2} \in [0.0008, 0.0013], \quad \sigma_{i,2} \in [0.0026, 0.0034]
  \]

- **Model-implied Persistence**
  \[
  (s_t = 1, r_t = 1) \in [0.7910, 0.8760], \quad (s_t = 1, r_t = 2) \in [0.7556, 0.8427] \\
  (s_t = 2, r_t = 1) \in [0.8485, 0.9256], \quad (s_t = 2, r_t = 2) \in [0.8048, 0.8947]
  \]
4-regime MSNK Model: Prob of Active Regime and High Volatility Regime

- Policy regime before the Volcker period and volatility regime are very similar to 2 regime cases
- Two episodes of passive policy regime after the Volcker period coexisting with the low volatility regime
Smoothed estimates of shocks

- Markup shock was high both in the mid 70s and the early 80s
- Policy shock was negative in the mid 70s but positive in the early 80s
Model Fit

- NK model with switching rule and shock volatility fits the data best based on log marginal likelihood.

- A fixed-regime specification with time-varying nonstationary inflation target substantially erodes model fit.

- Incorporating a drifting target to MSNK models does not change the big picture.
Prediction is done at the posterior mean of 4 regime model

Underprediction in the mid 70s and overprediction in the early 80s
4-regime MSNK Model: Model-Implied Inflation Persistence

Pattern of rising and falling persistence consistent with evidence in Cogley, Primiceri and Sargent (2008)
Summary

- Shifts in policy and volatility generate changes in inflation persistence
- Bayesian estimation of NK model with switching in the monetary policy coefficients and shock volatilities
- A rise and fall in the model-implied inflation persistence consistent with other empirical studies
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<th>Parameters</th>
<th>Prior 90% Interval</th>
<th>P1</th>
<th>Posterior 90% Interval</th>
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</table>
A Switching Fisherian Model of Inflation

► A simple example

\[ i_t = E_t \pi_{t+1} + r_t \]
\[ r_t = \rho r_{t-1} + \nu_t \]
\[ i_t = \alpha(s_t) \pi_t \]

\( s_t \in \{1, 2\} \) follows a Markov chain with \( p_{ij} = P[s_t = j | s_{t-1} = i] \)

► Policy is active if \( \alpha(s_t) > 1 \) and passive if \( \alpha(s_t) < 1 \)

► The model is summarized as

\[ \alpha(s_t) \pi_t = E_t \pi_{t+1} + r_t \]
The stacked system is

\[
\begin{bmatrix}
\pi_{1t} \\
\pi_{2t}
\end{bmatrix}
= A^{-1} P \begin{bmatrix}
E_t \pi_{1t+1} \\
E_t \pi_{2t+1}
\end{bmatrix} + \begin{bmatrix}
r_t \\
r_t
\end{bmatrix}
\]

where

\[
A = \begin{bmatrix}
\alpha_1 & 0 \\
0 & \alpha_2
\end{bmatrix}, \quad P = \begin{bmatrix}
p_{11} & 1 - p_{11} \\
1 - p_{22} & p_{22}
\end{bmatrix}
\]

Use standard tools applicable to constant-parameter RE models to

- compute solution
- determine uniqueness of stacked system
The Long-Run Taylor Principle

- The Long-Run Taylor Principle (LRTP): 
  - $\alpha_i > 1$, for some $i = 1, 2$
  - $\alpha_i > p_{ii}$ for $i = 1, 2$
  - $(1 - \alpha_2) p_{11} + (1 - \alpha_1) p_{22} + \alpha_1 \alpha_2 > 1$

- Passive policy can be substantial, but brief - or modest, and prolonged

- If the LRTP holds, there exists a unique uniformly bounded solution for the stacked system

- Intuition carries over to MSNK model
No known stacked system representation for the MSNK models with switching autoregressive coefficients

- Autoregressive parameters irrelevant for determinacy

Use determinacy restrictions from stacked system, but solve each MSNK model using MSV approach

- MSV and determinate solutions coincide
Minimum State Variable Solution

- Define state as \((r_t, s_t)\) and find MSV solutions
  - Posit regime-dependent rules
    \[\pi_t = a(s_t) r_t\]
    \[a(s_t) = \begin{cases} 
    a_1 & \text{for } s_t = 1 \\
    a_2 & \text{for } s_t = 2 
    \end{cases}\]
  - Expectations functions
    \[E[\pi_{t+1} | s_t = 1, r_t] = [p_{11}a_1 + (1 - p_{11})a_2] \rho r_t\]
    \[E[\pi_{t+1} | s_t = 2, r_t] = [(1 - p_{22})a_1 + p_{22}a_2] \rho r_t\]
We obtain a linear system in the unknown coefficients, $(a_1, a_2)$

$$A \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = b,$$

where

$$A = \begin{bmatrix} \alpha_1 - \rho p_{11} & -\rho (1 - p_{11}) \\ -\rho (1 - p_{22}) & \alpha_2 - \rho p_{22} \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
Minimum State Variable Solution

- Decision rules are:

\[ a_1 = a_1^F \left( \frac{1 + p_{12} a_2^F}{1 - p_{12} a_2^F p_{21} a_1^F} \right) \]

and

\[ a_2 = a_2^F \left( \frac{1 + p_{21} a_1^F}{1 - p_{12} a_2^F p_{21} a_1^F} \right) \]

- If \( p_{12} = p_{21} = 0 \), then \( a_i \) equal ‘fixed-regime’

\[ a_i^F = \frac{\rho}{\alpha_i - \rho p_{ii}}, \quad i = 1, 2 \]

- Basic intuition: \( \alpha_1 > \alpha_2 \Leftrightarrow a_1 < a_2 \)