Equilibrium Subprime Lending

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Financing contracts for households typically feature:

1. noncontingent repayment schedules
2. costly resolution of payment defaults

A large contracting literature rationalizes these features as a consequence of *ex post* asymmetric information. (Townsend (1979), Gale and Hellwig (1985)).
Introduction

- This paper studies the impact of *ex post* asymmetric information on debt capacities and asset prices in a dynamic endowment economy.
- We develop an equilibrium framework for the study of secured lending to impatient, information-problematic borrowers facing severe liquidity constraints.
- Our framework is analytically tractable.
- It delivers realistic orders of magnitude for debt capacities, default intensities, and trading volume in subprime mortgage markets.
Model

- Continuous time.
- Single perishable consumption good.
- A unit mass of assets - housing units.
- Two types of agents: a unit mass of households and several non-atomistic banks.
At date $t$, household $j$ cares for $q_{j,t}$ the number of housing units it occupies between $t$ and $t + dt$, and $n_{j,t} dt$ its consumption between $t$ and $t + dt$.

- Households maximize current home size, and current consumption when indifferent:
- With such preferences, mortgage constraints always bind.
There are discrete dates $\mathcal{N}_j$ with constant arrival intensity $\delta$ at which household $j$ moves exogenously (e.g., occupational change). "Exogenous termination (ET) dates".

Household $j \in [0, 1]$ endowed with an income stream $(l_{j,t})_{t \geq 0}$ such that $l_{j,t} = l_t \times i_{j,t}$.

Systematic component follows $\frac{dl_t}{l_t} = \phi \sigma dW_t$, $\phi \in [0, 1]$, and $\sigma > 0$.

Idiosyncratic component

\[
\begin{cases}
\forall t \in \mathcal{N}_j, i_{j,t} = 1 \\
\frac{di_{j,t}}{i_{j,t}} = \sqrt{1 - \phi^2 \sigma dW_{j,t}}, \text{ for } t \not\in \mathcal{N}_j
\end{cases}
\]

Thus, idiosyncratic income re-set to 1 at each ET date.
Banks are infinitely lived, risk-neutral, and discount the future at the rate $r > 0$.

Banks own an eviction technology. This means that a bank can transform an occupied home that it has financed into a vacant home.

Eviction comes at a cost equal to a fraction $\lambda$ of the home market value, where $\lambda \in (0, 1]$. 
Model-Market for vacant units

- Vacant units are perfectly divisible.
- To modify its home size from $q$ to $q' \neq q$, a household needs to move into $q'$ new units.
- Households face moving costs $c$. If household $j$ moves into $q$ new units at date $t$, only a fraction $\frac{q}{1+c}$ of these new units enters its date-$t$ preferences. Instantaneous cost.
- Banks and households are home-price takers.
Model-Contracts (1)

- Two contracting frictions:
  1. Households privately observe their income stream and can secretly consume.
  2. Households can vacate their homes and re-trade in mortgage and housing markets as they see fit.
Model-Contracts (2)

- Banks can commit to a contract.
- For realism, we study contracts that are enforceable by an agent who observes only the reports and actions of the borrower, not the aggregate state.
• Given households’ myopia, optimal contracts are simple in this environment:
  • Contracts are exclusive.
  • Households use loans to acquire vacant units that they occupy against the promise to meet future deterministic repayments.
  • The bank retains the rights to sell the home once it is vacant.
  • The bank commits to evict a household which does not keep its promise.

• In equilibrium, households report and repay the noncontingent contractual repayment until eviction, ET date, or voluntary termination to move to a bigger place.
With fixed-repayment contracts, banks quote at each date $t$ a loan-to-income ratio and a repayment ratio $\kappa \in [0, 1]$ such that if a household reports an income $I_t$ at the outset of the contract it will have to repay $\kappa I_t$ at all future dates $t + t'$. We first study and compare equilibria with fixed repayments and $P_t = P \times I_t$ for $\phi = 1$ and $\phi = 0$. 
Idiosyncratic Income Risk-Equilibrium Features

- $\phi = 0$. No aggregate uncertainty $\implies$ We solve for a steady-state with constant price $P$, and constant income $Idt$ and home supply $Sdt$ in the market between $t$ and $t + dt$.
- We obtain all the characteristics of the equilibrium as a function of the repayment ratio $\kappa \in (0, 1)$:
  
  1. Equilibrium home price $P$
  2. Home supply and income in the market between $t$ and $t + dt$
     $Idt$ and $Sdt$
  3. Total steady-state intensity of households arrival in the market decomposed into
     - Arrival because of default.
     - Arrival because they want a bigger place.
     - Arrival because of an ET date: $\delta$. 
Orders of magnitude

- If $r = 2\%$, $\sigma = 20\%$, $\delta = 5\%$ (20-year mortgage), $1 + c = 1.5$, then
- $\kappa = 50\% \times I_{j,t}$
- Arrival intensity in the market $\frac{\delta}{1 - p_\kappa - p_c} = 15\%$ (average effective contractual relationship of 6.5 years)
- Default intensity $= \frac{\delta p_\kappa}{1 - p_\kappa - p_c} = 2.3\%$
- These numbers are plausible for the subprime population.
Note that defaults increase in $\delta$ for $\delta$ small, and increase as $\chi = \frac{1}{1+c}$ increases. Equilibrium effect.
Systematic Income Risk

- $\phi = 1$. Incomes are identical. Thus trades for only two reasons: exogenous terminations and evictions. Each household always occupies one unit.
- Proposition 1

Let

$$\alpha = \sqrt{\frac{2(r + \delta)}{\sigma^2}} + \frac{1}{4} - \frac{1}{2}, \rho = 1 - \lambda \left(1 + \frac{\delta}{r}\right).$$

Unique equilibrium with fixed-repayment contracts and linear home prices. Repayment $\kappa$ is the unique solution of:

$$(\alpha + 1) \kappa^\alpha = 1 + \rho \alpha \kappa^{\alpha + 1}.$$  

Home price is:

$$P_t = \frac{1 - \kappa^\alpha}{1 - \rho \kappa^{\alpha + 1}} \times \frac{\kappa l_t}{r} = \frac{\alpha}{\alpha + 1} \times \frac{\kappa l_t}{r},$$

where $\frac{\kappa \alpha}{\alpha + 1}$ is also the loan-to-income ratio offered in the loan market.
Comparing systematic and idiosyncratic

- Assume $c = +\infty$ (infinite cost of moving). Equilibrium loan-to-income ratios and initial home prices are larger when income risk is diversifiable if and only if

$$\rho \geq 0 \iff \lambda \leq \frac{r}{r + \delta}.$$

- Intuition: systematic risk implies that home values are low after foreclosures because everybody defaults at the same time. But idiosyncratic income risk transforms this problem into a commitment problem because lucky households terminate their contracts when it is the most valuable to their bank.
Nonlinear Prices

- The only equilibrium price that satisfies the transversality condition is the linear one.
- Nonlinear exploding equilibrium price paths (housing bubbles and crashes) are such that:
  - \( \kappa(I) < \kappa_0 \) for all aggregate income \( I > 0 \), where \( \kappa_0 \) is the (constant) repayment ratio in the linear equilibrium. Intuition: banks minimize default risk to ride the bubble as long as possible.
  - Furthermore, 0 is an accumulation point of \( \kappa(I) \) as \( I \to +\infty \), and \( +\infty \) is an accumulation point of the loan-to-income ratio as \( I \to +\infty \).
- Thus leverage is procyclical (Almeida et al. (2006) and Lamont and Stein (1999))
- The repayment ratio is countercyclical: persistent effect of income shocks on default intensities.
Optimal contracts

- The fixed-repayment contracts have no reason to be optimal.
- Solving analytically for the optimal repayment schedule seems difficult.
- We solve for optimal repayment schedules that are log-linear and log-linear with one kink for the idiosyncratic case. (Closed forms)
Optimal contracts

Figure 6. Optimal repayment schedules for $r=2\%$, $\sigma=20\%$, $\delta=5\%$, and $\lambda=15\%$. 
Conclusion

- A simple, tractable model of equilibrium in a market for secured loans in the presence of tight borrowing constraints.
- Realistic orders of magnitude
- Future work:
  1. Workhorse model for more quantitative work using household level data
  2. Workhorse model for the study of other asset markets with highly leveraged investors.