Lumpy Capital, Labor Market Search & Employment Dynamics over Business Cycles

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“In times of recession, large employers disproportionately lose workers, while small companies, as a group, fare better.” (Kiviat 2009)
Cyclical Behaviour of the U.S. Job Creation & Job Destruction in Small and Large Firms

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<tr>
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<td>0.6192</td>
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<td>0.4498</td>
<td>-0.1781</td>
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Correlation with GDP

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| 6.81                        |

Data Source: Business Employment Dynamics (BED) data set U.S.
Chapter 2: Model and mechanism

- Labor market frictions $\rightarrow$ lumpy investment
  
  Small affected more

- Aggregate productivity increases $\rightarrow$
  labor market tighter $\rightarrow$ lumpy investment
  
  A tighter labor market deters more investment projects in small firms

  $\rightarrow$ Investment increases by more in large firms $\rightarrow$
  workers move from small to large firms
Literature

- Moscarini and Postel-Vinay (2008)
  - Workers move from small to large firms in late expansions
  - Different dynamic wage strategies
  - Lumpy investment
- Cooper, Haltiwanger and Wills (2006)
  - Employment adjustment is lumpy
  - Non-convex labor search costs (abstracts from capital)
Model: households

A representative household with a unit measure of members (Tractable, Shi 1997)

Preference:

\[ E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) - A N_t] \]

N: fraction of members are employed
A: marginal disutility of working

Unemployed workers search in a labor market
Model: production technology

\[ y = z k^a n^b \]

\( z \): aggregate productivity (Markov chain)

\text{DRS: } a + b < 1, \ a > 0, \ b > 0
Capital and investment

Have opportunity to invest with prob. $\psi$

Fixed cost $\xi$ : i.i.d., CDF $G(\xi)$

$$\begin{cases} 
  k' = (1 - \delta)k & \text{with probability } (1 - \psi) \\
  k' = (1 - \delta)k + i, \ i \geq 0 & \text{with probability } \psi 
\end{cases}$$
Labor search

Matching function: \[ M(\bar{v}, \bar{u}) \]

\[ M(\bar{v}, \bar{u}) = \min \{ \bar{v}, \bar{u}, \kappa \bar{v}^\gamma \bar{u}^{1-\gamma} \} \]

The average vacancy-filling rate:

\[ h = \frac{M(\bar{v}, (1 - N))}{\bar{v}} \]

Individual vacancy-filling rate for a firm with \( v \) vacancies:

\[ f(x) = C^x_v h^x v (1 - h)^{v-x} \quad x \in [0, 1] \]
State-contingent wage contract

\[
\begin{cases}
  w = \frac{A}{p} & \text{if } MP_L \leq \frac{A}{p} \\
  w = \frac{(MP_L + \frac{A}{p})}{2} & \text{if } MP_L > \frac{A}{p}
\end{cases}
\]

\(\frac{A}{p}\) is the disutility of working in terms of goods

\(MP_L\) is the marginal productivity of labor

\(p(z, \mu) = U'(c)\)
Time line

Aggregate shock $z_t$

Produce opportunity $(k, n)$

Investment decision $\xi$ is realized

Hire or layoff decision

Matching takes place

Wage contract

Period $t$

Aggregate shock $z_{t+1}$
Investment and employment decisions

- Aggregate state variables:
  - $z$: aggregate productivity
  - $\bar{\mu}$: distribution of capital and labor over firms

- Individual firm’s states: $k$, $n$, $\xi$

- Value functions:
  - $V^0(k, n; z_i, \bar{\mu})$
  - $V^1(k, n, \xi; z_i, \bar{\mu})$
  - $\tilde{V}^1(k, n; z_i, \bar{\mu})$

- Decisions:
  - Invest or not
  - Optimal level of capital
  - Hire or fire workers
Firm’s values

After investment shock

\[ \tilde{V}^1(k, n; z_i, \bar{\mu}) = zf(k, n) - wn + \tilde{\Delta}_{no} \]

\[ V^1(k, n, \xi; z_i, \bar{\mu}) = zf(k, n) - wn + \max(\tilde{\Delta}_f, \tilde{\Delta}_{no}) \]

The beginning of period expected value of a firm:

\[ V^0(k, n; z_i, \bar{\mu}) \equiv (1 - \psi)\tilde{V}^1(k, n; z_i, \bar{\mu}) + \psi \int_0^{\xi} V^1(k, n, \xi; z_i, \bar{\mu})G(d\xi) \]
Continuation value when not invest

\[ \tilde{\Delta}_{no} = \max_{v, f^i} \left[ -ev + \sum_{j=1}^{J} \pi_{ij} d_j(z, \bar{u}) \int_{0}^{1} V^0 ( (1 - \delta)k, n'; z_j, \bar{u}') F(dx) \right] \]

\[ n' = (1 - \varphi)n + vx - f^i \]

\( v \) : vacancies

\( f^i \) : firing

\( d \) : MRS between current and future consumption
Continuation value when invest

\[ \tilde{\Delta}_{no} = \max_{v, f} \left[ -ev + \right. \]

\[ \left. \sum_{j=1}^{J} \pi_{ij} d_j(z, \bar{\mu}) \int_{0}^{1} V^0((1 - \delta)k, n'; z_j, \bar{\mu}') F(dx) \right] \]

\[ \tilde{\Delta}_i = \max_{k_i} \left\{ -\xi - i + \max_{v, f} \left[ -ev + \right. \right. \]

\[ \left. \left. \left. \sum_{j=1}^{J} \pi_{ij} d_j(z, \bar{\mu}) \int_{0}^{1} V^0(k_i', n'; z_j, \bar{\mu}') F(dx) \right] \right\} \]

\[ \xi: \text{ fixed cost of investment} \]

\[ i: \text{ investment} \]
Recursive equilibrium

A recursive equilibrium is consists of a set of value functions \((W, V^0, V^1, \tilde{V}^1)\); a set of policy functions for the household \(C\) and \(\Lambda\); a set of policy functions for the establishments \(k^e, v, f^l\); a set of prices \(p\) and \(\bar{p}\); a set of average matching rate \(h\), and a set of distribution measures \(\tilde{\lambda}\) and \(\bar{\mu}\) such that:

1. Househould and establishments optimize;
2. The law of motions:
   \[
   N' = \int_S \int_0^1 \left[ (1 - \varphi) n(k,n;z, \bar{\mu}) + v(k,n;z, \bar{\mu}) x - f^l(k,n;z, \bar{\mu}) \right] dF(x) \mu(d[k \times n])
   \]
   \[
   K' = \int_S \left[ (1 - \delta) k(k,n;z, \bar{\mu}) + i(k,n;z, \bar{\mu}) \right] \mu(d[k \times n])
   \]
   \[
   \bar{\mu}' = \Gamma(z, \bar{\mu});
   \]
3. The share market clears, i.e. \(\Lambda(k,n,\lambda, N; z, \bar{\mu}) = \mu(k,n)\);
4. The goods market clears.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Targets or comment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preference parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time preference $\beta$</td>
<td>0.996</td>
<td>4% real interest rate</td>
</tr>
<tr>
<td>Intertemporal substitution $\theta$</td>
<td>0.4</td>
<td>Standard deviation of labor / output</td>
</tr>
<tr>
<td>Disutility from working $A$</td>
<td>1.44</td>
<td>60% employment-population ratio</td>
</tr>
<tr>
<td><strong>Production technology parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productivity shock persistence $\rho$</td>
<td>0.98</td>
<td>Prescott (1986)</td>
</tr>
<tr>
<td>Productivity shock standard deviation $\sigma$</td>
<td>0.0021</td>
<td>Veracierto (2008)</td>
</tr>
<tr>
<td>Capital share $a$</td>
<td>0.22</td>
<td>NIPA share of capital</td>
</tr>
<tr>
<td>Labor share $b$</td>
<td>0.64</td>
<td>NIPA share of labor</td>
</tr>
<tr>
<td><strong>Capital adjustment</strong></td>
<td></td>
<td></td>
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<tr>
<td>Capital depreciation rate $\delta$</td>
<td>0.008</td>
<td>Investment capital ratio 0.0221</td>
</tr>
<tr>
<td>Investment opportunity $\psi$</td>
<td>1/12</td>
<td>Investment duration 1 year</td>
</tr>
<tr>
<td>Capital adjustment cost upper bound $\zeta$</td>
<td>0.028$K$</td>
<td>Average adjustment cost 0.91% of $K$</td>
</tr>
<tr>
<td>Capital adjustment cost distribution $\beta_p$</td>
<td>1.2</td>
<td>18.6% investment spikes</td>
</tr>
<tr>
<td>Capital adjustment cost distribution $\beta_q$</td>
<td>0.8</td>
<td>free</td>
</tr>
<tr>
<td><strong>Labor market</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Matching technology $\kappa$</td>
<td>0.508</td>
<td>Vacancy duration of 45 days</td>
</tr>
<tr>
<td>Matching rate elasticity $\gamma$</td>
<td>0.7</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>Vacancy posting cost $e$</td>
<td>0.15</td>
<td>10% of one month wage bills</td>
</tr>
<tr>
<td>Exogenous job destruction rate $\varphi$</td>
<td>3.7%</td>
<td>3.7% job separation rate</td>
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Compare two steady states:
1% permanent increase in productivity

Benchmark model:

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<td>+1.93%</td>
<td>+0.63%</td>
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Why generates these in particular Mechanism
- Increase in labor market tightness
  - Large: invest, next period create job
  - Small: do not invest, next period destroy
What if there is no labor market search?
(Lumpy capital only)

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<td>+1.31%</td>
<td>+0.00%</td>
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<tr>
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<tr>
<td>+0.00%</td>
<td>+2.44%</td>
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Further contrast

- Benchmark model predicts the right signs
  - Investment rate increases by 5% in small and 14% in large firms
  - Workers move from small to large firms

- Lumpy capital model:
  - Investment increases more strongly in small firms
  - Opposite worker flows: from large to small firms

  ➔ Labour search frictions change investment behaviour in firms with different sizes
Dynamic equilibrium result

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<td>0.3332</td>
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<tr>
<td>Model</td>
<td>6.20</td>
<td>4.64</td>
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Business cycle features

- The model successfully predicts the cyclicality

- The model predicts a large volatility in small firms relative to large firms
  - Idiosyncratic risk in small firms (for example exit)
Conclusion

- Documents difference of job creation and job destruction in small vs. large firms

- A mechanism:
  - firm sizes can affect firms’ lumpy investment decision in the presence of labor search frictions
  - in turn, different investment rates in small and large firms affect worker flows between small and large firms