Growth and crisis, unavoidable connection?

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Motivation

- Financial crises can occur after prolonged periods of fast growth
- Crises and long run growth usually studied as independent objects
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- Crises and long run growth usually studied as independent objects
Contribution

- Construct open economy growth model with financial imperfections: financial crises natural outcome process of long run growth
- Provide framework to study effects of financial market regulation
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GDP Growth

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Current Account/GDP (cross country average)
Investment/GDP (cross country average)
Evidence

- Fast-growth associated with boom-bust cycles (Ranciere, Tornell, Westermann, 2008)

- Post- crises periods: low growth rates (Ree, Lee 2000)
Model features

- Aggregate production function: two stages of growth
  - Stage 1: Constant marginal returns to capital
  - Stage 2: Decreasing marginal returns to capital

  Uncertain “turning point” between stage 1 and stage 2

- Financial imperfection
  - Limited enforceability international debt contract: possibility of default.
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Aggregate production function of small open economy
Aggregate production function and Turning Point

\[ F(K) \]

Stage 1

Stage 2

uncertain

\[ K_0 \]

\[ \bar{K} \]

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Related Literature

Endogenous borrowing constraints and debt repudiation
- Eaton, Gersovitz (1981)
- Bulow, Rogoff (1989)
- Arellano (2008)
- Alvarez, Jermann (2000)
- Chatterjee, Corbae, Rios-Rull (2007)

Sudden stops, exogenous borrowing constraints & shocks
cycle/trend
- Mendoza (2008)
The model

- Environment and household’s problem
Environment

Continuous time

Two types of agents

- Unit measure, identical households.
- International investors.

Uncertainty

- Uncertainty on arrival turning point.
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State vector for household’s problem

- Aggregate state: marginal product of capital $r^k$
  - Stage 1: $r_t^k = \bar{r}$
  - Stage 2: $r_t^k < \bar{r}$

- Individual state
  - Capital and debt stocks $(k, b)$.
  - $f \in \{D, ND\}$: Default (D) or Non-Default (ND).

- State vector $(s; S) = (k, b, f; r^k)$. Value function $V(s; S)$. 

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HH’s utility

Initial condition \((s_0; S_0) = (k_0, b_0, ND; r^k_1)\)

\[
V(s_0; S_0) = \max E \left[ \int_0^\infty e^{-\rho t} \log c_t \ dt \right]
\]
Policies for HH

At any \((s, S)\),

- If \(f = ND\), HH can switch to default state \(f = D\).
- If \(f = ND\) and no default, HH chooses new net debt \(d\), investment \(x\)

\[
c + x = r^k k + d
\]

\[
\dot{k} = -\delta k + x
\]

\[
\dot{b} = r(s, S)b + d
\]

\[
b \leq m^*(S)k
\]

given leverage constraint \(m^*(S)\) and interest rate schedule \(r(s, S)\).

- If \(f = D\), autarky and output costs \(\xi \in (0, 1)\): HH chooses \(x^d\)

\[
c^d + x^d = (1 - \xi)r^k k
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Assume permanent punishment: default absorbing state.
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Endogenous determination of borrowing constraint

Borrowing constraints $m^*(S)$ “not too tight”, maximum leverage at $S$,

**Definition**

Endogenous borrowing constraint $m^*(S)$ satisfies for all $S$ and $k$

$$m^*(S) = \max\{m : V(k, m^*(S)k, ND; S) \geq V(k, 0, D; S)\}$$
Credit crunch and crisis

Growth and crisis, unavoidable connection?

\[ F(K) \]

\[ m^*(S) \]

\[ K \]

\[ m^*(S) \]

\[ F(K) \]

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The interest rate

In Stage 1

- Balanced growth $\bar{G}$.
- Assume $\bar{K}$ has Pareto distribution with parameter $\eta \in (0, 1)$

\[ \downarrow \]

Constant arrival rate Turning Point $\pi = \eta \bar{G}$

- Interest rate charged by international investors (risk neutral)

\[ r(s; \bar{r}) = \rho + \pi \]

international $\rho$ risk-free rate.
The interest rate

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\[ \Rightarrow \]

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international $\rho$ risk-free rate.
Characterize $m^*(S)$

$$V(k, m^*(S)k, ND; S) = V(k, 0, D; S)$$

$$\log c(S) + \frac{g(S)}{\rho} = \log c^d(S) + \frac{g^d(S)}{\rho}$$

In stage 1

$$\frac{\dot{k}(S)}{k(S)} = g(S) = \frac{\bar{r} - \delta - \rho - \pi}{1 - m^*_1}$$

$$g^d = (1 - \xi)\bar{r} - \delta - \rho$$
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Characterize $m^*(S)$
Non sustainable de-leveraging

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Sustainable de-leveraging

\[ F(K) \]

\[ m^*(S) \]

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Growth rate

$G(S')$

$K_0$  $\bar{K}$  $K$

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Regulate maximum leverage to avoid crisis?

$m(S)$

$K_0$ $\rightarrow$ $\bar{K}$ $\rightarrow$ $K$
Regulate maximum leverage to avoid crisis?

Proposition

*Equilibrium with endogenous borrowing constraints (constrained) Pareto optimal.*
Conclusions

- Useful not to separate issue of crises from long run growth
- Some crises natural outcome of development process in presence of financial frictions
- Framework useful to think about trade-offs faced when regulating financial markets
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Latvia

- **GD growth**
- **CA/GDP**

Growth and crisis, unavoidable connection?
Value Function: Not Default and Stage 2

\[ \rho V^{2,ND}(k, b, \tau) = \max_{x, d} \log c + V_{k}^{2,ND} \dot{k} + V_{b}^{2,ND} \dot{b} + V_{\tau}^{2,ND} \]

s.t. \[ c + x = r_{2}^{k}(\tau)k + d \]
\[ \dot{k} = x \]
\[ \dot{b} = r_{2}(k, b, \tau)b + d \]
\[ b \leq m_{2}(\tau)k \]

\( m_{2}(\tau) \) Borrowing constraint
\( d \) net new debt
Value Function: Default and Stage 2

\[ \rho V^{2,D}(k, \tau) = \max_{x^d} \log c^d + V^{2,D}_k k + V^{2,D}_\tau \]

s.t. \[ c^d + x^d = (1 - \xi)r^k_2(\tau)k \]
\[ \dot{k} = x^d \]
\[ \xi \in (0, 1) \]
Value Function: Not Default and Stage 1

\[(\rho + \pi) V^{1,ND}(k, b) = \max_{x,d} \log c + \pi V^{TP}(k, b) + V^{1,ND}_k k + V^{1,ND}_b \dot{b}\]

s.t. \[\begin{align*}
c + x &= r_1^k k + d \\
\dot{k} &= x \\
\dot{b} &= r_1(k, b)b + d \\
b &\leq m_1 k
\end{align*}\]

\(\pi\) arrival rate of TP
Value function at turning point, $\tau = 0$

At turning point borrowing constraint changes from $m_1$ to $m_2(0)$

- $b \leq m_2(0)k$

\[ V^{TP}(k, b) = V^{2,ND}(k, b, 0) \]
Credit crunch at the turning point

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- $b \leq m_2(0)k$
  \[ V^{TP}(k, b) = V^{2,ND}(k, b, 0) \]

- $b > m_2(0)k$, Default
  \[ V^{TP}(k, b) = V^{2,D}(k, 0) \]
Value function at turning point, $\tau = 0$

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