Estimating heterogeneous costs of participation in risky asset markets

Graciela Sanroman
Departamento de Economía, FCS, UDELAR
Motivation and main goal

- The literature on household portfolio choice is gaining richness and complexity. Two main conclusions arise from the recent research on this issue:
  - There is not a simple rule to select an optimal financial portfolio over the life cycle.
  - Actual household portfolio behaviour is extremely heterogeneous and most of this heterogeneity is given by the decision of whether to participate in the risky asset markets.

- The mail goal of this paper is to deal with these two issues simultaneously. We develop and estimate a dynamic structural model using household level panel data in order to assess the choice to participate in the risky financial asset markets.
Method

- We implement a simulation-based estimation method which requires solving the economic problem of each individual in the sample. There is not an available analytical solution to this problem, thus, our solution is based on numerical techniques.

- To our knowledge this is the first time that such a formal econometric approach is used in the field of household portfolio choice.

- We focus on the estimation of non-proportional participation costs to invest in risky asset markets. We proceed by assuming that households choose the optimal portfolio and calculate how much cost is needed in order to explain the observed households behaviour. We consider heterogeneous costs among education groups.
The data come from the Italian “Survey of Households Income and Wealth”. This is a unique dataset which has panel data on household wealth, income and consumption and also includes detailed information about financial holdings and the demographic characteristics of households members. Moreover, its quality have been proved to be good enough for our purposes.

The data: biannual and cover the years 1987 to 2004.
## Sample size

<table>
<thead>
<tr>
<th>Group</th>
<th>Description</th>
<th>No. Obs.</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group I</td>
<td>Households with zero financial holdings</td>
<td>7152</td>
<td>16</td>
</tr>
<tr>
<td>Group II</td>
<td>Households with zero risky asset holdings</td>
<td>23292</td>
<td>52</td>
</tr>
<tr>
<td>Group III</td>
<td>Households with positive risky asset holdings</td>
<td>14174</td>
<td>32</td>
</tr>
<tr>
<td>Total Sample</td>
<td></td>
<td>44618</td>
<td>100</td>
</tr>
</tbody>
</table>
### Sample statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Group I</th>
<th>Group II</th>
<th>Group III</th>
<th>Total Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Risky asset share (per unit)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.00</td>
<td>0.00</td>
<td>0.63</td>
<td>0.20</td>
</tr>
<tr>
<td>Stand. dev.</td>
<td>0.00</td>
<td>0.00</td>
<td>0.25</td>
<td>0.33</td>
</tr>
<tr>
<td>Median</td>
<td>0.00</td>
<td>0.00</td>
<td>0.68</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Financial Wealth at the end of the period (euros 2004)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0</td>
<td>12255</td>
<td>57132</td>
<td>24547</td>
</tr>
<tr>
<td>Stand. dev.</td>
<td>0</td>
<td>45763</td>
<td>111275</td>
<td>74427</td>
</tr>
<tr>
<td>Median</td>
<td>0</td>
<td>5796</td>
<td>30313</td>
<td>7707</td>
</tr>
<tr>
<td><strong>Non Financial Income (euros 2004)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>19776</td>
<td>28778</td>
<td>40991</td>
<td>31215</td>
</tr>
<tr>
<td>Stand. dev.</td>
<td>14379</td>
<td>17659</td>
<td>26071</td>
<td>21593</td>
</tr>
<tr>
<td>Median</td>
<td>16211</td>
<td>25001</td>
<td>35770</td>
<td>26530</td>
</tr>
<tr>
<td><strong>Previous participation in the risky asset markets</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.09</td>
<td>0.19</td>
<td>0.68</td>
<td>0.37</td>
</tr>
<tr>
<td>Stand. dev.</td>
<td>0.29</td>
<td>0.39</td>
<td>0.46</td>
<td>0.48</td>
</tr>
<tr>
<td>Median</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Age</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>54.1</td>
<td>51.9</td>
<td>53.2</td>
<td>52.6</td>
</tr>
<tr>
<td>Stand. dev.</td>
<td>15.0</td>
<td>14.2</td>
<td>13.0</td>
<td>14.0</td>
</tr>
<tr>
<td>Median</td>
<td>54.0</td>
<td>51.0</td>
<td>53.0</td>
<td>52.0</td>
</tr>
<tr>
<td><strong>Education (per cent)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than elementary sch.</td>
<td>14</td>
<td>6</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Elementary school</td>
<td>43</td>
<td>32</td>
<td>22</td>
<td>30</td>
</tr>
<tr>
<td>Middle school</td>
<td>27</td>
<td>31</td>
<td>26</td>
<td>29</td>
</tr>
<tr>
<td>High school</td>
<td>13</td>
<td>25</td>
<td>35</td>
<td>26</td>
</tr>
<tr>
<td>College and posgraduate</td>
<td>3</td>
<td>7</td>
<td>15</td>
<td>9</td>
</tr>
</tbody>
</table>
The existing literature on household portfolio choice shows a variety of evidence about households financial decisions (Campbell, 2006; Campbell and Viceira, 2002 and Guiso and Japelli, 2001):

- Households are extremely diverse in their portfolio choices,
- Lack of participation, i.e., the zero solution in the demand for the risky assets, accounts for most of this heterogeneity,
- The average real return on stocks and bonds has been higher than that on bank deposits,
- Wealthy people are more likely to hold stocks but a substantial proportion of wealthy people hold no stocks at all,
- Those who hold risky financial asset were more likely to have these types of holdings in the past.
Viceira (2001) and Cocco et al (2005) solve numerically a life cycle model using calibration techniques, this method allows them to simulate the individual optimal path for consumption and risky asset holdings under the assumption that household receives an exogenous, uncertain and uninsurable stream of labor income.

Figure 10 illustrates the shape of the optimal rule for the risky asset share depending on the financial wealth given the non-financial income stream by Viceira, Cocco et al and this work.
Figure 10: Optimal Rules for Risky Asset Shares

Optimal Risky Asset Shares

Financial wealth for a given future stream of non-financial income

Viceira (2001)  Cocco et al (2005)  This study
Figure 4: Risky Asset Shares by Cash-on-Hand/NF-Income Ratio

Figure 5: Average RAS by Cash-on-Hand/NF-Income Ratio
We follow Deaton (1991) consumption model and extend it by introducing the decision on the risky asset demand.

We assume:
- agent’s life horizon is finite
- a risk-averse agent
- derives utility from consumption no bequest motives
- only three assets in the economy: human capital, risk-less financial asset and risky financial asset
The agent’s problem can be briefly described as follows.

- In each time period $t$ the agent observes her income sources, the composition of her portfolio in the previous period and the set of investment opportunities available in the market.
- Then, based on her preferences and beliefs about future income and asset returns and taking into account that she is not permitted to borrow, the agent chooses her consumption expenditure $c_t$ (which also determines financial holdings) and the proportion of financial holdings to allocate to the risky asset $\alpha_t$.
- In order to select the optimal alternative the agent brings the future into the picture and builds contingent plans of consumption and investment for her remaining lifetime.
The optimization problem

The optimization problem can be written as

$$\max_{\{c_s, \alpha_s\}} E_t \left\{ \sum_{s=t}^{H} B^{s-t} u(c_s) \right\}$$

subject to constraints

To our knowledge, there is not any available analytical solution to this problem. However the optimal decision rules could be characterized in a generic way as functions of the state variables using Bellman principle of optimality.

$$V(s_t) = \max_{\delta_t=(c_t, \alpha_t) \in D} u(s_t, \delta_t) + B \int V(s_{t+1}) p(ds_{t+1}|s_t, \delta_t)$$

(P1')

with,

$$s_t = [W_t, \{nfi_s\}_{s=t,H}, a_t, I_t]$$
We need to solve the agent problem in order to evaluate her contribution to the pseudo-likelihood. But the problem has not an analytical solution. Thus, we use a numerical solution method. Imrohoroglu et al (1999); Rust (1994 and 1996); Carroll (2002); Haliassos and Michaelides (2001) and Storesletten et al. (2007).

The method can be briefly explained in three steps:

- **FIRST STEP**: Discretize the space of the state variables into J-locations (called grid points).
- **SECOND STEP**: Find the optimal control variables and evaluate the value function for each point of the grid.
- **THIRD STEP**: Obtain the optimal choice and the respective value function for any value of the state variable by interpolating results (extrapolating outside the grid).
We solve the model using backward induction. At each period we use the Euler’s conditions of the economic problem in order to obtain the optimal choice.

In the last period $H$ the policy function is trivial (the agent consumes all her cash on hand).

Then, in period $H-1$, for each grid point we use the policy function for the next period which depends on the state variables and the respective choice in $H-1$ and the state of nature in $H$, and compute the optimal policy rule for the current period.

We proceed iteratively until period $t$

The Euler’s equations of the model in each period $s$ are given by,

$$
E_s \left[ u' (c_s) - BR (x)_{s+1} u' (c_{s+1}) \right] = 0 \quad (3)
$$

$$
E_s \left[ A_s \left( R_{s+1} - R^f \right) u' (c_{s+1}) \right] = 0 \quad (4)
$$
The model determines the optimal choice of consumption and risky asset share:

\[ \{ c_{it}^*, \alpha_{it}^* \} \]  

(5)

It also yields

\[ \nu_{i0} = V(s_{it} \mid \alpha_{it}^* = 0) \]

and

\[ \nu_{i1} = V(s_{it} \mid 0 < \alpha_{it}^* \leq 1) \]
Indirect Inference

We estimate our structural dynamic programming model through indirect inference (Smith, 1990; Gourieroux et al, 1993; Keane and Smith, 2003): Indirect Inference is based on three mainstays:

- the estimation of an statistical model (auxiliary model) characterized by $r$ parameters $\theta$
- the solution through simulation of the economic model characterized by $k$ ($k \leq r$) parameters $\beta$
- a metric that allows to evaluate some objective function of the parameters of the auxiliary model $\theta$ as functions of the structural parameters $\beta$

The basic idea used here is to embed the solution of the structural model into the pseudo-likelihood of the auxiliary model (LR approach).
The actual data are
\[ \alpha_t = (\alpha_{1t}, \alpha_{2t}, ..., \alpha_{Nt}) \quad X_t = (x_{1t}, x_{2t}, ..., x_{Nt}) \]
the SHIW is a longitudinal dataset, thus \( \{ \alpha_{it}, x_{it} \}_{i=1,...,N; \, t=1,...,T} \).

The auxiliary statistical model specify a reduced-form model implicitly given by:
\[ \alpha_{it} = \alpha_{it}^* \left[ W_{it}, nfi_{it}, a_{it}, \alpha_{it-1}, Z_{it} \right] \quad (6) \]

The choice of the set of conditional variables is determined taking into account the state variables of the model, the expected sources of heterogeneity and the availability of information.

We focus on exploiting the information contained in the decision of participation using a logit model. The dependent variable:
\[ \delta_{it} = \begin{cases} 0 & \text{if } \alpha_{it}^* = 0 \\ 1 & \text{if } \alpha_{it}^* \in (0, 1] \end{cases} \quad (7) \]
The structural model, in period $t$, is given by:

$$\delta_i = 1 [v_{i1} = \max(v_{i1}, v_{i0})]$$

where

$$v_{i0} = V(s_{it} \mid \alpha_{it}^* = 0) = V(s_i \mid \delta_i = 0)$$

the value function under the assumption that the agent does not participate in the risky asset markets.

and

$$v_{i1} = V(s_{it} \mid 0 < \alpha_{it}^* \leq 1) = V(s_i \mid \delta_i = 1)$$

the value function under the assumption that the agent participates in the risky asset markets.

$s_i = (x_i, u_i)$ where $x$ are observable variables, $u$ unobservables, $\beta$ a vector of structural parameters and $\delta$ the observed individual choice.
Estimation Method: From Indirect Inference to Generalized Indirect Inference

- The dependent variable of our model, $\delta_i$, is a discrete random variable. Discrete random variables complicate the numerical calculation of the model because the objective surface is a step function due to any simulated choice $\delta_i^m$ is a step function of $\beta$ (Altonji et al, 2006).

- Keane and Smith (2003) propose to use a Generalized Indirect Inference approach, to smooth the objective surface. The key idea is to substitute the dependent variable $\delta_i$ with $\tilde{\delta}_i$,

$$
\tilde{\delta}_i = \frac{1}{1 + \exp \left[ \frac{v_{i0} - v_{i1}}{\lambda} \right]} \quad (8)
$$

- Note that if $v_{i0} = \max(v_{i1}, v_{i0})$ and $\lambda \to 0$ then $\tilde{\delta}_i \to \delta_i = 0$ and if $v_{i1} = \max(v_{i1}, v_{i0})$ and $\lambda \to 0$ then $\tilde{\delta}_i \to \delta_i = 1$.  

Summarizing the Estimation Method

- We observe \( \delta_{it}; W_{it}; (nfi_{is})_{s=t,t-1,t-2,...,1}; a_{it}; \delta_{it-1} \)
- We simulate a random process for the future income stream
  \[
  \{nfi_{is}^m\}_{i=1\ldots N, s=t+1,...,T}
  \]
- Then we solve the economic problem of the agent for a given value of the structural coefficients (\( \beta \)) compute
  \[
  v_{ij} = V (s_i, \beta | \delta_i) j = 0, 1;
  \]
  and obtain
  \[
  \tilde{\delta}^m(\beta) = (\tilde{\delta}^m_1(\beta), ..., \tilde{\delta}^m_N(\beta))
  \]
Afterwards we estimate

$$\tilde{\theta}^m(\beta) = \arg\max_\theta L(\theta; \tilde{\delta}^m(\beta), X)$$ \hspace{1cm} (9)

$$L(\theta; \tilde{\delta}^m(\beta), X) = \sum_{i=1}^{N} \tilde{\delta}^m_i(\beta) \log \Lambda (x_i' \theta)$$ \hspace{1cm} (10)

$$+ \left(1 - \tilde{\delta}^m_i(\beta)\right) \log \left[1 - \Lambda (x_i' \theta)\right]$$ \hspace{1cm} (11)

We repeat that procedure for \( m = 1, 2, \ldots M \) and compute

$$\tilde{\theta}(\beta) = \frac{1}{M} \sum_{m=1}^{M} \tilde{\theta}^m(\beta)$$ \hspace{1cm} (12)
Finally the estimation for the structural parameters $\beta$ comes from

$$\hat{\beta} = \arg \min_{\beta} \left[ L(\hat{\theta}; \delta, X) - L(\tilde{\theta}(\beta); \delta, X) \right]$$

where

$$L(\tilde{\theta}(\beta); \delta, X) = \sum_{i=1}^{N} \delta_i \log \Lambda \left(x_i'\tilde{\theta}(\beta)\right)$$

$$+ (1 - \delta_i) \log \left[1 - \Lambda \left(x_i'\tilde{\theta}(\beta)\right)\right]$$

(13)
The variance of \( \tilde{\beta} = \arg \max_{\beta} L(\tilde{\theta}(\beta); \delta, X) \) is given by

\[
\text{Var}(\tilde{\beta}) = \frac{1}{N} \left( \hat{D}' \hat{H} \hat{D} \right)^{-1} \hat{D}' \hat{W} \hat{D} \left( \hat{D}' \hat{H} \hat{D} \right)^{-1}
\]  

(15)

where \( \hat{D} \) is the \( r \times k \) matrix of the numerical partial derivatives evaluated at \( \hat{\beta} : \hat{D} = \frac{\partial \tilde{\theta}(\hat{\beta})}{\partial \beta'}. \)

- \( \hat{H} \) and \( \hat{W} \) are consistent estimations of \( H = \text{plim}_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \frac{\partial^2 l_i(\theta_0)}{\partial \theta \partial \theta'} \)

and \( W = \text{plim}_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \frac{\partial l_i(\theta_0)}{\partial \theta} \frac{\partial l_i(\theta_0)}{\partial \theta'} \)

- Notice that the auxiliary model is a logit model, thus we have

\[
\hat{H} = \frac{1}{N} \sum_{i=1}^{N} \hat{\lambda}_i x_i x_i' \text{ and } \hat{W} = \frac{1}{N} \sum_{i=1}^{N} \left( \delta_i - \hat{\Lambda}_i \right)^2 x_i x_i' \text{ where}
\]

\[
\hat{\Lambda}_i = \frac{\exp(x_i' \hat{\theta})}{1 + \exp(x_i' \hat{\theta})} \text{ and } \hat{\lambda}_i = \hat{\Lambda}_i \left( 1 - \hat{\Lambda}_i \right).
\]
Returns

- The agent supposes that the investment opportunity set in each current and future period is constant and there are two financial assets, one risk-less asset (cash and deposits) and another one risky (stocks and bonds).

- We assume that there are only two states of nature, bad and good, each with the same probability (50 percent). The risky asset gross return (denoted by $R_s$) is iid and its probability distribution is given by

  \[
  R_s = \begin{cases} 
  \frac{R}{\bar{R}} & \Pr = 0.5 \\
  \frac{\bar{R}}{R} & \Pr = 0.5 
  \end{cases}
  \] (16)

- We use the data in Pelizzon and Weber (2007) and compute both the expected value and the deviation of the excess return for the efficiency portfolio. $R$ and $\bar{R}$ are fixed at 0.86 and 1.25 respectively. The source data are reported in Table 3.

- Risk-less asset yields a constant gross after tax real return, $R^f$. $R^f$ is fixed in 1.02 following the estimation of the bank deposit rates in Italy by Panetta and Violi (1990) for the period 1981-1994.
Preference parameters are fixed at plausible values. We chose the subjective discounted value factor

\[ B = 0.96 \]

and the coefficient of relative risk aversion

\[ crra = 3 \]
Summarizing, the non-financial-income stream predictions are calculated using the following three equations,

\[
\hat{nfi}_{is} = x_{1is}' \pi_{1inc} + f_i \pi_{2inc} + t \pi_{3inc} + \hat{\eta}_{1i} + \hat{v}_{is} \tag{17}
\]

\[
\hat{v}_{is} = \sum_{q=1}^{2} \rho_s \hat{v}_{is-q} + \tilde{\epsilon}_{is} \tag{18}
\]

\[
\tilde{\epsilon}_{is} = \tilde{z}_{is} \times \sqrt{\hat{\eta}_{2i}} \tag{19}
\]

where \( s = t + 1, \ldots, H \) and \( nfi_{is} \) : nonfinancial income of the household; \( x_{1is}' \) : age and age square of the household’ head; \( \eta_{i1} \) : an unobservable time invariant individual characteristic that affect the nfi level; \( f'_i \) : dummies of: education, residence in the South and city size; \( t \) a polynomial in time and a dummy to 1998; \( \eta_{2i} \) an unobservable time invariant characteristic that affect the variance of the nfi process; and \( \tilde{z}_{is} \) is a pseudo-random number (normal standard).
Optimal Risky Asset Shares

Financial wealth for a given future stream of non-financial income

Figure 10: Optimal Rules for Risky Asset Shares

- Viceira (2001)
- Cocco et al (2005)
- This study
Figure 4: Risky Asset Shares by Cash-on-Hand/NF-Income Ratio

Figure 5: Average RAS by Cash-on-Hand/NF-Income Ratio
Simulated Results

Figure 13: Risky Asset Shares by Cash-on-Hand/NF-Income Ratio

Simulated Results

Figure 14: Average RAS by Cash-on-Hand/NF-Income Ratio
Main conclusions from simulation results are:

- The model is able to generate heterogeneous results. Moreover, it demonstrates that, in the presence of both borrowing restrictions and non proportional participation costs, the zero solution could be an optimal response.
- The age of the agent does not matter in order to select the optimal portfolio. (different than that of Cocco et al (2005), Viceira (2001) and Jagannathan and Kocherlakota (1999)).
- A highly non-linear relationship between the optimal risky asset shares and the cash-on-hand/non-financial income ratio is found. Results mimic the policy rule in Figure 10. (these results are in line with those of Cocco et al, 2005; Viceira, 2001; Campbell, 2006).
The zero solution could be optimal even for large values of the cash-on-hand/non-financial income ratio or the non-financial income level.

The average optimal risky asset share is an increasing function of both the cash-on-hand/non-financial income ratio and the non-financial income level.

To the majority of those on the bottom of the cash-on-hand/non-financial income ratio distribution is optimal not to invest in the risky asset markets even if the participation cost is very small.

The same applies for those in the bottom of the non-financial income distribution.
Empirical Results

- We estimate a simpler version of the model assuming that the non-proportional participation costs are given exclusively by a single opportunity cost.
- This cost depends on the individual level of non-financial income of the period, that is, it could be modelled as

$$G_{is} = \begin{cases} 0 & \text{if } \alpha_{is} = 0 \\ g^e \times nfi_{is} & \text{if } \alpha_{is} > 0 \end{cases}$$

- We allow $g$ to varies among groups. In particular, we split the sample into four categories elementary school, secondary school, high school and college or post-graduated.
Table 9: Generalized Indirect Inference estimation of participation costs (in percentage of non-financial income) By education group (CRRA=3)

<table>
<thead>
<tr>
<th>Education group</th>
<th>No. Observations</th>
<th>Estimated Cost</th>
<th>Robust Std. Error</th>
<th>Robust Stat-t</th>
</tr>
</thead>
<tbody>
<tr>
<td>College and postgraduate</td>
<td>270</td>
<td>0.175</td>
<td>0.061</td>
<td>2.8</td>
</tr>
<tr>
<td>High School</td>
<td>964</td>
<td>1.650</td>
<td>0.325</td>
<td>5.1</td>
</tr>
<tr>
<td>Secondary</td>
<td>926</td>
<td>3.350</td>
<td>0.346</td>
<td>9.7</td>
</tr>
<tr>
<td>Elementary</td>
<td>795</td>
<td>6.000</td>
<td>0.420</td>
<td>14.3</td>
</tr>
</tbody>
</table>
Table 10: Implicit monetary participation costs (in euros 2006)
By education group (CRRA=3)

<table>
<thead>
<tr>
<th>Education group</th>
<th>No. Observations</th>
<th>Estimated Cost (1)</th>
<th>Average Non-financial Income (2)</th>
<th>Implicit Monetary Costs (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>College and postgraduate</td>
<td>270</td>
<td>0.175</td>
<td>59076</td>
<td>103</td>
</tr>
<tr>
<td>High School</td>
<td>964</td>
<td>1.650</td>
<td>43574</td>
<td>719</td>
</tr>
<tr>
<td>Secondary</td>
<td>926</td>
<td>3.350</td>
<td>34550</td>
<td>1157</td>
</tr>
<tr>
<td>Elementary</td>
<td>795</td>
<td>6.000</td>
<td>18765</td>
<td>1126</td>
</tr>
</tbody>
</table>

(1) In percent of non-financial income
(2) In euros 2006
Empirical Results (cont.)

- The estimated cost, in percentage of the non-financial income attains:
  - 0.175 percent for college and postgraduate (103 euros)
  - 1.65 percent for high school (719 euros)
  - 3.35 percent for secondary school (1157 euros)
  - 6.00 percent for elementary school (1126 euros)

- These results are comparable with (US data):
  - Attanasio and Paiella (2008): average lower bound of 0.7 percent of income.
  - Paiella (2006): lower bound of the participation cost at 130 dollars
Empirical Results (cont.)

- We make a sensitivity analysis in order to investigate how the coefficient of risk aversion influences estimated costs.
- We consider three alternative values for the coefficient of risk aversion:
  - 1.7
  - 3
  - 6
- This analysis is interesting for at least two reasons:
  - let us to investigate how great the variations of the estimated costs under different values of the risk aversion coefficient are.
  - the data give us some insight into how the coefficient of risk aversion varies among education groups.
Empirical Results (cont.)

- From theory: two opposite effects of the coefficient of relative risk aversion:
  - the higher the risk aversion the lower will be the optimal risky asset share for a given value of financial wealth (given the whole stream of non-financial income)
  - more prudent consumers will accumulate significantly more wealth

- For a given stream of non-financial income, the non-proportional participation cost that rationalizes non-participation increases with the absolute amount of financial wealth.

- Thus the theoretical relation between the coefficient of risk aversion and the non-proportional participation cost is ambiguous
Table 11: Generalized Indirect Inference estimation of participation costs (in percentage of non-financial income) by education group. CRRA = 1.7, 3 and 6

<table>
<thead>
<tr>
<th>Education group: College and Postgraduated</th>
<th>No. Observations: 270</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA</td>
<td>Pseudo Log-Likelihood</td>
</tr>
<tr>
<td>1.7</td>
<td>-174.90</td>
</tr>
<tr>
<td>3</td>
<td>-175.13</td>
</tr>
<tr>
<td>6</td>
<td>-180.79</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Education group: High School</th>
<th>No. Observations: 964</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA</td>
<td>Pseudo Log-Likelihood</td>
</tr>
<tr>
<td>1.7</td>
<td>-629.38</td>
</tr>
<tr>
<td>3</td>
<td>-627.35</td>
</tr>
<tr>
<td>6</td>
<td>-629.96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Education group: Secondary School</th>
<th>No. Observations: 926</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA</td>
<td>Pseudo Log-Likelihood</td>
</tr>
<tr>
<td>1.7</td>
<td>-576.48</td>
</tr>
<tr>
<td>3</td>
<td>-572.99</td>
</tr>
<tr>
<td>6</td>
<td>-568.20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Education group: Elementary School</th>
<th>No. Observations: 795</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA</td>
<td>Pseudo Log-Likelihood</td>
</tr>
<tr>
<td>1.7</td>
<td>-449.13</td>
</tr>
<tr>
<td>3</td>
<td>-443.91</td>
</tr>
<tr>
<td>6</td>
<td>-435.67</td>
</tr>
</tbody>
</table>
Table 12: Implicit monetary participation costs (in euros 2006)
By education group. CRRA = 1.7, 3 and 6

<table>
<thead>
<tr>
<th>Education group: College and Postgraduated</th>
<th>No. Observation: 270</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA</td>
<td>Pseudo Log-Likelihood</td>
</tr>
<tr>
<td>------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>1.7</td>
<td>-174.90</td>
</tr>
<tr>
<td>3</td>
<td>-175.13</td>
</tr>
<tr>
<td>6</td>
<td>-180.79</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Education group: High School</th>
<th>No. Observation: 964</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA</td>
<td>Pseudo Log-Likelihood</td>
</tr>
<tr>
<td>------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>1.7</td>
<td>-629.38</td>
</tr>
<tr>
<td>3</td>
<td>-627.35</td>
</tr>
<tr>
<td>6</td>
<td>-629.96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Education group: Secondary School</th>
<th>No. Observation: 926</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA</td>
<td>Pseudo Log-Likelihood</td>
</tr>
<tr>
<td>------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>1.7</td>
<td>-576.48</td>
</tr>
<tr>
<td>3</td>
<td>-572.99</td>
</tr>
<tr>
<td>6</td>
<td>-568.20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Education group: Elementary School</th>
<th>No. Observation: 795</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA</td>
<td>Pseudo Log-Likelihood</td>
</tr>
<tr>
<td>------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>1.7</td>
<td>-449.13</td>
</tr>
<tr>
<td>3</td>
<td>-443.91</td>
</tr>
<tr>
<td>6</td>
<td>-435.67</td>
</tr>
</tbody>
</table>

(1) In percent of non-financial income  (2) In euros 2006
Empirical Results (cont.)

- We find that the estimated participation costs vary slightly with the CRRA coefficient.
- The costs turn out to be positive and significant at the 5 percent in almost all cases (CRRA 1.7, 3 and 6 respectively):
  - “College and post-graduate”: 0.14 (at 10 percent), 0.175 and 0.185
  - “High School”: 1.55, 1.65 and 1.85
  - “Secondary”: 3.5, 3.35 and 3.15
  - “Elementary”: 5.8, 6 and 6.1

We compute the equivalent costs measured in terms of euros of 2006. We also observe that the variations in the estimated cost within each education groups are relatively small.
Empirical Results (cont.)

The empirical evidence shows that:

- in general, the higher the CRRA the higher the participation costs but does not hold for the group of “Secondary School”
- neither the theoretical model nor empirical evidence allows us to draw conclusions about the sign of the relationship between the risk aversion coefficient and participation costs, at least not in the context of our model and our data.

Another interesting result emerges from the relationship between pseudo-loglikelihood and the coefficient of risk aversion:

- the maximum of the pseudo-likelihood within each education group is attained for different values of the RRA coefficient suggesting that the CRRA is decreasing with education.
Part of the contribution of this paper is methodological: to our knowledge this is the first time that such a formal econometric approach is used in the field of the household portfolio choice.

Another contribution of this paper is to show that the standard errors of the model have a well defined analytical solution.
Our simulated results support some conclusions of recent literature on the issue of portfolio choice over the life cycle but reject others:

- The optimal risky asset share is a highly non-linear function of the cash-on-hand/non-financial income ratio. Results seem to mimic the expected policy rule illustrated in Figure 10. However the average RAS increase over the coh/nfi distribution.
- The same applies for the non-financial income but in this case there is not a clear pattern from Figure 15.
- Our results do not exhibit any systematic relation with respect to the age of the agent.
We estimate an average for a single per-period non-proportional participation cost in risky asset markets (previous literature estimates lower bounds) and we also consider heterogeneous costs among education groups:

- We found that non proportional participation costs in risky asset markets are positive and significant.
- We conclude that they vary a lot between the education of the household head: the estimated cost for college and post-graduated is 0.175 percent of non-financial income but is 6 percent for the elementary school group. The results for secondary and high school groups are 3.35 and 1.65 percent of non-financial income respectively.
We also make a sensitivity analysis by fixing the coefficient of risk aversion at 1.7, 3 and 6:

- We find that estimated costs vary slightly
- We do not find conclusive evidence about the sign of the relationship between the coefficient of risk aversion and participation costs
- We found some evidence that indicates that risk aversion exhibits a negative relationship with respect to the education of the head of the household
Future extensions (1)

- To estimate the value of the coefficient of risk aversion for the different education level groups.
- To estimate how participation costs vary among other sources of heterogeneity:
  - geographic regions,
  - previous participation in risky asset markets
  - time periods
- To analyze how sensitive the results are to variations in parameters like:
  - the subjective rate of discount
  - the moments of asset returns distribution
- The utility function: the Epstein-Zin specification
Some more difficult extensions could also be done:

- different types of costs such opportunity and monetary, entry and monitoring
- housing
- investments in owner enterprises
- uncertainty in the future stream of non-financial income (disaster events)

To use another auxiliary (statistical) model (consumption and risky asset shares).