Exchange Rates and Fundamentals: A Generalization

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INTRODUCTION

GENERALIZING THE EW HYPOTHESIS

ECONOMETRICS OF THE DSGE-PVM
   State Space Systems
   Estimation Methods
   Priors

DSGE-PVM ESTIMATES

CONCLUSION
Economists’ ire raised by floating exchange rates.
The Exchange Rate Problem

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- Systematically beat a random walk in out-of-sample forecast competition?
The Exchange Rate Problem

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- Since Meese & Rogoff (1983), train of models have tried ... in vain?
Economists’ ire raised by floating exchange rates.

Systematically beat a random walk in out-of-sample forecast competition?

Since Meese & Rogoff (1983), train of models have tried … in vain?

Reconcile what is useful forecasting model and equilibrium exchange rate model?
Before DSGE models ... present-value model is workhorse to study exchange rates in equilibrium.
Introduction

An Equilibrium Exchange Rate Model

- Before DSGE models . . . present-value model is workhorse to study exchange rates in equilibrium.

- Standard-PVM is a hybrid monetary model with liquidity preference MD, UIRP, and PPP.
An Equilibrium Exchange Rate Model

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Before DSGE models ... present-value model is workhorse to study exchange rates in equilibrium.

Standard-PVM is a hybrid monetary model with liquidity preference MD, UIRP, and PPP.

Standard-PVM also found lacking by data.

Meese (1986) example: reject PVM cross-equation restrictions and bubbles, but exchange rates are persistent (i.e., unit root).
Engel & West (2005) hypothesize PVM predicts exchange rate approximates random walk.
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Need fundamentals $\sim I(1)$ and PVM discount factor approaching one.
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Implication is permanent shocks dominate exchange rate movements.
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Need fundamentals $\sim I(1)$ and PVM discount factor approaching one.

Implication is permanent shocks dominate exchange rate movements.

Corollary: fundamentals do not Granger-cause exchange rate, although PVM generates equilibrium in currency markets.
Introduction

Why Generalize Engel-West Hypothesis?

- DSGE model is current workhorse model of open economy macro.
Why Generalize Engel-West Hypothesis?

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- Can EW hypothesis apply to open economy monetary DSGE models?
Introduction

Why Generalize Engel-West Hypothesis?

- DSGE model is current workhorse model of open economy macro.

- Can EW hypothesis apply to open economy monetary DSGE models?

- If yes, what are empirical implications?
Contributions of Paper

- Generalize EW hypothesis with DSGE-PVM equivalent to standard-PVM.
- Generalize EW hypothesis with five propositions.
- Suggest another interpretation to EW hypothesis: eliminate common serial correlation in exchange rate and fundamental, then PVM discount factor $\rightarrow 1$.
- Floating rate CDN$/US$ exchange rate is random walk or dominated by permanent shocks.
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Floating rate CDN$/US$ exchange rate is random walk or dominated by permanent shocks.
Table 1: List key elements of standard- and DSGE-PVMs.
Table 1: Summary of Standard PVM and DSGE-PVM

<table>
<thead>
<tr>
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<th><strong>Standard-PVM</strong></th>
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<tbody>
<tr>
<td><strong>ECM(0):</strong></td>
<td>(13) $\Delta e_t - \frac{1 - \omega}{\omega} X_{t-1} = (1 - \omega) \sum_{j=0}^{\infty} \omega^j [E_t - E_{t-1}] z_{t+j}$.</td>
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<td><strong>EW Equation:</strong></td>
<td>(14) $\Delta e_t = \zeta(\omega) \nu_t + (1 - \omega) \sum_{j=0}^{\infty} \omega^j E_{t-1} \Delta z_{t+j}$.</td>
</tr>
<tr>
<td><strong>Parameters:</strong></td>
<td>$\omega \equiv \frac{\phi}{1 + \phi}$ = Discount Factor, $\phi =$ Money Demand Interest Rate Semi-Elasticity, $\psi =$ Money Demand Income Elasticity.</td>
</tr>
<tr>
<td><strong>Fundamentals:</strong></td>
<td>$X_t = e_t - z_t$, $z_t = m_t - \psi y_t$, $m_t =$ Cross Country Money, $y_t =$ Cross-Country Output.</td>
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<td><strong>ECM(0):</strong></td>
<td>(13) $\Delta e_t - \frac{1 - \kappa}{\kappa} X_{DSGE,t-1}$</td>
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<td>$= (1 - \kappa) \sum_{j=0}^{\infty} \kappa^j [E_t - E_{t-1}] \left{ m_{t+j} - c_{t+j} \right}$.</td>
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<td>$+ (1 - \kappa) \sum_{j=0}^{\infty} \kappa^j E_t \left{ \Delta m_{t+j} - \Delta c_{t+j} \right}$.</td>
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<td><strong>Parameters:</strong></td>
<td>$\kappa \equiv \frac{1}{1 + r^<em>}$ = Discount Factor, $r^</em> =$ Steady State Real World Interest Rate.</td>
</tr>
<tr>
<td><strong>Fundamentals:</strong></td>
<td>$X_{DSGE,t} = e_t - m_t + c_t$, $m_t =$ Cross-Country Money, $c_t =$ Cross-Country Consumption.</td>
</tr>
</tbody>
</table>
Introduction

Summary Tables

- Table 1: List key elements of standard- and DSGE-PVMs.
- Table 2: Summarize propositions.
Table 2: Summary of Propositions for Standard- and DSGE-PVMs


Proposition 2: Currency Returns Are an ECM(0).

Proposition 3: Exchange Rate Approximates a Martingale as $B \rightarrow 1$.

Proposition 4: VECM(0) Imply Common Trend and Common Cycle for Exchange Rate and Fundamental.

PVM of exchange rates

\[ e_t = (1 - B) \sum_{j=0}^{\infty} B^j E_t z_{t+j}. \]
Generalizing the EW Hypothesis

Standard- and DSGE-PVMs (almost) Identical

- PVM of exchange rates

\[ e_t = (1 - B) \sum_{j=0}^{\infty} B^j E_t z_{t+j}. \]

- Standard-PVM: \( B \equiv \omega \) tied to \( \phi \) and \( z_t = m_t - \gamma_t. \)
PVM of exchange rates

\[ e_t = (1 - B) \sum_{j=0}^{\infty} B^j E_t z_{t+j}. \]

- Standard-PVM: \( B \equiv \omega \) tied to \( \phi \) and \( z_t = m_t - \gamma_t \).
- DSGE-PVM: \( B \equiv \kappa \) tied to \( r^* \) and \( z_t = m_t - c_t \).
Engel and West (2005) show $e_t$ approximates random walk as $B \rightarrow 1$. 

Assume $z_t \sim I(1)$, $\Delta z_t = \zeta(L)\upsilon_t$, $\upsilon_t \sim WN$. 

Apply Weiner–Kolmogorov prediction formula.
Engel and West (2005) show $e_t$ approximates random walk as $B \rightarrow 1$.

Assume $z_t \sim I(1)$, $\Delta z_t = \zeta(L)\nu_t$, $\nu_t \sim WN$. 
Engel-West Hypothesis Setup

- Engel and West (2005) show $e_t$ approximates random walk as $\mathcal{B} \rightarrow 1$.

- Assume $z_t \sim I(1)$, $\Delta z_t = \zeta(L) \nu_t$, $\nu_t \sim \mathcal{W} \mathcal{N}$.

- Apply Weiner–Kolmogorov prediction formula.
Claim $\text{plim}_B \rightarrow 1[\Delta e_t - a\zeta(1)\nu_t] = 0$. 

PVM and some algebra produce $\Delta e_t = \zeta(B)\nu_t + (1 - B)\sum_{j=0}^{\infty} B^j E_t - 1 \Delta z_t + j$. 

Let $B_p \rightarrow 1$ to verify EW hypothesis.
Claim $\lim_{B \to 1} [\Delta e_t - a\zeta(1)\nu_t] = 0$.

PVM and some algebra produce

$$\Delta e_t = \zeta(B)\nu_t + (1 - B) \sum_{j=0}^{\infty} B^j E_{t-1} \Delta z_{t+j}.$$ 

Wold serially correlated innovation

component
Claim $\text{plim}_B \rightarrow 1 [\Delta e_t - a\zeta(1)v_t] = 0.$

PVM and some algebra produce

$$\Delta e_t = \zeta(B)v_t + (1 - B) \sum_{j=0}^{\infty} B^j E_{t-1} \Delta z_{t+j}.$$  

Wold serially correlated innovation component

Let $B \xrightarrow{p} 1$ to verify EW hypothesis.
Standard PVM and cointegration

\[ X_t = e_t - z_t = \sum_{j=1}^{\infty} B^j E_t \Delta z_{t+j}. \]
Standard PVM and cointegration

\[ \chi_t = e_t - z_t = \sum_{j=1}^{\infty} B^j E_t \Delta z_{t+j}. \]

**Proposition 1**: \( e_t \) and \( z_t \) cointegrate

PVM and Cointegration

- Standard PVM and cointegration

\[ \chi_t = e_t - z_t = \sum_{j=1}^{\infty} B^j E_t \Delta z_{t+j}. \]

- **Proposition 1**: \( e_t \) and \( z_t \) cointegrate

- ECM generates cycles in \( e \) that reflect factors driving \( e \) to long-run PPP \( \Rightarrow \) Mark (1995).
Standard PVM and a bit of algebra gives

\[
\Delta e_t - \left( \frac{1 - B}{B} \right) x_{t-1} = (1 - B) \sum_{j=1}^{\infty} B^j [E_t - E_{t-1}] z_{t+j-1}.
\]
Currency Returns $\sim$ ECM

- Standard PVM and a bit of algebra gives

$$\Delta e_t - \left(\frac{1 - B}{B}\right) \chi_{t-1} = (1 - B) \sum_{j=1}^{\infty} B^j [E_t - E_{t-1}] z_{t+j-1}.$$

- **PROPOSITION 2**: PVM predicts $\Delta e_t \sim$ ECM with no lags of $\Delta e_t$ and $\Delta z_t$. 
Generalizing the EW Hypothesis

**Currency Returns ~ ECM**

- Standard PVM and a bit of algebra gives

\[
\Delta e_t - \left( \frac{1 - B}{B} \right) \chi_{t-1} = (1 - B) \sum_{j=1}^{\infty} B^j [E_t - E_{t-1}] Z_{t+j-1}.
\]

- **Proposition 2**: PVM predicts $\Delta e_t \sim$ ECM with no lags of $\Delta e_t$ and $\Delta z_t$.

- $\chi_{t-1}$ only source of serial correlation in $\Delta e_t$. 
**Proposition 3:** $B \rightarrow 1 \Rightarrow E_t e_{t+1} = e_t$. 

Exchange rate approaches a martingale $\Rightarrow$ verify EW hypothesis.

Why? Eliminate only source of serial correlation in $\Delta e_t = \Rightarrow$ EW hypothesis.
Proposition 3: $B \rightarrow 1 \Rightarrow E_t e_{t+1} = e_t$.

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A Generalization of the EW Hypothesis

- **Proposition 3**: $B \rightarrow 1 \Rightarrow E_t e_{t+1} = e_t$.

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- Why? Eliminate only source of serial correlation in $\Delta e_t \Rightarrow$ EW hypothesis.
Let $\Delta z_t \sim ECM(0)$, $\Rightarrow \Delta q_t = [\Delta e_t \ \Delta z_t]'$ is VEMC(0)
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$$\Delta q_t = \begin{bmatrix} 1 - \frac{B}{\eta} \\ \frac{B}{\eta} \end{bmatrix} X_{t-1} + \begin{bmatrix} \zeta(B) v_t \\ u_{\Delta z,t} \end{bmatrix}.$$ 

Linear combination of $\Delta e_t$ and $\Delta z_t$

$$\Rightarrow \bar{\beta}' \Delta q_t = \bar{\beta}' u_t, \quad \bar{\beta}' = \begin{bmatrix} 1 - \frac{1 - B}{B\eta} \end{bmatrix}.$$
Let $\Delta z_t \sim \text{ECM}(0)$, $\Rightarrow \Delta q_t = [\Delta e_t \ \Delta z_t]'$ is VEMC(0)

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Linear combination of $\Delta e_t$ and $\Delta z_t$

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**Proposition 4A**: $\{\Delta e_t, \Delta z_t\}$ share a common feature, which is $\mathcal{WN}$. 

**COMMON FEATURE RESTRICTION**
Proposition 1: $\{e_t, z_t\}$ share a common trend.
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Proposition 4A: \( \{ e_t, z_t \} \) share a common cycle.
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Proposition 4A: \( \{e_t, z_t\} \) share a common cycle.

Proposition 4B: \( \{e_t, z_t\} \) possess a common trend-common cycle decomposition.

Proposition 5: Common serial correlation in $\Delta e$ and $\Delta z$ is wiped out with common feature vector
\[
\bar{\beta} = \left[1 - \frac{1 - B}{B\eta}\right] \implies B \rightarrow 1 \text{ verifies EW hypothesis.}
\]
**Proposition 5:** Common serial correlation in $\Delta e$ and $\Delta z$ is wiped out with common feature vector 
\[
\bar{\beta} = [1 - \frac{1 - B}{B \eta}] \implies B \rightarrow 1 \text{ verifies EW hypothesis.}
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- Common serial correlation in $\Delta e$ and $\Delta z$ is $\chi_{t-1}$.  

**Generalizing Exchange Rates and Fundamentals**
Table 3: Reduced Form Evidence

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- Reject $e$ and $z$ cointegrate $\Rightarrow P_1$. 

- Not reject $\{e, z\}$ share a common feature $\Rightarrow P_4$. 

- Silent on EW hypothesis.
Table 3: Reduced Form Evidence

- Reject \( e \) and \( z \) cointegrate \( \implies P1 \).
- Reject \( e \) and \( z \) VAR lag length \( = 1 \) \( \implies VECM(0) \).
### Table 3: Reduced Form Evidence


- Reject $e$ and $z$ cointegrate $\Rightarrow$ P1.

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VAR methods cannot test EW hypothesis.
‘Structural’ Estimation of DSGE-PVM

- VAR methods cannot test EW hypothesis.
- DSGE-PVM has deep structure that can permanent-transitory decompositions of cross-country money and consumption.
‘Structural’ Estimation of DSGE-PVM

- VAR methods cannot test EW hypothesis.
- DSGE-PVM has deep structure that can permanent-transitory decompositions of cross-country money and consumption.
- DSGE-PVM as UC model

\[ e_t = \mu_t - a_t + (1 - \kappa) \sum_{j=0}^{\infty} \kappa^j E_t \left\{ \tilde{m}_{t+j} - \tilde{c}_{t+j} \right\}. \]
Permanent-Transitory Decompositions

- $\mu_t \sim \mathcal{RW}$, $\tilde{m}_t = \epsilon_{\tilde{m},t} + \sum_{i=1}^{k_{\tilde{m}}} \alpha_i \epsilon_{\tilde{m},t-i}$,
PERMANENT-TRANSITORY DECOMPOSITIONS

- \( \mu_t \sim RW, \ \tilde{m}_t = \varepsilon_{\tilde{m},t} + \sum_{i=1}^{k_{\tilde{m}}} \alpha_i \varepsilon_{\tilde{m},t-i} \),

- \( a_t \sim RW, \ \tilde{c}_t = \sum_{i=1}^{k_{\tilde{c}}} \theta_i \tilde{c}_{t-i} + \varepsilon_{\tilde{c},t} \).
Permanent-Transitory Decompositions

- $\mu_t \sim RW$, $\tilde{m}_t = \varepsilon_{\tilde{m},t} + \sum_{i=1}^{k_{\tilde{m}}} \alpha_i \varepsilon_{\tilde{m},t-i}$,

- $a_t \sim RW$, $\tilde{c}_t = \sum_{i=1}^{k_{\tilde{c}}} \theta_i \tilde{c}_{t-i} + \varepsilon_{\tilde{c},t}$.

- DSGE-PVM restricted 2-trend, 2-cycle state vector

\[ S_t = \begin{bmatrix} \mu_t & a_t & \varepsilon_{\tilde{m},t} & \varepsilon_{\tilde{m},t-1} & \ldots & \varepsilon_{\tilde{m},t-k_{\tilde{m}}} & \tilde{c}_t & \tilde{c}_{t-1} & \ldots & \tilde{c}_{t-k_{\tilde{c}}} \end{bmatrix} \]
2-Trend, 2-Cycle State Equation System

\[
S_{t+1} = \begin{bmatrix}
\mu^* \\
a^* \\
0 \\
\vdots \\
0
\end{bmatrix} + \begin{bmatrix}
1 & 0 & \ldots & 0 & 0 & \ldots & 0 \\
0 & 1 & \ldots & 0 & 0 & \ldots & 0 \\
0 & 0 & \ldots & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & 0 & \theta_1 & \ldots & \theta_{k_{\tilde{c}}}
\end{bmatrix} S_t + \boldsymbol{\varepsilon}_{t+1},
\]

where

\[
\boldsymbol{\varepsilon}_{t+1} = \begin{bmatrix}
\varepsilon_{\mu,t+1} \\
\varepsilon_{a,t+1} \\
\varepsilon_{\tilde{m},t+1} \\
0_{1\times k_{\tilde{m}}} \\
\varepsilon_{\tilde{c},t+1} \\
0_{1\times (k_{\tilde{c}}-1)}
\end{bmatrix}',
\]

and \( \Omega = \mathbf{E}\varepsilon_{t+1}\varepsilon_{t+1}' \), and companion matrix of AR\((k_{\tilde{c}})\) is \( \Theta \).
### 2-Trend, 2-Cycle Observer Equation System

\[
y_t = \begin{bmatrix}
1 & \pi_{e,a} & \delta_{\tilde{m},0} & \delta_{\tilde{m},1} & \ldots & \delta_{\tilde{m},k_{\tilde{m}}} & \delta_{\tilde{c},0} & \ldots & \delta_{\tilde{c},k_{\tilde{c}}-1} \\
1 & 0 & 1 & \alpha_1 & \ldots & \alpha_{k_{\tilde{m}}} & 0 & \ldots & 0 \\
0 & 1 & 0 & 0 & \ldots & 0 & 1 & 0 \ldots & \ldots
\end{bmatrix} S_t
\]

- Factor loadings on \( \varepsilon_t, \varepsilon_{t-1}, \ldots, \varepsilon_{t-k_{\tilde{m}}} \) are

\[
\delta_{\tilde{m},i} = (1 - \kappa) \sum_{j=i}^{k_{\tilde{m}}} \kappa^{j-i} \alpha_j, \quad i = 0, \ldots, k_{\tilde{m}}.
\]
**2-Trend, 2-Cycle Observer Equation System**

\[ y_t = \begin{bmatrix} 1 & \pi_{e,a} & \delta_{\sim m,0} & \delta_{\sim m,1} & \cdots & \delta_{\sim m,k_{\sim m}} & \delta_{\sim c,0} & \cdots & \delta_{\sim c,k_{\sim c} - 1} \\ 1 & 0 & 1 & \alpha_1 & \cdots & \alpha_{k_{\sim m}} & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots \end{bmatrix} S_t \]

- Factor loadings on \( \varepsilon_t, \varepsilon_{t-1}, \ldots, \varepsilon_{t-k_{\sim m}} \) are

\[
\delta_{\sim m,i} = (1 - \kappa) \sum_{j=i}^{k_{\sim m}} \kappa^{j-i} \alpha_j, \quad i = 0, \ldots, k_{\sim m}.
\]

- Factor loadings on \( \tilde{c}_t, \ldots, \tilde{c}_{t-k_{\sim c}} \) are elements of row vector

\[
\delta_{\tilde{c},i} = - (1 - \kappa) s_{\tilde{c}} \left[ I_{k_{\tilde{c}}} - \kappa \Theta \right]^{-1}, \quad s_{\tilde{c}} = [1 \ 0_{1 \times k_{\tilde{c}} - 1}].
\]
UC 2-trend, Money and Consumption Cycle Observer Systems

- UC 2-trend, money cycle observer equation

\[ y_t = \begin{bmatrix}
1 & \pi_{e,a} (1 - \pi_{c,m}) \delta_{m,0} & (1 - \pi_{c,m}) \delta_{m,1} & \ldots & (1 - \pi_{c,m}) \delta_{m,k_m} \\
1 & 0 & 1 & \alpha_1 & \ldots & \alpha_{k_m} \\
0 & 1 & \pi_{c,m} & \pi_{c,m} \alpha_1 & \ldots & \pi_{c,m} \alpha_{k_m}
\end{bmatrix} S_{m,t}. \]
UC 2-trend, Money and Consumption Cycle Observer Systems

- UC 2-trend, money cycle observer equation

\[ y_t = \begin{bmatrix} 1 & \pi_{e,a} & (1 - \pi_{c,\tilde{m}}) \delta_{\tilde{m},0} & (1 - \pi_{c,\tilde{m}}) \delta_{\tilde{m},1} & \ldots & (1 - \pi_{c,\tilde{m}}) \delta_{\tilde{m},k_{\tilde{m}}} \\ 1 & 0 & 1 & \alpha_1 & \ldots & \alpha_{k_{\tilde{m}}} \\ 0 & 1 & \pi_{c,\tilde{m}} & \pi_{c,\tilde{m}} \alpha_1 & \ldots & \pi_{c,\tilde{m}} \alpha_{k_{\tilde{m}}} \end{bmatrix} S_{\tilde{m},t}. \]

- UC 2-trend, consumption cycle observer equation

\[ y_t = \begin{bmatrix} 1 & \pi_{e,a} & (1 - \pi_{m,\tilde{c}}) \delta_{\tilde{c},0} & \ldots & (1 - \pi_{m,\tilde{c}}) \delta_{\tilde{c},k_{\tilde{c}}-1} \\ 1 & 0 & \pi_{m,\tilde{c}} & \ldots & 0 \\ 0 & 1 & 1 & \ldots & 0 \end{bmatrix} S_{\tilde{c},t}. \]
Likelihood Approach

- State space of Kalman filter to evaluate likelihood,

\[ \mathcal{L}(\mathbf{y}_t \mid \Gamma_{2,i,\kappa}, UC_{2,i,\kappa}), \quad \mathbf{y}_t = [e_t \ m_t \ c_t]' . \]

\( \Gamma_{2,i,\kappa} = \text{parameter vector.} \)

\( UC_{2,i,\kappa} = 6 \text{ models,} \)

all have 2 trends: \( \{\mu_t \ a_t\} \),

\( i = 2 \text{ cycles: } \{\tilde{m}_t \ \tilde{c}_t\}, \text{ only } \tilde{m}_t \text{ or } \tilde{c}_t \text{ cycle.} \)

\( \kappa = 1 \text{ or } \kappa \in [0.9, 0.999] . \)
Bayesian Approach

- Approximate KF likelihood with MCMC generated by random walk MH simulator
  ⇒ Rabanal and Rubio-Ramírez (JME, 2005).
Bayesian Approach

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  \[\Rightarrow\text{Rabanal and Rubio-Ramírez (JME, 2005).}\]

- Generate 1.5 million posterior draws, toss first 0.75 million as ‘burn-in’ across five sets of initial conditions for six UC models.
Bayesian Approach

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  ⇒ Rabanal and Rubio-Ramírez (JME, 2005).

- Generate 1.5 million posterior draws, toss first 0.75 million as ‘burn-in’ across five sets of initial conditions for six UC models.

- Compute marginal likelihoods of six UC models
One Economic Prior

- Tables 4 and 5 list priors on UC model P–T processes and factor loadings.

\[ \kappa \sim \text{Inv-Gamma}, \text{mean} = 0.988, \text{std} = 0.0383. \]

Impose EW prior on \( \kappa \in [0.9, 0.999] \), else toss draw out.
Tables 4 and 5 list priors on UC model P–T processes and factor loadings.

Calibrate $\kappa = 1$ for $UC_{2,\tilde{c},\kappa=1}$ models

$\Rightarrow e \sim RW$ driven only by $\mu$ and $a$. 

$\kappa \sim Inv-Gamma$, mean $= 0.988$, std $= 0.0383$. Impose EW prior on $\kappa \in [0.9, 0.999]$, else toss draw out.
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- Calibrate $\kappa = 1$ for $UC_{2,\tilde{c},\kappa=1}$ models
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RESULTS

Results

- Tables 4 and 5 list posteriors of UC models.
DSGE-PVM Estimates

**RESULTS**

- Tables 4 and 5 list posteriors of UC models.
- Figure 1: only $UC_2,\tilde{c},\kappa$ model estimates of $\kappa$ move off prior and toward one.
Figure 1: Prior and Posterior PDFs of DSGE-PVM Discount Factor

- Prior of DSGE-PVM Discount Factor
- Posterior 2-Trend, 2-Cycle Model
- Posterior 2-Trend, M-Cycle Model
- Posterior 2-Trend, C-Cycle Model

(PDF deflated by 0.1)
RESULTS


- Tables 4 and 5 list posteriors of UC models.

- Figure 1: only $UC_2,\tilde{c},\kappa$ model estimates of $\kappa$ move off prior and toward one.

- Figure 2: $UC_2,\tilde{c},\kappa$ produces relatively smooth CDN$/US$ exchange rate trend and cycle.
Figure 2: CDN$/US$ Exchange Rate Trend and Cycle, 1976Q1 - 2004Q4

-10 -8 -6 -4 -2 0 2 4 6 8

ln[CDN$/US$ Ex Rate]

2-Trend, 2-Cycle Trend
2-Trend, C-Cycle Trend

Ex Rate Cycle from
2-Trend, 2-Cycle Model
Ex Rate Cycle from
2-Trend, C-Cycle Model
Permanent Shocks Dominate CDN$/US$ Exchange Rate when $\kappa \in [0.9, 0.999]$

- Tables 7 and 8: Shocks to $\mu$ and $a$ dominate unconditional variance decompositions and FEVDs of CDN$/US$ exchange rate.
Permanent Shocks Dominate CDN$/US$ Exchange Rate when $\kappa \in [0.9, 0.999]$

- Tables 7 and 8: Shocks to $\mu$ and $\alpha$ dominate unconditional variance decompositions and FEVDs of CDN$/US$ exchange rate.

- Figure 4: As $\kappa \rightarrow 1$, smoother and less persistent transitory CDN$/US$ exchange rate.
Figure 4: CDN$/US$ Ex Rate Cycles at Different DSGE-PVM Discount Factors

Discount Factor, 16th Percentile = 0.943

Discount Factor, 84th Percentile = 0.988

Discount Factor = 0.999

Discount Factor, 16th Percentile = 0.943

Discount Factor, 84th Percentile = 0.988

Discount Factor = 0.999

Discount Factor, 16th Percentile = 0.994

Discount Factor, 84th Percentile = 0.9987

Discount Factor = 0.999

2-Trend, 2-Cycle UC Model

2-Trend, C-Cycle UC Model
Bayesian Model Evaluation

- Data favor $\hat{\mathcal{L}}(\mathcal{Y}_t | \Gamma_2, \tilde{c}, \kappa=1, \ U C_2, \tilde{c}, \kappa=1) = -24.76$. 
Data favor $\hat{L}(Y_t \mid \Gamma_2, \tilde{c}, \kappa=1, \; UC_2, \tilde{c}, \kappa=1) = -24.76$.

When $\kappa \in [0.9, 0.999]$, data favor $\hat{L}(Y_t \mid \Gamma_2, \tilde{c}, \kappa, \; UC_2, \tilde{c}, \kappa) = -29.88$. 
Data favor $\hat{L}(Y_t \mid \Gamma_2, \tilde{c}, \kappa=1, UC_2, \tilde{c}, \kappa=1) = -24.76$.

When $\kappa \in [0.9, 0.999]$, data favor $\hat{L}(Y_t \mid \Gamma_2, \tilde{c}, \kappa, UC_2, \tilde{c}, \kappa) = -29.88$.

Scale $\hat{L}(Y_t \mid \Gamma_2, \tilde{c}, \kappa=1, UC_2, \tilde{c}, \kappa)$ by $\exp(5.12)$ to be indifferent between it and $UC_2, \tilde{c}, \kappa=1$. 
Bayesian Model Evaluation

- Data favor $\hat{L}(\mathcal{Y}_t | \Gamma_2, \tilde{c}, \kappa=1, \ UC_2, \tilde{c}, \kappa=1) = -24.76$.

- When $\kappa \in [0.9, 0.999]$, data favor $\hat{L}(\mathcal{Y}_t | \Gamma_2, \tilde{c}, \kappa, \ UC_2, \tilde{c}, \kappa) = -29.88$.

- Scale $\hat{L}(\mathcal{Y}_t | \Gamma_2, \tilde{c}, \kappa=1, \ UC_2, \tilde{c}, \kappa)$ by $\exp(5.12)$ to be indifferent between it and $UC_2, \tilde{c}, \kappa=1$.

- Data favors CDN$/US$ exchange rate is random walk or permanent shocks dominate.
Contributions of Paper

- Generalize EW Hypothesis to open economy DSGE models $\Rightarrow$ DSGE-PVM.
- Generalize EW hypothesis to eliminate cycle in exchange rate and fundamental.
- Estimates support EW Hypotheses.
- Puzzle that transitory shocks, especially ‘mean reverting’ money fundamentals, are disconnected from CDN$/US$ exchange rate.
Conclusion

Contributions of Paper

▶ Generalize EW Hypothesis to open economy DSGE models $\Rightarrow$ DSGE-PVM.

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GENERALIZING EXCHANGE RATES AND FUNDAMENTALS

CONCLUSION

CONTRIBUTIONS OF PAPER

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