Mortgage Innovation and the Foreclosure Boom

Dean Corbae
University of Texas at Austin

Erwan Quintin
Federal Reserve Bank of Dallas

June 3, 2009

Preliminary and Incomplete!
Recent trends in US housing

- **Homeownership rates**
- **Quarterly foreclosure rates**
- **House prices, CS-index**

### Charts:
1. **FRMs as a fraction of all mortgages**
   - 1998: 85%
   - 2000: 75%
   - 2002: 70%
   - 2004: 65%
   - 2006: 60%
2. **Quarterly foreclosure rates, all mortgages**
   - 1998: 0.2%
   - 2000: 0.4%
   - 2002: 0.6%
   - 2004: 0.8%
   - 2006: 1.0%
   - 2008: 1.0%
3. **Quarterly foreclosure rates by mortgage type**
   - Prime FRM
   - Prime ARM
   - Subprime FRM
   - Subprime ARM
4. **Shares of Mortgage stock**
   - Subprime FRM
   - Subprime ARM
5. **Homeownership rates**
   - 1998: 65%
   - 2000: 66%
   - 2002: 67%
   - 2004: 68%
   - 2006: 69%
6. **House prices, CS-index**
   - 1998: 80
   - 2000: 85
   - 2002: 90
   - 2004: 95
   - 2006: 100
   - 2008: 105
Motivation

- The importance of mortgages with low initial payments has increased.
- So have foreclosure rates . . .
- . . . particularly among borrowers who hold non-standard mortgages . . .
- . . . and in the “subprime” segment . . .
- . . . especially coincident with the downturn in house prices.

Our Question: How much of the recent rise in foreclosure rates can non-standard mortgages account for? The answer depends on selection and separation.
A model of housing

- Heterogenous agent model where agents choose to rent or finance a house purchase by choosing among a set of mortgage contracts.
- Mortgage holders may default because:
  1. they can’t afford current payments (involuntary default);
  2. their home equity is negative (voluntary default);
- Mortgage terms reflect default risk, hence vary with income and asset position at the origination of the loan.

⇒ Our model is consistent with the fact that mortgage terms vary a lot across borrowers, even among those who opt for the same mortgage types.
Outline of Experiments

- When only standard FRMs are available, households with low earnings and low assets cannot afford to buy houses (ss).
- Once IOMs are introduced, those high risk agents select into low downpayment, backloaded contracts and lenders price these mortgages accordingly (separating the good risks from the bad) (ss).
- Foreclosures are amplified during the transition path in response to an unanticipated price decline since home equity drops on existing mortgage contracts (trans).
Summary of Results

- Introducing IOMs causes default rates to rise by 45%.
- IOMs account for nearly 64% of overall defaults.
- Default rates on IOMs are twice as high as on FRMs.
- IOMs amplify the amount of default in response to an unanticipated price downturn by over 6%.
- The model with both IOMs and FRMs account for over 2/3 of foreclosure rates in the data during the transition.
Literature on Foreclosures

• Garriga and Schlagenhaft (2009).
  • Pool borrowers within mortgage types so cannot separate prime vs subprime within a contract (recall Figure 1)
  • As in Chatterjee et al (2007), we do not have cross subsidization (i.e. no cream skimming consistent with competitive lending).

• Guler (2008).
  • Studies the impact of an innovation to the screening technology on steady state default rates.
Environment

- Time is discrete and infinite.
- There is a continuum of households and a financial intermediary.
- Young agents become mid-aged with probability $\rho_M$, mid-aged agents become old with probability $\rho_O$, old agents die with probability $\rho_D$.
- Until old, agents earn stochastic income $y$ drawn from a Markov chain. Old agents earn $y_O$ with certainty.
- At birth agents have zero assets and draw initial income from the invariant distribution of the MC.
• Agents value consumption $c_t$ and housing services $s_t$ according to:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, s_t)$$

where $U$ satisfies standard assumptions.

• Agents can save at rate exogenously given risk free rate $r_t > 0$ in period $t$. 
Housing

- In any period $t$, agents can rent quantity $h_1 > 0$ of housing services at rate $R_t$.
- When agents become middle-aged, they are given the one-time option to purchase quantity $h \in \{h_2, h_3\}$ of housing services for unit price $q_t$, where $h_3 > h_2 > h_1$.
- Homeowners face idiosyncratic shocks to their housing capital. A fraction $\lambda$ of agents who own a house of size $h = h_3$ or $h = h_2$ see the quantity of capital they own fall to $h_2$ or $h_1$ (to account for foreclosures not associated with job loss when housing prices are non-decreasing).
- The service flows from housing quantity $h$ are
  \[ s_t = h \left[ 1\{h=h_1\} + (1 - 1\{h=h_1\})\theta \right], \quad \theta > 1. \]
- Houses of size $h$ carry maintenance costs $\delta h$.
- Agents can sell their house in any given period, but are then constrained to be renters for the remainder of their life.
- Old must sell their house.
Financial intermediary

- Stores savings with return $r_t$ at date $t$.
- Can transform quantity $k \geq 0$ of deposits into quantity $Ak$ of housing capital.
- Rents and sells housing capital. Rented capital bears maintenance cost $\delta$ in each period.
- Issues all mortgages. Mortgages carry administrative cost $\phi$. 
Mortgage contracts

- FRMs have constant payments every period.
- IOMs are back loaded.

FRMs have constant payments every period. IOM contracts

IOM contracts
Timing

1. Youth:
   - Receive age shock and perfect signal of income realization.
   - Make savings decision.

2. Middle-age:
   - Receive age shock and perfect signal of income realization.
   - New mid-aged agents make home-buying and mortgage choice decision.
   - Existing homeowners may receive a devaluation shock and decide whether to default or sell.
   - Make mortgage or rental payments as well as savings decisions.

3. Old:
   - Newly old agents sell their house if they own one.
   - Receive death shock or income.
   - Make (dis)saving decision.
Young agents

- State: \( \omega = (a, y) \)
- The value function \( V_Y(\omega) \) for a young agent solves

\[
V_Y(a, y) = \max_{a' \geq 0} U(c, h_1) \\
+ \beta E_{y'|y} [(1 - \rho_M)V_Y(a', y') + \rho_M V_M(a', y', \ldots)]
\]

subject to:

\[
c + a' = y + a(1 + r) - Rh_1.
\]
After introduction of IOMs, average savings of newly mid-aged falls by one third from 0.77 to 0.53.
Distribution of hhs contract choice

Asset poor agents select into IOMs while asset rich agents opt for FRMs.
Evolution of home equity

Principal balance over time

Home equity

$q_{h_2^n(b(n; \kappa = \text{FRM}, 20\%, h_2))}$

$q_{h_2^n(b(n; \kappa = \text{IOM}, 20\%, h_2))}$

$q_{h_1^n(b(n; \kappa = \text{FRM}, 20\%, h_2))}$

$q_{h_1^n(b(n; \kappa = \text{IOM}, 20\%, h_2))}$
FRM vs IOM Default Hazards for Median Income/Assets

IOM default hazard higher than FRM since
- backloading means net equity is low longer (vol.)
- mortgage payments are higher (invol.)
## Default frequencies by mortgage type

<table>
<thead>
<tr>
<th></th>
<th>voluntary</th>
<th>involuntary</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>FRM only</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRM</td>
<td>0.88</td>
<td>1.33</td>
</tr>
<tr>
<td><em>FRM+IOM</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRM</td>
<td>0.72</td>
<td>1.46</td>
</tr>
<tr>
<td>IOM</td>
<td>0.91</td>
<td>3.43</td>
</tr>
</tbody>
</table>

Even FRM involuntary default is higher in FRM+IOM economy since median savings are lower.
Parameterization

- We choose parameters so that a benchmark economy with FRMs only matches the relevant features of the US economy prior to 2002.
- Model period is three years.
- for preferences, we specify for all \((c, h) > (0, 0)\),

\[
U(c, s) = \psi \log(c) + (1 - \psi) \log(s).
\]
Income Process

- Calibrated from the PSID 1996 and 1999.
- The incomes of mid-aged agents $y^M \in \{0.2737, 1, 2.4057\}$ with the median normalized to 1. The transition matrix is
  \[
  \begin{bmatrix}
  0.7845 & 0.1781 & 0.0374 \\
  0.1647 & 0.6607 & 0.1746 \\
  0.0508 & 0.1612 & 0.7880
  \end{bmatrix}
  \]
- The incomes of young agents $y^Y \in \{0.2664, 0.7218, 1.6306\}$ with transition matrix
  \[
  \begin{bmatrix}
  0.6788 & 0.2363 & 0.0849 \\
  0.2503 & 0.5055 & 0.2442 \\
  0.0709 & 0.2583 & 0.6709
  \end{bmatrix}
  \]
- Cross-sectional variance and autocorrelation of log income is 0.45 and 0.76. This is in line with the (three-year) estimates 0.4 and 0.86 by Krueger and Perri (2005).
Table: Benchmark parameterization with $\alpha = 1$

<table>
<thead>
<tr>
<th>Par.</th>
<th>Description</th>
<th>Par. Val.</th>
<th>Target</th>
<th>Tar. Val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_M$</td>
<td>Avg. length of young</td>
<td>0.25</td>
<td>20-31 years old</td>
<td>12 years</td>
</tr>
<tr>
<td>$\rho_O$</td>
<td>Avg. length of mid-aged</td>
<td>0.10</td>
<td>32-61 years old</td>
<td>30 years</td>
</tr>
<tr>
<td>$\rho_D$</td>
<td>Avg. length of old</td>
<td>0.125</td>
<td>62-85 years old</td>
<td>24 years</td>
</tr>
<tr>
<td>$r$</td>
<td>Storage returns</td>
<td>0.12</td>
<td>Annual risk-free rate</td>
<td>4%</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Maintenance rate</td>
<td>7.5%</td>
<td>Annual Residential depre. (Harding et al.,07)</td>
<td>2.5%</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Downpayment on FRMs</td>
<td>0.20</td>
<td>Avg. LTV for FRM mortgages (AHS)</td>
<td>0.85</td>
</tr>
<tr>
<td>$T$</td>
<td>Maturity length of mortgages</td>
<td>10</td>
<td>AHS</td>
<td>30 years</td>
</tr>
<tr>
<td>$n^{IOM}$</td>
<td>Interest-only period for IOM</td>
<td>3</td>
<td>MBA</td>
<td>5-10 years</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Owner-occupied premium</td>
<td>100</td>
<td>Homeownership rates (Census Bureau)</td>
<td>67%</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Housing shock probability</td>
<td>0.02</td>
<td>Foreclosure rates (MBA)</td>
<td>3.75%</td>
</tr>
<tr>
<td>$A$</td>
<td>Housing technology TFP</td>
<td>0.67</td>
<td>Avg. LTY at loan origination (AHS)</td>
<td>0.83</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.75</td>
<td>Avg. ex-housing asset-to-income ratio (98 SCF)</td>
<td>0.74</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Mortgage service cost</td>
<td>0.06</td>
<td>Avg. FRM yields (FHFB)</td>
<td>24%</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Foreclosing costs</td>
<td>0.70</td>
<td>Loss-incidence estimates (Hayre el al.,08)</td>
<td>50%</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Utility share on consumption</td>
<td>0.5</td>
<td>Avg. housing expenditure</td>
<td>0.2</td>
</tr>
<tr>
<td>$h_3$</td>
<td>Size of luxury house</td>
<td>2.4</td>
<td>Avg. rent-to-income ratios for rich hhs</td>
<td>0.6</td>
</tr>
<tr>
<td>$h_2$</td>
<td>Size of regular house</td>
<td>1</td>
<td>Avg. rent-to-income ratios for avg. hhs</td>
<td>0.25</td>
</tr>
<tr>
<td>$h_1$</td>
<td>Size of rental unit</td>
<td>0.40</td>
<td>Avg. rent-to-income ratios for poor hhs</td>
<td>0.15</td>
</tr>
</tbody>
</table>
# Steady State Statistics

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>FRM only</th>
<th>FRM+IOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homeownership rate</td>
<td>67.00</td>
<td>72.25</td>
<td>74.35</td>
</tr>
<tr>
<td>Foreclosure rates</td>
<td>3.5–4.5</td>
<td>2.21</td>
<td>3.20</td>
</tr>
<tr>
<td>Avg. LTY</td>
<td>0.83</td>
<td>1.33</td>
<td>1.47</td>
</tr>
<tr>
<td>Avg. LTV</td>
<td>0.85</td>
<td>0.80</td>
<td>0.90</td>
</tr>
<tr>
<td>Avg. ex-housing asset/income ratio</td>
<td>0.74</td>
<td>0.70</td>
<td>0.65</td>
</tr>
<tr>
<td>Avg. FRM yields</td>
<td>24</td>
<td>19.37</td>
<td>19.28</td>
</tr>
<tr>
<td>Loss-incidence estimates</td>
<td>0.50</td>
<td>0.46</td>
<td>0.39</td>
</tr>
<tr>
<td>Avg. housing expenditure share</td>
<td>0.20</td>
<td>0.23</td>
<td>0.24</td>
</tr>
<tr>
<td>Avg. Rent-to-income for rich hh</td>
<td>0.6</td>
<td>0.65</td>
<td>0.64</td>
</tr>
<tr>
<td>Avg. Rent-to-income for avg. hh</td>
<td>0.25</td>
<td>0.23</td>
<td>0.24</td>
</tr>
<tr>
<td>Avg. Rent-to-income for poor hh</td>
<td>0.15</td>
<td>0.13</td>
<td>0.14</td>
</tr>
</tbody>
</table>

[› stats def.][1]  [› SS def.][2]
Comparing mortgage rates for big and small houses shows they rise with borrower’s loan-to-income ratios as in the data.
Yield schedule, IOM

IOM rates are higher than FRM at all possible asset/income ratios.
If we define subprime as the bottom 30% of hhs who obtained the highest mortgage interest rate, then there are both prime (with avg yield 21.16%) and subprime (with avg yield 21.54%) IOMs but only prime (with avg yield 19.28%) FRMs.
## Contract Moments

<table>
<thead>
<tr>
<th></th>
<th>Data (2004 SCF)</th>
<th>FRM+IOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>CV(yield) for FRMS</td>
<td>0.153</td>
<td>0.016</td>
</tr>
<tr>
<td>CV(yield) for other</td>
<td>0.341</td>
<td>0.010</td>
</tr>
<tr>
<td>( \rho(yield, income) ) on FRMs</td>
<td>-0.12</td>
<td>-0.924</td>
</tr>
<tr>
<td>( \rho(yield, income) ) on other</td>
<td>-0.18</td>
<td>-0.606</td>
</tr>
<tr>
<td>( \rho(yield, networth) ) on FRMs</td>
<td>-0.03</td>
<td>-0.10</td>
</tr>
<tr>
<td>( \rho(yield, networth) ) on other</td>
<td>-0.14</td>
<td>-0.65</td>
</tr>
</tbody>
</table>
The Value of Innovation

• Q: How much would a newborn in FRM only economy be willing to pay to be in the FRM+IOM economy?

• Let \( \{k_L, k_M, k_H\} \) be the consumption-equivalent welfare changes associated with the introduction of IOMs for agents born with income \( y \in \{y_L, y_M, y_H\} \).

• We find that:

\[
\begin{align*}
 k_L &= 0.1841 \\
 k_M &= 0.0833 \\
 k_H &= 0.0020
\end{align*}
\]

• The average welfare gain is 9% (this large number may be sensitive to preference parameterization).
FRM+IOM economy response to price drop can account for more than 2/3 of foreclosures and default rates are amplified by 6% over FRM only economy.
Intermediary losses following an unexpected price shock

Losses are steeper when IOMs are present.
Summary

- Introducing IOMs causes default rates to rise by 45%.
- IOMs account for nearly 64% of overall defaults.
- Default rates on IOMs are twice as high as on FRMs.
- IOMs amplify the amount of default in response to an unanticipated price downturn by over 6%
- The model with both FRMs and IOMs can account for more than 2/3 of observed foreclosures during transition.
To do

• Estimate parameters via SMM in FRM+IOM economy.
• Endogenize prices ($\alpha < 1$)
  • homeownership rise will generate price rise.
  • welfare gain not definite.
• Add aggregate uncertainty.
Middle-aged agents

\[
\mu_M(A, y, H, h, n; \kappa) = \rho_M \int_{\Omega_Y} \mathbb{1}_{\{(H, h, n)=(0, h_1, 0)\}} \mathbb{1}_{\{a'_Y(\omega) \in A\}} \Pi(y|\omega) d\mu_Y(\omega) \\
+ (1 - \rho_0) \int_{\Omega_M} \mathbb{1}_{\{(H'(\omega)=H, n(\omega)=n-1, a'_M(\omega) \in A\}} \Pi(y|\omega) P(h|\omega) d\mu_M(\omega) \\
\times \left\{ \mathbb{1}_{\{n(\omega)=0, \Xi(\omega)=\kappa\}} + \mathbb{1}_{\{n(\omega)>0, \kappa=\kappa(\omega)\}} \right\}
\]
\[ \mu_O(A) = (1 - \rho_D) \int_{\Omega_O} 1_{\left\{ a'_O(\omega) \in A \right\}} d\mu_O(\omega) \]
\[ + \rho_O \int_{\Omega_M} 1_{\left\{ a'_M(\omega) + \max\{ H'(\omega)[qh(\omega) - b(n+1,\kappa)],0 \} \in A \right\}} d\mu_M(\omega) \]
## Share of overall default rates

<table>
<thead>
<tr>
<th></th>
<th>voluntary</th>
<th>involuntary</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FRM only</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRM</td>
<td>0.88</td>
<td>1.33</td>
<td>2.20</td>
</tr>
<tr>
<td><strong>FRM+IOM</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRM</td>
<td>0.38</td>
<td>0.77</td>
<td>1.15</td>
</tr>
<tr>
<td>IOM</td>
<td>0.43</td>
<td>1.61</td>
<td>2.04</td>
</tr>
<tr>
<td>Total</td>
<td>0.81</td>
<td>2.38</td>
<td>3.20</td>
</tr>
</tbody>
</table>
• The rate is truncated since the household default probability is too high for the bank to break-even at any mortgage rate below the rate at which the mortgage payment in the first period is so high that the budget set is empty.

• In that period (i.e. when $n = 0$), the budget set is empty when $c = a' = 0$ and

$$m(0; \zeta, r^\zeta) > y_0 + (a_0 + \iota - vq\bar{h} \cdot 1\{\zeta = FRM\})(1 + r).$$

Since $m(0; \zeta, r^\zeta, h_0)$ is strictly increasing in $r^\zeta$, we know there is an interest rate $\overline{r}^\zeta$ that depends on $y_0$ and $a_0$ such that for any $r > \overline{r}^\zeta$ the bank cannot break even.
\[ V_{FRM+IOM}^{FRM}(0, y_i) = E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t^{FRM}(1 + k_i), s_t^{FRM}) \right] \]

\[ = E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left\{ \ln(c_t^{FRM}) + \ln(1 + k_i) + \ln(s_t^{FRM}) \right\} \right] \]

\[ = V_{FRM}(0, y_i) + \frac{\ln(1 + k_i)}{(1 - \beta)} \]

\[ \implies 1 + k_i = \exp((1 - \beta) \left[ V_{FRM+IOM}^{FRM}(0, y_i) - V_{FRM}^{FRM}(0, y_i) \right]) \]

We can then calculate \( k_i \) for each income level:

\[ 1 + k_L = \exp ((1 - 0.75) [(-2.4098) - (-3.0858)]) = 1.1841 \]

\[ 1 + k_M = \exp ((1 - 0.75) [(-0.5332) - (-0.8533)]) = 1.0833 \]

\[ 1 + k_H = \exp ((1 - 0.75) [0.8966 - 0.8887]) = 1.0020 \]
Important legal considerations

- **1980**: Depository Institutions Deregulation and Monetary Control Act (DIDMCA) preempts state interest rate caps.
- **1982**: Alternative Mortgage Transaction Parity Act (AMTPA) permits the use of variable interest rates and balloon payments.
- Equal Credit Opportunity Act: “...ensures an equal chance to obtain credit. This does not mean all consumers who apply for credit get it: ...income, expenses, debt, and credit history are considerations for creditworthiness.”
- Anti-deficiency laws: borrower is not responsible for any deficiency. The banks cannot attach to household’s assets. Some states have them (CA, ...), others don’t.
Housing Market Clearing Condition

The market for housing capital clears provided

\[
\int_{\Omega_{M}} h \{ H' = 1, h(\omega) = h \} d \mu_{M} - \int_{\Omega_{M}} h' \{ H' = 1 \} P(h' | \omega) d \mu_{M} = A k^{\alpha}
\]

- In equilibrium the production of new housing capital must equal the housing capital lost to devaluation.
- Both the rental and owner-occupied markets clear since the intermediary is willing to accommodate any allocation of total housing capital by the arbitrage condition.
• Loss-incidence estimates

$$\min\{(1 - \chi)qh, b\}$$
Value function for a mid-aged agents with mortgage

\[ V_M(a, y, 1, h, n; \kappa) \]

\[ = \max_{c \geq 0, a' \geq 0, (H', D', D^V, S) \in \{0, 1\}^4} U(c, (1 - H')h_1 + H'(1_{h=h_1} + \theta 1_{\{h \neq h_1\}})h) \]

\[ + (1 - H') \beta E_{y'|y} [(1 - \rho_O) V_M(a', y', 0, h_1, n + 1; \emptyset) + \rho_O V_O(a')] \]

\[ + H' \beta E_{(y', h')|(y, h)} \left[ (1 - \rho_O) V_M(a', y', 1, h', n + 1; \cdot) \right. \]

\[ \left. + \rho_O V_O(a' + \max \{qh - b(n + 1; \kappa), 0\}) \right] \]

subject to:

\[ c + a' = y + (1 + r)(a + \iota + (1 - H') \max((1 - (D' + D^V)\chi)qh - b(n; \kappa), 0)) \]

\[ - H'(m(n; \kappa) + \delta h) - (1 - H')Rh_1 \]

\[ D' = 1 \text{ if and only if } y + (a + \iota)(1 + r) - m(n; \kappa) - \delta h < 0 \]

\[ D^V = 1 \text{ if } H' = 0 \text{ and } qh - b(n; \kappa) < 0 \]

\[ S = 1 - H' - D' - D^V \]
Definition of net profit

\[ NF = \int_{\omega} \left\{ \left( 1 - S(\omega) - D^V(\omega) - D^I(\omega) \right) \cdot b(n; \kappa)(r^c - r - \phi) \\
+ D^V(\omega) \cdot \left( -\left( b(n; \kappa) - (1 - \kappa)qh \right) \right)(1 + r + \phi) \\
+ D^I(\omega) \cdot \min(0, -\left( b(n; \kappa) - (1 - \kappa)qh \right))(1 + r + \phi) \right\} d\mu(\omega) \]
Distribution of young agents

Let \((n_L, n_M, n_H)\) be the invariant income distribution implied by the income process. The invariant distribution \(\mu_Y\) on \(\Omega_Y\) solves, for all \(y \in \{y_L, y_M, y_H\}\) and \(A \subset \mathbb{R}^+\):

\[
\mu_Y(A, y) = \mu_0 \mathbb{1}_{0 \in A, y = y_j} n_j + (1 - \rho_M) \int_{\omega \in \Omega_Y} \mathbb{1}_{a_Y(\omega) \in A} \Pi(y | \omega) d\mu_Y(\omega)
\]
Newly middle-aged agents $n = 0$

- **State:** $\omega = (a, y, H, h, n; \kappa)$
- The value function $V_M(a, y, 1, h, 0; \emptyset)$ for a newly middle-aged agent solves

\[
V_M(a, y, 1, h, 0; \emptyset) = \max_{c \geq 0, a' \geq 0, H' \in \{0, 1\}, \kappa \in K(\omega_0)} U(c, (1 - H')h_1 + H'\theta h_0)
+ (1 - H')\beta E_{y'}[y] [(1 - \rho_O)V_M(a', y', 0, h_1, 1; \emptyset) + \rho_O V_O(a')]
+ H'\beta E_{(y', h')}[(y, h_0)] \left[ (1 - \rho_O)V_M(a', y', 1, h', 1; \kappa) + \rho_O V_O(a' + \max \{qh_0 - b(1; \kappa), 0\}) \right]
\]

subject to:

\[
c + a' = y + (1 + r)(a + \iota - H'\nu \mathbb{1}_{\{\zeta = \text{FRM}\}} qh_0), \\
-H'(m(0; \kappa) + \delta h_0) - (1 - H')Rh_1, \\
a + \iota \geq H'\nu \mathbb{1}_{\{\zeta = \text{FRM}\}} qh_0,
\]

where $K(\omega_0)$ is the set of mortgage contracts available with typical element $\kappa = (\zeta, r^\zeta, h_0)$. 

[Back to dist. contracts]
Intermediary decision making

- The intermediary's value function is denoted $W(\omega)$. It is given by

1. If the mid-age household is currently a homeowner and the mortgage is not paid off so that $\omega = (a, y, 1, h, n; \kappa)$ with $n \in (0, T - 1]$

$$W^K(\omega) = (D^I(\omega) + D^V(\omega)) \min\{(1 - \chi)qh, b(n; \kappa)\} + S(\omega)b(n; \kappa) + (1 - D^I(\omega) - D^V(\omega) - S(\omega)) \left(\frac{m(n; \kappa)}{1 + r + \phi} + E_{\omega' | \omega} \left[\frac{W^K(\omega')}{1 + r + \phi}\right]\right)$$

where $S(\omega) = 1$ if household sells the house.
2 If the household has just turned mid-age and its budget set is not empty so that \( \omega_0 = (a_0, y_0, 0, h_1, \ldots) \) and
\[
\begin{align*}
y_0 + (a_0 + 1 - 1 \{\zeta = FRM\} \nu h_0 q)(1 + r) - m(0; \kappa) - \delta h_0 & \geq 0 \\
a_0 - 1 \{\zeta = FRM\} \nu h_0 q & \geq 0
\end{align*}
\]
then
\[
W^\kappa(\omega_0) = \frac{m(0; \kappa)}{1 + r + \phi} + E_{\omega' | \omega_0} \left[ \frac{W^\kappa(\omega')}{1 + r + \phi} \right]
\]

3 In all other cases, \( W(\omega) = 0 \).
Zero profit condition

Then the zero profit condition on a loan contract \( \kappa = (\zeta, r^\zeta, h_0) \) with state \( \omega_0 = (a_0, y_0, \ldots) \) is \( W^\kappa(\omega_0) \) can be written

\[
W^\kappa(\omega_0) - (1 - \nu 1_{\{\zeta=FRM\}})qh_0 = 0.
\]
Intermediary decision making

- Arbitrage between renting and selling houses implies

\[ q = \sum_{t=1}^{+\infty} \frac{R - \delta}{(1 + r)^t}, \]

which determines rental payments \( R \).

- Housing capital investment \( k \) maximizes profits:

\[ Ak^\alpha q - k. \]
Steady state equilibrium

1. Agent policies are optimal given all prices.
2. The intermediary’s output of new housing capital in each period is optimal given \( q \).
3. Per capita profits associated with housing capital production are \( \iota \).
4. The allocation of housing capital to rental and the owner-occupied market is optimal for the intermediary.
5. The market for housing capital clears every period.
6. The intermediary expects to make zero profit on all mortgages. In other words, condition holds for all \( \omega_0 \in \Omega_M \) and all mortgages in \( K(\omega_0) \).
7. The distribution of states is invariant given pricing functions and agent policies.
Fixed-rate mortgages

- Agents with assets $a$ and income $y$ can purchase a house of size $h$ by selecting a mortgage type $\zeta = \{FRM, IOM\}$ with yield $r^\zeta(a, y, h)$.
- FRMs require down payments $\nu hq_t$, where $\nu \in (0, 1)$, and fixed payments for $T$ periods:

$$m^{FRM,t} = \frac{r^{FRM,t}}{1 - (1 + r^{FRM,t})^{-T}}(1 - \nu)hq_t, \quad \forall n \in \{0, T - 1\}$$
Interest-only mortgages

• IOMs require no downpayment.
• Interest payments of \( r^{IOM} h_q t \) for first \( n^{IOM} \) periods.
• Fixed payments for the remaining \( T - n^{IOM} \) periods.

\[
m^{IOM,t}_n = \begin{cases} 
  h q r^{IOM,t} & \text{if } n < n^{IOM} \\
  \frac{r^{IOM,t}}{1 - (1 + r^{IOM,t})^{-1} - (T - n^{IOM})} h q & \text{if } n \geq n^{IOM}
\end{cases}
\]