Market Selection

Leonid Kogan, Stephen Ross, Jiang Wang and Mark M Westerfield

May 2009
The Market Selection Hypothesis

■ **Old Version**: (Friedman 1953): Agents who trade based inaccurate beliefs will lose money, and they will eventually be driven out of the market. Prices will come to reflect actual underlying probabilities.

- Theoretically, one can construct counter examples even with competitive, complete markets (Kogan, Ross, Wang, and Westerfield [2004], Yan [2008], etc.).
- Empirically, cognitive biases are widespread, including among traders who manage large accounts (Ito [1990], Odean [1999], etc.).

■ **New Version**: Agents with more accurate beliefs are better able to exploit market trading opportunities to increase their wealth. Prices may never come to reflect actual underlying probabilities, but that only reflects trading or consumption constraints.

- Supporting models: Blume and Easley (2008), Sandroni (2000).
- Most people, especially practitioners, have a strong feeling that this *should* be true. If not, what is the value to good information?
Our Contribution

- The Market Selection Hypothesis – that markets generically favor investors with more accurate beliefs – is not true.
  
  It is true for a specific class of bounded models, but not otherwise.

- We unify a literature of examples and counter-examples.
  
  We present general conditions for long term survival and price impact.

- Survival of inaccurate traders is determined by a comparison of belief differences to risk attitudes.

- Price impact of inaccurate traders exists if consumption shares are sufficiently volatile.
  
  Price impact is from consumption volatility, while survival is from consumption level.

- Extension to state-dependent preferences (habit, etc).
The Economy

- There is a sequence of endowments: $D_t$.
  $t \in [0, \infty)$, possibly discrete, possibly continuous.

- Complete Markets.

- Two traders, $A$ and $B$, both with utility function $U(x)$, time discount rate $\rho$.

- Different beliefs. Probability measures $A$ and $B$: $E^A[Z_t] = E \left[ \xi_t^A Z_t \right]$

  $\xi_t = \frac{\xi_t^B}{\xi_t^A}$ is the relative probability weight

  “$\xi_t \to 0$” or “$|\ln(\xi_t)| \to \infty$” mean that along the true path of the economy, $B$ has zero probability weight relative to $A$. Separation of beliefs.
Pareto Optimality: $\frac{U'(C_A,t)}{U'(C_B,t)} = \xi_t$.

Higher probability weight means more consumption and a lower marginal utility.

How are differences in $U'(C)$ reflected in differences in $C$?

Risk aversion: $\gamma(C) = -\frac{CU'''(C)}{U''(C)}$

$$\frac{\Delta C}{\Delta \ln(U'(C))} \approx \frac{C}{\gamma(C)}$$

Pareto Optimality can be re-written as:

$$-\ln(\xi_t) = \int_{C_{B,t}}^{C_{A,t}} \frac{\gamma(x)}{x} dx$$

Differences in Beliefs = Separation in Consumption Allocations, Modified by Risk Aversion
Definition: Agent $\mathbb{B}$ survives if $\frac{C_{\mathbb{B}, t}}{D_t} \not\to 0$.

Survival:

$$\lim_{t \to \infty} \frac{\gamma(D_t)}{|\ln(\xi)|} = 0 \implies \mathbb{B} \text{ becomes extinct}$$

$$\lim_{t \to \infty} \frac{\gamma(D_t)}{|\ln(\xi)|} = \infty \implies \mathbb{B} \text{ survives with } \frac{1}{2} \text{ of total consumption}$$

Risk Preferences vs. Beliefs: Reluctance to trade versus gains from trade.

Bound $\gamma$ or bound $D$, then separation of beliefs ($|\ln(\xi_t)| \to \infty$) guarantees $\mathbb{B}$ becomes extinct. Otherwise, anything can happen...
Examples

- Look at three different economies with the same utilities and belief differences. Uncertainty in each economy is given by a Brownian Motion and belief differences about the drift are constant.

\[
\gamma(x) = x^\alpha, \quad 0 < \alpha < 1
\]

\[
\xi_t = \exp \left( -\frac{1}{2} \delta^2 t + \delta B_t \right)
\]

- Three Endowments:

  - Fast Growth: (Exponential) \( D_t = \exp (\mu t + \sigma B_t) \)
  - Medium Growth: (Approx. Polynomial) \( D_t = \left( \frac{|\ln(\xi_t)|}{\alpha^{-1} - X_t^\alpha |\ln(\xi_t)|^{-\alpha}} \right)^{1/\alpha} \)
  - Slow Growth: (Approx. Linear) \( D_t = \ln(1 + \exp (\mu t + \sigma B_t)) \)
- First step: Take Preferences and plug them into the Pareto Optimality condition

\[-\ln(\xi_t) = \int_{C_B,t}^{C_A,t} \frac{\gamma(x)}{x} dx\]

- Then, fix \(\frac{C_B,t}{D_t}\) and plot \(\xi_t\) as a function of \(D_t\).
Then, add the actual $D_t$ and $\xi_t$ results from the three economies ...
Price Impact

- Stochastic Discount Factor: $M_t = e^{-\rho t} U'(C_{A,t})$.

- Reference economy prices are for an economy where both agents have probability measure $A$: $M_t^*$.

- Definition: Agent $\mathbb{B}$ has no price impact if

$$\lim_{t \to \infty} \frac{M_{t+s}^*}{M_t^*} = 1$$

Look from $t$ to $t + s$, not from 0 to $t$. Relative Prices.
Transformation:

\[
\lim_{t \to \infty} \int_{C_{A,t+s}}^{D_{t+s}} \frac{\gamma(x)}{x} dx - \int_{C_{A,t}}^{D_t} \frac{\gamma(x)}{x} dx = 0
\]

- Changes in consumption allocations, rather than levels.
- Bound \( \gamma \) or bound \( D \) and there is no price impact (like no survival).
  Otherwise, look at

\[
\operatorname{Prob}\left[ \limsup_{t \to \infty} |\ln(\xi_{t+s}) - \ln(\xi_t)| > \epsilon \right]
\]

- Price impact without survival is generic!
  - When \( D \) is large, \( C_{B,t} \) can be non-trivial and volatile and still have \( \frac{C_{B,t}}{D_t} \to 0 \) (no survival).
  - If \( C_{B,t} \) is volatile, \( U'(C_{A,t} = D_t - C_{B,t}) \) is different from \( U'(C_{A,t} = D_t) \) (price impact).
State-Dependent Preferences

- New Utility function: \( U(C, H) \). Same differences in beliefs.

- Earlier results extend to the new setting.

  Example: \( \frac{\gamma(D_t)}{\ln(\xi_t)} \) becomes \( \frac{\gamma(D_t, H_t)}{\ln(\xi_t)} \).

- Mechanism for generating unbounded risk aversion or consumption volatility.
Conclusion

- The Market Selection Hypothesis – that markets generically favor investors with more accurate beliefs – is not true.

- We present general conditions for long term survival and price impact that can be extended to the case of state-dependent preferences.

- Survival: Separation of beliefs implies differences in consumption allocations, but shrunk by risk aversion.

  Price Impact: Volatility in consumption allocations causes changes in relative prices.

  Price impact by small players is generic.