Aggregate Uncertainty and Learning in a Search Model
Preliminary

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Introduction

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  - Get a good quickly at a low price
  - Learn about the market conditions
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eBay: How much to bid today depends on the expected price tomorrow
Introduction

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- Our model: Bidding in auctions over time with aggregate uncertainty about the state of the market
- eBay: How much to bid today depends on the expected price tomorrow
- What is the implication of learning for
  - duration (unemployment) and prices?
  - optimal information policy?
  - common vs private values?
Our Contribution

- Agents learning about the relative number of buyers to sellers (market tightness)
- Tractable model with learning about *endogenous* price distribution
- How aggressively to pursue an object given future opportunities
- We use a simple model where sellers offer auctions
- Knowledge about future opportunities is dispersed among bidders
- Some but not full similarity to standard common value auctions
Model Overview

State is $w \in \{L, H\}$ and fixed throughout. Agents’ prior is uniform. Analyzing steady state. In every period $t$:

- **Stock**: A mass $M_B^w$ of buyers, $M_S^w$ of sellers with unit demand and unit supply. Common valuations $v = 1$ and costs $c = 0$.
- **Type**: $z = (i, \theta) \in [0, 1]^2$ distributed by $\Gamma^w$
  - $i$ is a payoff irrelevant index
  - $\theta$ is the belief, $\theta = \text{prob}[w = H]$
- **Sellers matched with random number $N \in \{0, 1\ldots\}$ of buyers**
- **$N$ is unobserved and $f[N, \mu] = e^{-\mu} \mu^N / N!$, $\mu = M_B^w / M_S^w$**
- **Buyers bid in a second price auction with exogenous reserve $r$; bids not revealed**
- **Exit**: Winners leave with good. Losers leave with probability $(1 - \delta)$
- **Entry**: A mass one of sellers and a mass $d_H$ or $d_L$ of buyers
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- Add an index $i \in [0, 1]$ to all buyers with belief $\theta = \theta_0$, 
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  - The Set $Z$
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3. strictly increasing bidding function $\beta(z)$ that is optimal if the others also follow strategy $\beta$
Equilibrium Characterization

- \( \tilde{z} \) is the type after losing with bid \( \beta(z) \) with original belief \( \theta(z) \).
- Such a type has belief: \( \theta_1 = \tilde{\theta}(z, Z_{(1)} \geq z) \).
- \( \theta_1 \) is the actual belief that drives incentives tomorrow.
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- Let \( \theta_2 \) be the posterior conditional on being tied,
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Proposition: There exists an almost everywhere unique symmetric equilibrium in strictly increasing bidding strategies; payoffs \( V(\theta) \) are unique and for almost all \( z \)

\[
\beta(z) = \nu - \delta W(\theta_1, \theta_2).
\]
Intuition

- $W(\alpha, \psi)$ the value of behaving as if one has belief $\alpha$, with actual belief $\psi$
- bidder with type $z$ is indifferent between winning against someone with type $z$ or losing in a second price auction
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- but the bidder bids according to belief \( \theta_1 \) in the next auction
- Continuation value is then \( \delta W(\theta_1, \theta_2) \)
- Therefore

\[
\beta(z) = v - \delta W(\theta_1, \theta_2).
\]
Revealing Bids Hurts the Seller

- Assumed that sellers do not reveal any information about bids
- Suppose other sellers hide the bids
- Continuation values and distributions as above
- For a given seller, which one of these two auctions is better:
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Continuation values and distributions as above

For a given seller, which one of these two auctions is better:

Second price auction without any announcement or

Second price auction with revealing the winning bid

Revelation of the winning bid after the auction

but whether it is revealed is announced beforehand

so continuation values are known to be affected beforehand
Revealing Bids Hurts the Seller

We can show that it is never optimal for a seller to unilaterally reveal bids after the auction:

**Theorem**

The buyers always bid (weakly) less in the auction with bid revelation. Therefore, it never pays for a seller to reveal the winning bid in his auction unilaterally.
Because information is valuable, revealing it increases continuation values and depresses current bids.
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But the above result shows that this is not the case.

Problem: obtaining information changes future bids and thus continuation values are not fixed.
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- Because information is valuable, revealing it increases continuation values and depresses current bids.
- Seems possible to reduce the model to a static common value auction by substituting in continuation values.
- But the above result shows that this is not the case.
- Problem: obtaining information changes future bids and thus continuation values are not fixed.
- Linkage principle may thus fail in our setup.
- For this: revelation *before* the auction must be studied.
Ex-ante information revelation and the linkage principle

- Linkage principle may thus fail in our setup
- For this: information revelation *before* the auction is studied
- we have a numerical example where it is better *not* to reveal info even ex-ante
- trade-off:
  - for given continuation values info revelation increases competition (linkage effect)
  - info revelation increases continuation values depressing current bids (learning effect)
- parameter values determine which effect is stronger
Conclusion

- Stylized model of search with learning about aggregate uncertainty
- Motivating examples: e-Bay auctions or labor markets
- Provide an existence proof relying on new techniques
- Relate our model to cv auctions
- If a common value element enters through dynamic interaction, a static common value auction may lead to misleading intuition