Emerging Markets in an Anxious Global Economy.*

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Abstract

We provide a theory of pricing for emerging asset classes, like emerging markets, that are not yet mature enough to be attractive to the general public. Our model provides an explanation for the volatile access of emerging economies to international financial markets and for several stylized facts we identify in the data during the 1990’s. We present a general equilibrium model with incomplete markets and endogenous collateral and an extension encompassing adverse selection. We show that contagion, flight to liquidity and issuance rationing can occur in equilibrium during what we call global anxious times.

Keywords: Margin, leverage cycle, liquidity preference, collateral value, informational volatility, contagion, portfolio effect, flight to liq-

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*This paper is based on an essay in the 2005 Yale dissertation of the first author, written under the supervision of the second author. The empirical regularities that motivate the paper, most of the simulations, and the two fundamental models of this paper originated with the first author and appear in the dissertation. Part of the conceptual framework and a number of the analytical motifs of this paper can be found in the published and unpublished work of the second author.

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1 Introduction

Since the 1990's emerging markets have become increasingly integrated into global financial markets, becoming an asset class. However, contrary to what was widely predicted by policy makers and economic theorists, these changes have not translated into better consumption smoothing opportunities for emerging economies. The access to international markets itself has turned out to be very volatile and, even worse, emerging economies with sound fundamentals are the ones who seem to suffer more during periods of low debt issuance. We suggest that this disheartening picture is a symptom of an unfinished integration into global financial markets.

The goal of this paper is to present a theory of asset pricing that will shed light on the problems of “emerging” assets (like emerging markets) that are not yet mature enough to be attractive to the general public. Their marginal buyers are liquidity constrained investors with small wealth relative to the whole economy, who are also marginal buyers of other risky assets. We will use our theory to argue that the periodic problems faced by “emerging” asset classes are sometimes symptoms of what we call a global anxious economy rather than of their own fundamental weaknesses.

We distinguish three different conditions of financial markets: (i) the normal economy, when leverage is high but the liquidity preference is low; (ii) the anxious economy when leverage is curtailed and the liquidity preference is high, and the general public is anxiously selling risky assets to more confident natural buyers; and (iii) the crisis or panicked economy when many formerly leveraged natural buyers are forced to liquidate or sell-off their positions to a reluctant public, often going bankrupt in the process. A recent but growing literature on leverage and financial markets has concentrated on crises or panicked economies. We concentrate on the anxious economy (a much more frequent phenomenon) and provide an explanation with testable implications for (i) contagion, (ii) flight to liquidity and (iii) differential issuance rationing.
Our theory provides a rationale for the stylized facts present in emerging markets, and perhaps also explains some price behavior of other “emerging” asset classes like the US sub-prime mortgage market.

In Section 2 we look at issuance and spread behavior of emerging market and high yield bonds during the six year period 1997-2002, which includes the fixed income liquidity crisis of 1997-98. This crisis lasted for a few months, or about 4% of the sample period. Our estimates show, however, that during 20% of this period, primary markets for emerging market bonds were closed. Traditionally, periods of abnormally low access have been explained by showing that weak emerging market fundamentals were responsible (stressing the demand of funds side). This paper will argue that closures are often a symptom of an anxious global economy. We will provide a theory for how shocks in other globally traded sectors like high yield can be transmitted to emerging markets even during less dramatic times than crises like the one in 1998. Recent empirical evidence also points to the supply side of funds (see Calvo et al. (2004) and Fostel and Kaminsky (2007)).

We describe three stylized facts present in the data. First, emerging market and high yield bonds show spread correlation (of 33% on average) even though their payoffs would seem to be uncorrelated. In particular, during emerging market closures there is an increase in spreads and volatility for both assets. Second, although emerging market spreads increase during closures, the behavior across the credit spectrum within the asset class is not the same: high-rated emerging market spreads increase less than low-rated emerging market spreads. Third, during closures the drop in issuance is not uniform either: high-rated emerging market issuance drops more than low-rated emerging market issuance. Issuance from emerging countries with sound fundamentals suffers more, even though high rated spreads change much less.

The starting points for our analysis are Geanakoplos (2003) and Fostel (2005). The first paper described what we now call the leverage cycle. Bad news not only reduces the value of assets, but it also gives rise to expectations of high volatility, which leads forward looking lenders to set higher margins, which contracts buying and thus causes more price declines. In normal times the endogenous equilibrium leverage is too high, in crises times equilibrium leverage is too low. The second paper extended the leverage cycle to an
economy with multiple assets and introduced what we now call the *anxious economy*.

In section 3 we introduce our notion of the *anxious economy*. This is the state when bad news lowers expected payoffs somewhere in the global economy (say in high yield), increases the expected volatility of ultimate high yield payoffs, and creates more disagreement about high yield, but gives no information about emerging market payoffs. A critical element of our story is that bad news not only increases uncertainty, it also increases heterogeneity. When the probability of default is low there cannot be much difference in opinion. Bad news raises the probability of default and also the scope for disagreement. Investors who were relatively more pessimistic before become much more pessimistic afterward. One might think of the anxious economy as a stage that is frequently attained after bad news, and that occasionally devolves into a sell-off if the news grows much worse, but which often (indeed usually) reverts to normal times. After a wave of bad news that lowers prices, investors must decide whether to cut their losses and sell, or to invest more at bargain prices. This choice is sometimes described on Wall Street as whether or not to catch a falling knife.

For simplicity we suppose agents are divided into a small group of optimists, representing the natural buyers of the assets, and a large group of pessimists, representing the general public. Both groups are completely rational, forward looking, expected utility maximizers, but with different priors. Heterogeneity is important because it means that the valuation of an asset depends critically on what a potentially small segment of the economy thinks of it. Even if the asset is small relative to the size of the whole economy, it might be significant relative to the wealth of the segment of the population most inclined to hold it. If markets were complete, then in equilibrium everyone on the margin would be equally inclined to hold every asset. But with incomplete markets it may well happen that assets are entirely held by small segments of the population.

In this context, the first question that the model tries to solve is the following: If the bad news only affects one sector, say high yield, will asset prices in sectors with independent payoffs like emerging markets be affected? This is not only a pressing problem for emerging markets. In 2007 the sub-prime mortgage market may suffer losses on the order of $250 billion, which
is tiny compared to the whole economy. Could this have a big effect on other asset prices? In other words, is contagion possible in equilibrium?

We show in Section 3 that when the economy is reducible to a representative agent, the answer is no. We also show that if the economy has heterogeneous investors but complete markets, and if optimists’ wealth is small relative to the whole economy, then the answer is still no.

At the end of section 3, we show that in an economy with heterogeneous investors and incomplete markets (that limit borrowing), it is possible to get contagion without leverage. In the anxious economy emerging market bonds will fall in value in tandem with the high yield bonds, even though there is no new information about them. This fall derives from a portfolio effect and a consumption effect. The consumption effect arises when consumption goes down and marginal utility of consumption today goes up, lowering the relative marginal utility of all assets promising future payoffs. The portfolio effect refers to the differential dependence of portfolio holdings on news. After the bad news, the pessimistic investors abandon high yield, and the optimists take advantage of the lower prices to increase their investments in high yield. When the optimists increase their investment in high yield, they must withdraw money from somewhere else, like emerging markets and consumption. This causes the price of emerging market bonds to fall. For the fall to be big, it is important that optimists were substantial holders of emerging market bonds, that the pessimists will not easily replace them without a substantial price inducement, and that the pessimists are willing to purchase high yield bonds, at least after good news.

This theoretical mechanism is compatible with the recent evolution of the emerging market investor base. Emerging market bonds are still not a mature enough asset class to become attractive to the general public (the pessimists), and at the same time the marginal buyers of these assets are crossover investors willing to move to other asset classes like high yield. The proportion of crossover investors was negligible before 1997 but by 2002 accounted for more than 40% of the investor base.

A popular story is that leverage (say in high yield) causes bigger losses after bad news, which causes leveraged investors to sell other assets (like emerging markets), which causes contagion. This story implicitly relies on incomplete markets (otherwise leverage is irrelevant) and on heterogeneous
agents (since there must be borrowers and lenders to have leverage). The popular story is a sell-off story during panicked economies. The most optimistic buyers are forced to sell off their high yield assets, and more assets besides, holding less of high yield after the bad news than before. Asset trades in the anxious stage thus move in exactly the opposite direction from the crisis stage. In the anxious economy it is the public that is selling in the bad news sector, and the most optimistic investors who are buying.

In the popular story there are usually defaults and bankruptcies (since the high yield holdings were not enough to meet margin calls). But these events are rare, happening once or twice a decade. Our data describes events with ten to twenty times the frequency, happening roughly twice a year. To explain our data on emerging market closures we tell a story that places liquidity and leverage on center stage, but which does not have the extreme behavior of the sell-off. We describe an anxious economy, not a panicked economy.

To study the role of leverage in contagion we introduce our model of general equilibrium with endogenous collateral in section 4. In the anxious economy, we just saw that the optimists have more opportunities to spend money, which increases their need for liquidity. Our model introduces the liquidity preference to quantify their need: it is the amount they would be willing to pay above the riskless interest rate in order to borrow more money even if they knew they were going to pay back all they owed for sure. In the anxious stage their liquidity preference is high. The model also introduces a pricing lemma which shows that the price of an asset can always be decomposed as the sum of its payoff value and its collateral value to any agent who holds it. Ownership of an asset not only gives the holder the right to receive future payments (reflected in the payoff value) but also enables the holder to use it as collateral to borrow more money. We define the collateral value of an asset to any agent as the product of his liquidity preference and the asset’s collateral capacity, that is the amount of money that can be borrowed using the asset as collateral.

Every asset’s collateral capacity is determined in equilibrium by endogenous margins requirements. We find that margins endogenously rise between the normal state in which the economy begins, and the anxious stage reached after bad news. Together with the portfolio and consumption effects, this creates a higher liquidity preference. The increase in liquidity preference in the
anxious economy tends to raise the collateral value of assets, and thus might work against the contagion. Indeed, we find that in contrast to the crisis economy, leverage makes asset prices higher in the anxious economy than they would have been without leverage. Nevertheless, prices fall more with leverage; not because leverage leads to asset under-valuation in the anxious economy after bad news (as in the crisis economy), but because leverage leads to asset over-valuation in the normal economy before bad news comes. Thus our model rationalizes the contagion of Stylized Fact 1, and the role of leverage, but through a mechanism different from the usual sell-off story characteristic of panicked economies.

The second question is: Why isn’t the fall in prices of emerging market bonds uniform? In the anxious economy asset prices generally fall, but assets with higher collateral capacities, and thus higher collateral values (or lower margins), fall less. We call this phenomenon flight to liquidity since in our model we define the liquidity of an asset by its collateral capacity. During flight to liquidity investors with heightened liquidity preference prefer to buy assets that enable them to borrow money more easily (lower margins). The other side of the coin is that investors choose to sell first those assets on which they cannot borrow money (higher margins); this raises the most cash since the sales revenue net of loan repayment is higher. We might thus equally call this “flight from illiquidity.” Moreover, the model provides the following testable implication. We show that even when two assets have the same information volatility, margins during normal times will be different and can predict which assets are the ones that will suffer more during future flight to liquidity episodes. Traditionally the deterioration in price of low quality assets is explained in terms of “flight to quality” which in our model corresponds to movements in payoff values. We show the presence of a different and complementary channel originating almost exclusively from liquidity considerations. Our second result rationalizes Stylized Fact 2 since low-rated emerging market bonds exhibit higher margins than high-rated emerging market bonds.

Finally, the third question the paper aims to answer is why the fall in issuance during closures in not uniform. To address this, section 5 extends our first model to encompass the supply of emerging market assets as well as asymmetric information between countries and investors. The departing point here is Dubey and Geanakoplos (2002), which developed techniques to
incorporate adverse selection-signalling into a general equilibrium model. In our paper we extend our general equilibrium model with incomplete markets and collateral with the same goal. We show that flight to liquidity combined with asymmetric information between investors and countries lead to differential issuance rationing. "Good" type country assets are better collateral than "bad" type country assets. During episodes of global anxiety and high liquidity preference, the price differential between asset types increases. When investors cannot perfectly observe these types only a drastic drop in good quality issuance removes the incentive of bad types to mimic good types, maintaining the separating equilibrium. In a world with no informational noise, spillovers from other markets and flight to liquidity may even help "good" issuance. However, with some degree of informational noise between countries and investors, good quality assets suffer more. Our third result rationalizes stylized Fact 3.

The first result in the paper is related to a big literature on contagion. Despite the range of different approaches there are mainly three different kinds of models. The first kind blends financial theories with macroeconomic techniques, and seeks for international transmission channels associated with macroeconomic variables. Examples of this approach are Goldfajn and Valdes (1997), Agenor and Aizenman (1998), Corsetti, Pesenti and Roubini (1999) and Pavlova and Rigobon (2006). The second kind models contagion as information transmission. In this case the fundamentals of assets are assumed to be correlated. When one asset declines in price because of noise trading, rational traders reduce the prices of all assets since they are unable to distinguish declines due to fundamentals from declines due to noise trading. Examples of this approach are King and Wadhwani (1990), Calvo (1999), Calvo and Mendoza (2000), Cipriani and Guarino (2001) and Kodres and Pritsker (2002). Finally, there are theories that model contagion through wealth effects as in Kyle and Xiong (2001). When some key financial actors suffer losses they liquidate positions in several markets, and hence this sell-off generates price co-movement. The last two approaches have in common a focus exclusively on contagion as a financial market phenomenon, abstracting from macroeconomic variables, as does our paper. Our explanation is complementary with all those studies.

Our second result is related to a big literature on liquidity. Under supply of liquidity and liquidity crises were studied in several papers like Geanako-
plos (1997), Holmstrom and Tirole (1998), Caballero and Krishnamurthy (2001), Morris and Shin (2004) and Fostel and Geanakoplos (2004). Flight to Liquidity was modeled by Vayanos (2004) who gets it in a model where an asset’s liquidity is defined by its exogenously given transaction cost. Brunnermeier and Pedersen (2007) model flight to liquidity in the tradition of modeling liquidity in Grossman and Miller (1988). In their paper, market liquidity is the gap between fundamental value and the transaction price and they show how this market liquidity interacts with funding liquidity (given by trader’s capital and margin requirements). In our paper we model an asset’s liquidity as its capacity as collateral to raise cash. Hence, our liquidity preference arises from endogenously determined time varying margin requirements in equilibrium.

The third result is related to the tradition of credit rationing as in Stiglitz and Weiss (1981). Also, to an increasing literature that tries to model asymmetric information within general equilibrium like Gale (1992), Bisin and Gottardi (2006) and Rustichini and Siconolfi (2007). We treat it in a framework of perfect competition following the techniques of Dubey-Geanakoplos (2002) through pooling of promises. The result is also related to several papers in the sovereign debt literature that have worked under the assumption of asymmetric information between investors and countries, as in Alfaro and Kanuczuk (2005) and Catão, Fostel and Kapur (2007).

Finally, our model is related to a vast literature that explains financial crises, sudden stops, and lack of market access in emerging market economies. The sovereign debt literature as in Bulow-Rogoff (1989), stresses moral hazard and reputation issues. The three “generations” of models of currency crises explain reversals in capital flows by pointing to fiscal and monetary causes as in Krugman (1979), to unemployment and overall loss of competitiveness as in Obstfeld (1994) and to banking fragility and overall excesses in financial markets as in Kaminsky and Reinhart (1999) and Chang and Velasco (2001). Others explore the role of credit frictions to explain sudden stops as in Calvo (1998) and Mendoza (2004). Others focus on balance sheet effects as in Krugman (1999), Aghion et al.(2004), Schneider and Tornell (2004), and finally on the interaction of financial and goods markets as in Martin and Rey (2006) to mention a few.
2 Stylized Facts

Following Fostel (2005) we look at Emerging Market issuance of dollar-denominated sovereign bonds covering the period 1997-2002. The data we use is obtained by Dealogic, which compiles daily information on issuance at the security level. We define a Primary Market Closure\(^1\) as a period of 3 consecutive weeks or more during which the weekly primary issuance over all Emerging Markets is less than 40 percent of the period’s trend. As shown in table 1, market closures are not rare events. During this period, there were 13 market closures which implies that 20.29% of the time primary markets of emerging market bonds were closed. Finally, while some of the closures seemed associated with events in emerging countries, others seemed to correspond with events in mature economies.

During the same period, we look at the secondary markets of Emerging Markets and US High Yield bonds. We use daily data on spreads from the JPMorgan index EMBI+ for Emerging Markets and the Merrill Lynch index for US High Yield. Data for Emerging Market spreads disaggregated by credit ratings is available at weekly frequency from Merrill Lynch indexes.\(^2\) We will describe now three stylized facts present in the data during this period.

**Fact 1: Emerging Market and US High Yield Spread Correlation**

Emerging Markets and US High Yield exhibit a positive spread correlation, and in particular around closures both exhibit increasing spread and volatility behavior.

The average correlation during the period is .33. Figure 1 shows average spread behavior for both assets from 20 days before to 20 days after the beginning of a typical closure. The increasing spreads around closures is also true for 20-day rolling volatility as shown in figure 2. This increasing pattern is robust across all closures in the sample and to different rolling windows specifications.

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\(^1\)We follow IMF (2003).

\(^2\)Although spreads at issuance, which reflect the actual cost of capital, may be the most relevant for the issuer, portfolio managers arguably follow spreads in secondary markets more closely. Also, these spreads available at higher frequency may reflect subtle changes in global investing conditions more accurately than lower frequency data.
Fact 2: Credit Rating and Emerging Market Spreads

Although Emerging Market spreads increase around market closures, the behavior across the credit spectrum within the asset class is not uniform: high-rated Emerging Market spreads increase less than low-rated Emerging Market spreads. By low-rated we mean all sub-investment grade bonds, i.e. everything below or equal BB.

Figure 3 shows the average weekly percent change in spreads around closures for different Emerging Markets ratings. On average low-rated spreads increase more than high-rated spreads, and this behavior is robust across closures as well.

Fact 3: Credit Rating and Emerging Markets Primary Issuance

During primary market closures the drop in issuance is not uniform across the credit spectrum: high-rated Emerging Markets issuance drops more than low-rated Emerging Markets issuance. While high-rated issuance accounts for 23% during normal times, it only accounts for 12% during closures. Hence during crises, emerging market economies with sound fundamentals seem to suffer more (issue less). One may argue that we should expect this behavior since precisely those good fundamentals allow countries to look for alternative sources of financing during bad times. However, this drastic reduction in issuance is puzzling when considered jointly with the behavior in spreads described before: high-rated issuance decreases more than low-rated issuance despite the fact that high-rated spreads increase less than low-rated spreads.

Finally, given the ad-hoc nature of the definition of market closures, we conduct a robustness check for different thresholds and trend specifications. Of course, the number of closures varies, but it never becomes less than 10 or more than 14. And more importantly, all three stylized facts are still remarkably robust to all these different specifications.\textsuperscript{3}

\textsuperscript{3}Results are available from the authors upon request.
3 The Problem

3.1 The Anxious Economy

We introduce the theoretical problem motivated by the empirical section through a simple example described in figure 4. Consider a world with four instruments: a single consumption good, a high yield asset $H$, and two emerging market assets $E$ of differing quality, $E^G$ and $E^B$ (good and bad type of emerging markets). Asset payoffs are denominated in units of the single consumption good. These payoffs come in the terminal nodes, and are uncertain.

Agents have riskless initial endowments $e$ of the consumption good at each node. While agents are endowed with $H$, they need to buy $E^G$ and $E^B$ from emerging countries, which at each state enter the market and decide their issuance.

We shall suppose that news arrives at $D$ which makes everyone believe that $H$ is less likely to be productive, but which gives no information about $E^G$ and $E^B$. After $U$, (which occurs with probability $q$) the output of $H$ is 1 for sure, but after $D$ the output of $H$ can be either 1, with probability $q$, or $H < 1$, with probability $1 - q$. The output of $E^G$ ($E^B$) is either 1, with probability $q$, or $G$ ($B$), with probability $1 - q$, irrespective of whether $U$ or $D$ is reached and independent from the output of $H$. $H, G$ and $B$ can be interpreted as recovery values in the case of asset default and are such that $H < 1, B < G < 1$.

We call state $D$ the anxious economy. This is the state when bad news lowers expected payoffs in high yield (our proxy for the global economy), increases the expected volatility of ultimate high yield payoffs, and creates more disagreement about high yield, but gives no information about emerging market payoffs. This stands in sharp contrast with traditional financial models, where asset values are modeled by Brownian motions with constant volatilities. At $U$ the uncertainty about $H$ is resolved, but at $D$ it becomes greater than ever. State $D$ will not turn out be a crisis situation because agents get a new infusion of endowments $e$. 
In discussing asset price changes we must keep in mind how much news is arriving about payoff values. We would expect asset prices to be more volatile if there is a lot of news about their own payoff, and to be less volatile or even flat if there is no news. In our setup there is an acceleration of news over time, and eventually more news about \( E^B \) than about \( E^G \). There are situations when this kind of uncertainty is natural, for example, if everyone can see that a day is approaching when some basic uncertainty is going to be resolved.\(^4\)

To be precise, for each asset \( A \) and each node \( s \), let us define \( E_s(A) \) as the expected terminal delivery of \( A \) conditional on having reached \( s \). Similarly, define the *informational volatility* at \( s \), \( V_s(A) \), as the standard deviation of \( E_\alpha(A) \) over all immediate successors \( \alpha \) of \( s \). Then \( E_1(E^G) = q1 + (1-q)G = E_U(E^G) = E_D(E^G) \). Thus \( V_1(E^G) = 0 \). There is no information about the payoffs of \( E^G \) between periods 1 and 2. Similarly \( E_1(E^B) = q1 + (1-q)B = E_U(E^B) = E_D(E^B) \). Thus \( V_1(E^B) = 0 \). However, \( 0 < V_U(E^G) = V_D(E^G) < V_U(E^B) = V_D(E^B) \).

Naturally the price of \( H \) falls from 1 to \( D \) and is lower at \( D \) than at \( U \) since the bad news lowers its expected payoff. However, the expected payoff of \( E^G \) (and \( E^B \)) is exactly the same at \( U \) and at \( D \), as is its information volatility.

1. Why should the prices of \( E^G \) and \( E^B \) fall from 1 to \( D \) and be lower at \( D \) than at \( U \) (even without a shock to them)? We will refer to this problem as *Contagion*.

2. Why should the price of \( E^B \) fall more than the price of \( E^G \) from 1 to \( D \)? And why the gap in prices between \( U \) and \( D \) should be bigger for \( E^B \) than for \( E^G \)? We will refer to this problem as *Differential Contagion*. Moreover, is there a market signal at time 1 that can predict which asset will perform worse at \( D \)?

3. Why should emerging market issuance fall from 1 to \( D \), but more importantly, why should the issuance of \( E^G \) fall more than the issuance

\(^4\) At the present time everyone can see that a year from now subprime mortgages from the bad 2006 vintage will reset and then it will be revealed how bad the defaults will be.
of $E^K$? And why the gap in issuance between $U$ and $D$ should be bigger for $E^G$ than for $E^K$? We will refer to this problem as Differential Issuance Rationing.\footnote{Though what we see in the data corresponds to movements from 1 to $D$, from a theoretical point of view it makes sense to compare with the counterfactual state $U$ as well.}

Answers to problems 1, 2 and 3 will help rationalize stylized facts 1, 2 and 3 respectively. The first model in section 4 will focus on contagion and differential contagion while the second model in section 5 will focus on issuance rationing. Hence, until section 5 we will assume a fixed supply of emerging market assets. Before introducing the first model, let us go back to our example and attempt to gain intuition about what is involved in solving the first two problems within standard models.

### 3.2 Representative Agent

For a moment, let us abstract from different types of emerging market assets and consider only two assets, $E$ (Emerging Market) and $H$ (High Yield), with independent payoffs as discussed before.\footnote{Equivalently, assume that $G = B$, so there is no difference between emerging market assets.} Intuitively, since $E$ and $H$ are independent assets, one would expect uncorrelated price behavior in equilibrium. And in fact, this intuition is correct in certain cases as we will discuss now.

Consider an economy with a representative investor with logarithmic utility who does not discount the future. Simulation 1 calculates equilibrium prices using the following parameter values\footnote{Sections 3.4 and 4.2.1 will extensively discuss the choice of parameter values.}: the recovery values are $E = .1$ and $H = .2$, initial endowments are $e = 2020$ in every node, beliefs are given by $q = .9$ and finally the agent is endowed with 2 units of each asset at the beginning. The first part of table 2 shows that the price of $H$ falls at $D$ since its expected output is lower. But the equilibrium price of $E$ is slightly higher at $D$ than at $U$, so $E$ and $H$ are actually slightly negatively correlated. There is no contagion. The reason for this is very simple: at $D$ future consumption is lower than at $U$ since $H$ is less productive, so the marginal utility for future output like from $E$ is slightly higher.
3.3 Heterogeneous Agents and Complete Markets

Let us extend the previous model to allow for heterogeneous agents. Agents will differ in beliefs and wealth. There are “optimists” who assign probability \( q^O = 0.9 \) and “pessimists” who assign probability \( q^P = 0.5 \) to good news about \( H \) and \( E \). Because of their beliefs, optimists have a higher opinion at 1 about \( H \) than pessimists do. While optimists think \( H \) will pay fully with probability \( 1 - (1 - q^O)^2 = 0.99 \), pessimists only attach probability \( 1 - (1 - q^P)^2 = 0.75 \) to the same event. At \( D \) their opinions about \( H \) fully paying diverge even more, \( q^O = 0.9 > q^P = 0.5 \). This growing dispersion of beliefs after bad news is not universal, but is plausible in some cases and will be important to our results.

Initial endowments are \( e^O = 20 \) and \( e^P = 2000 \) for optimists and pessimists respectively in all states. Each type of investor owns 1 unit of each asset at the beginning. The rest of the parameters are as in simulation 1.

Markets are complete in the sense that all Arrow securities are present, and hence \( E \) and \( H \) can be expressed as linear combination of those.

The second part of table 2 shows that prices exhibit only a tiny degree of contagion. The reason for any contagion is that with complete markets, agents are able to transfer wealth to the states which they think are relatively more likely. Therefore, at \( D \) prices reflect more the pessimist preferences (and hence may be slightly lower than at \( U \)). However, as we make pessimists richer and richer, this type will become close to a representative agent and all prices will reflect his preferences. In the limit contagion will disappear as shown by simulation 1. We will see that with incomplete markets, making pessimists richer will not kill contagion; in fact it makes contagion worse.

3.4 Incomplete Markets and Heterogeneous Agents

3.4.1 Contagion, Portfolio Effect and Consumption Effect

Simulations 1 and 2 show that contagion without correlated fundamentals is not a general phenomenon. The first example illustrates the need for some kind of agent heterogeneity while the second highlights the need for market incompleteness. In the next example we will assume both. Agents
are heterogeneous. As before, they differ in beliefs and endowments which are given by $q^0 = .9, q^P = .5, e^O = 20$ and $e^P = 2000$ respectively. Each type of investor starts with 1 unit of each asset $E$ and $H$ at the beginning and trades these assets thereafter.

But now markets are assumed to be incomplete. Agents can only trade the physical assets $E$ and $H$, and the consumption good. Arrow securities are assumed not present and agents are not allowed to borrow. Given that $D$ is followed by four states, two assets are not enough to complete markets. But even at 1 markets are incomplete due to the presence of short sales constraints.\footnote{Markets are incomplete means there is a node at which agents, at equilibrium prices, cannot create all the Arrow securities that span the dimension of the set of successors states.}

Let us take a moment to discuss parameter values before presenting simulation 3. As before, we assume that $H$’s recovery value is bigger than $E$’s, in particular $H = .2$, $E = .1$. This constitutes a realistic assumption since in general the recovery value from a domestic firm is bigger than the one from foreign countries due to the absence of international bankruptcy courts. As above, investors have logarithmic utilities and do not discount the future. We think of optimists as the class of investors who find emerging markets an attractive asset class, whereas pessimists are thought of as the “normal” public who invest in the US stock market. While the market for emerging markets bonds accounted for approximately 200 billion dollars, the US stock market accounted for approximately 20 trillion dollars by the end of 2002. Hence we have given pessimists 100 times the wealth of optimists.

Results for simulation 3 are shown in tables 3, 4 and 5. Prices for $E$ and $H$ rise at $U$ and fall at $D$, displaying contagion. Along the path from 1 to $D$ of bad news about $H$, the price of $H$ naturally falls, declining 19% from .9 to .74. The price of $E$ falls as well from 1 to $D$, even when there was no specific bad shock to it. It goes from .8 to .73, a decline of 8.6%. The difference in prices between $U$ and $D$ for $H$ is 26.25% and for $E$ is 15.7%.

Why does $E$ fall in price in the anxious economy? First, because of a portfolio effect. Second, because of a consumption effect.
What is crucial in the portfolio effect is that optimists hold more of $H$ after bad news than after good news about $H$. At $U$ news are so good that both types agree about $H$ and optimists end up holding none of it. However, at $D$, when asset volatility has gone up, the difference in opinion increases, so optimists see a special opportunity and end up holding all $H$. Given constant wealth, they have relatively less wealth to spend on $E$ and on consumption. The reduction in the demand for $E$ naturally lowers its price. Equivalently, the portfolio effect generates a consumption effect: consumption goes down (by 9%) and marginal utility goes up from $U$ to $D$, reducing the marginal utility of $E$ relative to consumption. Thus, the price of $E$ mimics the price of $H$. Since the price at 1 is an average of the prices at $U$ and $D$, the portfolio effect also implies that the price falls from 1 to $D$.

The portfolio and consumption effects also explain why the fall of 26.25% in the price of $H$ from $U$ to $D$ is bigger than the fall in its expected payoff of 8%.

Investor heterogeneity and market incompleteness are what generate the portfolio and consumption effects; without them contagion may well disappear. Heterogeneous beliefs make emerging market assets less attractive to the “normal public”, modeled here as pessimists, but extremely attractive to another class of investors, modeled here as optimists. Contagion arises when these optimistic investors become “crossover” investors, spending part of their capital on high yield bonds. This portfolio effect is in line with important changes that have taken place in the investor base for emerging market assets in recent years. In particular, the proportion of crossover investors was negligible before 1997 but by 2002 accounted for more than 40%. The correlation between emerging markets and US high yield spreads was negligible before 1997 and was on average 33% in the period 1997 to 2002 covered in this paper.

On the other hand, while leveraged investors such as hedge-funds accounted for 30% of all activity in emerging markets in 1998, this share declined to 10% by 2002. The impact of hedge funds, through their leveraged positions, on contagion has received substantial attention in both academic and official communities and there is an agreement that this decline has contributed to an easing of contagion and volatility more recently.

\footnote{See IMF (2003).}
Simulation 3 shows that portfolio and consumption effects are enough to generate contagion without leverage (although the fall in $E$ from 1 to $D$ was only half of the fall from $U$ to $D$). Since it is usual to associate contagion with leverage, we will introduce collateral, and hence leverage, in section 4. It will turn out that leverage will reduce contagion as measured by a fall from $U$ to $D$, but it will generate a bigger price crash from 1 to $D$.

3.4.2 Differential Contagion

Consider our example with 3 assets, $H$, $E^G$ and $E^B$, $B < G$. Are the portfolio and consumption effects enough to generate not only contagion but differential contagion across emerging market assets of differing quality in the anxious economy?

Simulation 4 calculates the equilibrium for the same parameter values as before except the recovery values which now are $H = .2, G = .2$ and $B = .05$ (the emerging market asset $E$ with recovery value $1$ is replaced by a good emerging market asset with higher recovery value, $2$, and a bad emerging market asset with a lower recovery value, $0.05$). Tables 6, 7 and 8 present the results. As in simulation 3, the portfolio and consumption effects generate contagion. However, assets of different quality get hit in the same way creating an homogeneous fall in prices. Therefore, simulation 4 shows the need of something more to solve the second problem of differential contagion than agent heterogeneity and market incompleteness. The model developed in section 4 will provide a framework to attack both problems of contagion and differential contagion.

4 Model I: Collateral General Equilibrium

So far we have not allowed agents to borrow; they were very limited in how much they could spend on buying what they thought were underpriced assets. Letting the agents use assets as collateral to borrow money enables them to take more extreme positions, which will have important consequences for asset pricing.

Standard General Equilibrium theory with incomplete markets does not include collateral. We present a model of collateral equilibrium adapted from
Geanakoplos (1997), Geanakoplos and Zame (1998) and Geanakoplos (2003). Though our model is not as general, it enables us to address our three questions by including two critical features from the more general theory. First, agents are never required to deliver more than the value of their collateral and second, collateral levels needed to back a given promise are endogenously determined in equilibrium.

4.1 The Model

4.1.1 Time and Uncertainty

The model is a finite-horizon general equilibrium model, with $t = 1, \cdots, T$. Uncertainty is represented by a tree of date-events or states $s \in S$, including a root $s = 1$. Each state $s \neq 1$ has an immediate predecessor $s^*$, and each non-terminal node $s \in S \setminus S_T$ has a set $S(s)$ of immediate successors. Each successor $\tau \in S(s)$ is reached from $s$ via a branch $\sigma \in B(s)$; we write $\tau = s\sigma$. We denote the time of $s$ by the number of nodes $t(s)$ on the path from 1 to $s$. For instance, in our example in figure 4 we have that the immediate predecessor of $UU$ is $UU^* = U$. The set of immediate successors of $U$ is $S(U) = \{UU, UD\}$. Each of these successors is reached from $U$ via a branch in the set $B(U) = \{U, D\}$. Finally, the time of $U$ is $t(U) = 2$.

4.1.2 Assets and Collateral

A financial contract $k$ consists of both a promise and collateral backing it, so it is described by a pair $(A_k, C_k)$. Collateral consists of durable goods, which will be called assets. The lender has the right to seize as much of the collateral as will make him whole once the loan comes, due but no more.

This paper will focus on a special type of contract. In each state $s$ its promise is given by $\phi_s \cdot \bar{1}$, where $\bar{1} \in R^{S(s)}$ stands for the vector of ones with dimension equal the number of successors of $s$. The contract $(\phi_s \cdot \bar{1}, C)$ promises $\phi_s$ units of consumption good in each successor state and is backed by collateral $C$. If the collateral is big enough to avoid default, the price of this special contract is given by $\phi_s/(1 + r_s)$, where $r_s$ is the riskless interest rate. Now, let us be more precise about how the collateral levels are determined.
There is a single consumption good \( x \in \mathbb{R}_+ \). Each asset \( j \in J \) delivers a dividend of the good \( D_{sj} \) in each state \( s \in S \). The set of assets \( J \) is divided into those assets \( j \in J^c \) that can be used as collateral and those assets \( j \in J \setminus J^c \) that cannot. We shall assume that households are only allowed to issue at each state a non-contingent promise backed by collateral so large that payment is guaranteed. Thus, holding one unit of collateralizable asset \( j \in J^c \) in state \( s \) permits an agent to issue \( \phi_s \) promises to deliver one unit of the consumption good in each immediate successor state \( t \in S(s) \), such that

\[
\phi_s \leq \min_{t \in S(s)} [p^t_j + D_{tj}]
\]  

The maximum promise backed by one unit of \( j \) is given by its minimum yield (its price plus the deliveries) in the immediate future states, ruling out the possibility of default in equilibrium. \(^{11}\) Notice that the collateral capacity of an asset \( j \) at \( s \) is endogenous, depending on the equilibrium prices \( p^t_i, t \in S(s) \). \(^{12}\)

Now we are in the position to define one of the key concepts in the paper. Buying 1 unit of \( j \) on margin at state \( s \) means: selling a promise of \( \min_{t \in S(s)} [p^t_j + D_{tj}] \) using that unit of \( j \) as collateral, and paying \( (p^t_s - \frac{1}{1+r_s} \cdot \min_{t \in S(s)} [p^t_j + D_{tj}] ) \) in cash. The margin of \( j \) at \( s \) is,

\[
m^j_s = \frac{p^t_s - \frac{1}{1+r_s} \cdot \min_{t \in S(s)} [p^t_j + D_{tj}]}{p^t_j}
\]

The margin is given by the current asset price net of the amount borrowed using the asset as collateral, as a proportion of the price, i.e., the cash requirement needed to buy the asset today as a proportion of its price. We will denote as leverage the inverse of the margin. The margin requirement is not only endogenous but also a forward looking variable; it depends on the current price and on how the asset is going to be priced in the future. These

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\(^{10}\) Considering a single consumption good greatly simplifies notation without loss of generality since the focus here will be primarily on asset prices.

\(^{11}\) This will not represent a problem, in fact it will make the argument stronger: even in the absence of default, there will be inefficiencies in international financial markets.

\(^{12}\) Geanakoplos (2003) showed that with heterogeneous priors and two successors states, even if agents were allowed to use \( j \) to collateralize any promise of the form \( \lambda \tilde{\lambda} \), they would never choose \( \lambda > \min_{t \in S(s)} [p^t_j + D_{tj}] \).
facts will be of great importance, in particular, they will have a big effect on asset pricing as discussed below.

4.1.3 Investors

Each agent \( i \in I \) is characterized by a utility, \( u^i \), a discounting factor, \( \delta^i \) and subjective probabilities, \( q^i \). We assume that the Bernoulli utility function for consumption in each state \( s \in S \), \( u^i : R_+ \rightarrow R \) is continuous, concave and monotonic. Agent \( i \) assigns subjective probability \( q^i_s \) to the transition from \( s^* \) to \( s \). (Naturally \( q^i_1 = 1 \)). Letting \( \bar{q}^i_s \) be the product of all \( q^i_t \) along the path from 1 to \( s \), the von-Neumann-Morgenstern expected utility to agent \( i \) is

\[
U^i = \sum_{s \in S} \bar{q}^i_s \delta^i_t u^i(x_s) \tag{3}
\]

Each investor \( i \) begins with an endowment of the consumption good \( e^i_s \in R_+ \) in each state \( s \in S \), and an endowment of assets at the beginning \( y^i_{1s} \in R^J_+ \). We assume that all assets and the consumption good are present, \( \sum_{i \in I} y^i_{1s} >> 0 \) and \( \sum_{i \in I} e^i_s > 0, \forall s \in S \). Given prices \( ((p_s, r_s), s \in S) \), each agent \( i \in I \) decides consumption, \( x_s \), asset holdings, \( y_{sj} \), and borrowing (lending), \( \phi_s \), in order to maximize utility (3) subject to the budget set defined as

\[
B^i(p, r) = \{(x, y, \phi) \in R^S_+ \times R^{SJ}_+ \times R^S : \forall s \}
\]

\[
(x_s - e^i_s) + \sum_{j \in J} p^i_s(y_{sj} - y^i_{sj}) \leq \frac{1}{1 + r_s} \phi_s - \phi^*_s + \sum_{j \in J} y^i_{sj} D_{sj} \]

\[
\phi_s \leq \sum_{j \in J^c} y_{sj} \min_{t \in S(s)} [p^i_t + D_{tj}]
\]

In each state \( s \), expenditures on consumption minus endowments of the good, plus total expenditures on assets minus asset holdings carried over from the last period, can be at most equal to the money borrowed selling promises, minus the payments due at \( s \) from promises done in the previous period, plus the total asset deliveries. Notice that there is no sign constraint

\footnote{The consumption good is the numeraire, so \( p^*_s = 1 \).}
on $\phi_s$; a positive (negative) $\phi_s$ indicates the agent is selling (buying) promises or in other words, borrowing (lending) money. In the last line, the collateral constraint, the total amount of promises made at $s$ cannot exceed the total collateral capacity of all collateralizable asset holdings.

4.1.4 Collateral Equilibrium

A Collateral Equilibrium in this economy is a set of prices and holdings such that

$$((p, r), (x^i, y^i, \phi^i)_{i \in I}) \in R^S_J \times R^S_S \times (R^S_+ \times R^S_+ J \times R^S)^I : \forall s$$

$$\sum_{i \in I} (x^i_s - e^i_s) = \sum_{i \in I} \sum_{j \in J} y^i_{s,j} D_{sj}$$

$$\sum_{i \in I} (y^i_{s,j} - y^i_{s,j}^*) = 0 : \forall j$$

$$\sum_{i \in I} \phi^i_s = 0$$

$$(x^i, y^i, \phi^i) \in B^i(p, r)$$

$$(x, y, \phi) \in B^i(p, r) \Rightarrow U^i(x) \leq U^i(x^i) : \forall i$$

Markets for the consumption good, assets and promises clear in equilibrium, and agents optimize their utility constrained to their budget set as defined above. A Collateral Equilibrium always exists under all the described assumptions in this model as shown by Geanakoplos and Zame (1998). As is well known, this is not true for the standard General Equilibrium model with incomplete markets since equilibrium may fail to exist without a bound on short sales; the best result in the standard model is only generic existence. Collateral requirements fix this problem since they place (an endogenous) bound on short sales.

4.1.5 Asset Pricing

As mentioned above, endogenous margin requirements have a huge effect on asset prices. An asset’s price reflects its future returns, but also its ability to be used as collateral to borrow money. Consider a collateral equilibrium
in which an agent $i$ holds an asset $j$ at state $s \in S$, $y_{sj}^i > 0$ and suppose he consumes a positive amount in each state. Suppose that asset $j$ cannot be used as collateral, then

$$p^i_s = PV^i_s = \frac{\sum_{\sigma \in B(s)} q^i_{s\sigma} [p^i_{s\sigma} + D^i_{s\sigma}] du^i(x^i_{s\sigma})/dx}{du^i(x^i_s)/dx}$$

(4)

The price equals the *Payoff Value*, $PV^i_s$, i.e. the normalized expected marginal utility of its future payoff. This equation remains true if $j$ can be used as collateral but the collateral constraint for $i$ is not binding at $s$. In that case the first order condition for borrowing also holds,

$$\frac{1}{1 + r_s} = \frac{\sum_{\sigma \in B(s)} q^i_{s\sigma} du^i(x^i_{s\sigma})/dx}{du^i(x^i_s)/dx}$$

(5)

However, when an asset can be used as collateral, and the collateral constraint is binding, the situation for holding $j$ and loans is quite different. Agent $i$ cannot take out an additional loan unless he holds additional collateral. Thus, even if the marginal disutility of repaying the loan is less than the marginal utility of the money borrowed, it may just be impossible to borrow more money:

$$\frac{1}{1 + r_s} > \frac{\sum_{\sigma \in B(s)} q^i_{s\sigma} du^i(x^i_{s\sigma})/dx}{du^i(x^i_s)/dx}$$

(6)

Similarly, buying $j$ makes it possible to take out a loan, hence $i$ may buy $j$ on margin even when the expected marginal utility of $j$ is less than its price. Then, we might well have that

$$p^j_s > PV^j_s = \frac{\sum_{\sigma \in B(s)} q_{s\sigma}^j [p_{s\sigma}^j + D_{s\sigma}^j] du^i(x_{s\sigma}^j)/dx}{du^i(x_s^j)/dx}$$

(7)

Denote the difference between the left and right hand side in equation (6) by the *Liquidity Preference* of $i$ at $s$, $LP^i_s$, which gives a measure of how much agent $i$ values liquidity at $s$. Denote the debt of agent $i$ backed by a marginal unit of asset $j$ as
\( \phi_{sj}^i = \begin{cases} 
0 & \text{if } j \notin J^c \text{ or if the collateral constraint} \\
\min_{t \in S(s)}[p_t^j + D_t^j] & \text{is not binding at } s \text{ for } i \end{cases} \)

We define the Collateral Value of \( j \) at \( s \) as

\[ CV_{s}^j = LP_{s}^i \cdot \phi_{sj}^i \quad (8) \]

The collateral value reflects the asset’s efficiency as liquidity provider. This efficiency depends on two things. First, the asset’s collateral capacity measured as the amount of promises that can be backed by a marginal unit of the asset, \( \phi_{sj}^i \). Second, on how valuable is this collateral capacity to agent \( i \) measured by the liquidity preference, \( LP_{s}^i \). In fact, the following holds.

Pricing Lemma

Suppose that \( y_{sj}^i > 0 \). Then,

\[ p_s^j = PV_s^j + CV_s^j \]

this is, the price equals the sum of the Payoff value and Collateral Value.

Proof: The first order condition that obtains if \( i \) holds asset \( j \) is that the marginal utility of the cash payment necessary to buy \( j \) is equal to the expected marginal utility of the unencumbered payoff, i.e. the return on \( j \) less the repayment of the debt. Then in equilibrium the following holds

\[ p_s^j - 1 + r_s \phi_{sj}^i = \frac{\sum_{\sigma \in B(s)} q_{\sigma s}^i [p_{\sigma sj} + D_{\sigma sj} - \phi_{sj}^i] du^i(x_{\sigma sj}^i) / dx}{du^i(x_s^i) / dx} \quad (9) \]

14These concepts relate to the standard concept of fundamental value of an asset in the following way. Define the Fundamental Value of an asset \( j \) at \( s \) as

\[ FV_s^j = \sum_{\gamma \in \Gamma(s)} q_{\gamma s}^j D_{\gamma sj} du^i(x_{\gamma sj}^i) / dx \]

where \( \Gamma(s) \) is the set of all the successors (not only immediate) and \( q_{\gamma s}^j \) is the product of all \( q_{s'}^i \) along the path from \( s \) to \( \gamma \). If the asset cannot be used as collateral, then \( p_s^j = PV_s^j = FV_s^j \). However, if the asset can be used as collateral, then \( p_s^j > PV_s^j > FV_s^j \).
The pricing lemma follows from the definitions of $PV^j_s$ and $CV^j_s$ and equation (9).

Another convenient way to state the pricing lemma is as follows. Let

$$\mu^i_s \in R^{S(s)}$$ be the vector of marginal utilities weighted by state probabilities

$$\mu_{s\sigma}^i = \frac{q_{s\sigma} i \cdot dx(x_{s\sigma})/dx}{du(x_{s\sigma})/dx}, \sigma \in B(s).$$

Let $1 \in R^{S(s)}$ the vector of ones with dimension equal the number of successors of $s$. Let $A^j_s \in R^{S(s)}$ be the vector of payoffs of asset $j$ in each state following $s$, $A^j_{s\sigma} = [p^j_{s\sigma} + D^j_{s\sigma}]$. Then, the payoff value of $j$ at $s$ is given by

$$PV^j_s = \mu^i_s \cdot A^j_s$$ (10)

and the liquidity preference of $i$ at $s$ is given by

$$LP^i_s = \frac{1}{1 + r_s} - \mu^i_s \cdot 1$$ (11)

From equations (8), (10) and (11) the pricing lemma can be stated as

$$p^j_s = \mu^i_s \cdot A^j_s + \left( \frac{1}{1 + r_s} - \mu^i_s \cdot 1 \right) \cdot \phi^i_{s\sigma}$$ (12)

4.2 Contagion and Collateral Values

In this section we will extend our example in order to understand the role of collateral in contagion. Simulation 5 solves the equilibrium for the same assets and investor characteristics as in simulation 3, except that now $E$ can be used as collateral to borrow money, and hence can be leveraged. For simplicity, we will assume that this is not the case for $H$. Table 9 presents the equilibrium prices. As before, there is contagion due to the portfolio effect. Is there something different this time?

The conventional wisdom is that leverage causes agents to lose more money during crises, making asset prices even lower. On the contrary, we find that during the anxious economy the asset prices are higher than they would have been without collateral. Yet leverage still causes bigger price crashes.
The Pricing Lemma will explain this. Table 10 provides disaggregated information about price components, liquidity preference and margin requirements in equilibrium at each node. In non-collateral simulation 3, \( E \) cannot be used as collateral and the price equals its payoff value, whereas in collateral simulation 5 the price also includes collateral values.

At \( U \) the price in both simulations is almost the same. First, the payoff values are high and essentially the same because optimists’ marginal utilities \( \mu \) are high and nearly the same. Second, the collateral value in simulation 3 is zero. In simulation 5 it is small because, after good news at \( U \), the liquidity preference is small, and the collateral capacity is low in the second period (the maximum promise equals \( .1 = \min\{1, .1\} \)).

At \( D \) the payoff values are low since the portfolio and consumption effects have caused \( \mu \) to go down, as we saw before. The “wealth effect” implicit in other models has almost no bite in the anxious stage at \( D \): it is true that leverage at \( 1 \) has a negative consumption effect at \( D \), since it causes optimists to lose more money. But this is almost exactly offset by a positive consumption effect due to the possibility of borrowing again. The fall in consumption from \( U \) to \( D \) of 9% we already saw in simulation 3 is barely worsened to 10% by leverage in simulation 5. Hence, the payoff value at \( D \) is only slightly lower with collateral than without. However, the collateral value becomes significant. This is because the interest rates are nearly the same and so the same decrease in \( \mu \) that made the payoff value to go down causes the liquidity preference to go up, from .03 at \( U \) to .2 at \( D \) (see equation (11)). This effect explains why the price at \( D \) is bigger when there is collateral, and hence explains why the gap between \( U \) and \( D \) is smaller with collateral.\(^{15}\)

At \( 1 \) the price is higher with collateral than without for three reasons. First, the payoff value is higher than it was without collateral due to a con-

\[^{15}\]One may wonder if leverage could destroy contagion at \( D \) since the collateral value might rise enough to offset the fall in the payoff values. From equation (12) we can see that a lower \( \mu \) reduces the payoff value, but increases the liquidity preference and hence the collateral value. However, the change in collateral value cannot ever offset the changes in payoff value. Rearranging equation (12) we get that

\[
p_j^i = \mu_i^j \cdot (A_j^s - \phi_{isj} \cdot \bar{1}) + \frac{1}{1 + r_s}
\]

if the interest rate does not change, a lower \( \mu \) unambiguously lowers the price of the asset since \( A_j^s - \phi_{isj} \cdot \bar{1} > 0 \) from equation (1).
sumption effect: borrowing at 1 allows bigger consumption, increasing $\mu$. Second, the payoff value is also higher due to the presence of future collateral values, which raises future prices. Third, the collateral value is high, even though the liquidity preference is only moderate, because the collateral capacity is high (since the asset values at $U$ and $D$ are still high).

Collateral has a bigger effect on prices at 1 than at $D$ because it operates through three channels instead of only one. The presence of these channels explains why contagion, measured as the fall from 1 to $D$, is bigger when there is collateral.

To sum up there are three important points. First, leverage is not necessary for contagion to occur in equilibrium as shown by simulation 3. Second, collateral, through leverage, generates a bigger price crash. Third, the drop in asset prices is not due to asset under-valuation during anxious times but due to asset over-valuation during normal times.\textsuperscript{16}

Finally, both simulations provide a solution to the first problem and in particular rationalize Stylized Fact 1. Even without problems in Emerging Market fundamentals, a bad shock to the High Yield sector could have negative spillovers on Emerging Markets.

4.2.1 Robustness

The fundamental source of contagion is the portfolio effect, namely bad news about $H$ gives optimists an opportunity to hold it at attractive levels, reducing the money they can put into $E$. In simulation 5 optimists hold no $H$ after good news at $U$, and all of $H$ after bad news at $D$. Table 11 provides portfolio holdings and consumption at each node.

This corner solution gives an extreme form of the portfolio effect. One may wonder how robust contagion is to other regimes, where for example pessimists and optimists may both be marginal buyers of all the assets. Different parameter values will change asset holdings in equilibrium, allowing us

\textsuperscript{16}In the conventional story $H$ is leveraged and the bad news about $H$ induces investors who are leveraged to sell $E$, causing the its price to fall more than if there had been no leverage. However, when simulation 5 was extended to allow $H$, not only $E$, to be used as collateral, all three conclusions remained intact. In particular, the price of $E$ is higher at $D$ when both assets can be used collateral than when not.
to explore this question. It turns out that the simulation is not just a fluke. In fact, contagion is quite robust to other parameter choices. Two crucial parameters are investors’ beliefs and wealth. So, let us keep the rest of the values at the original levels and fix \( q^O = .9 \) and \( e^O = 20 \). Define \( q^O - q^P \) as the disagreement and \( e^P - e^O \) as wealth gap between investors. Figure 5 presents a grid of simulations. In all the regions numbered from 1 to 11 contagion holds in equilibrium. The different regions correspond to different regimes in terms of asset holdings and whether collateral constraints are binding or not. Region 1 corresponds to simulation 5. But contagion holds also in less extreme portfolio regimes. For example in regime 8 optimists and pessimists both hold \( H \) in both states in the second period, but still optimists hold more \( H \) at \( D \) than at \( U \) so the portfolio effect is still present. The only regions in which contagion breaks down are the two lower regions 12 and 13 where \( q^P = .8999 \) and \( q^O - q^P \) is near zero. Of course, at the origin we are back to the case of a representative agent. Table 12 describes all these regimes showing at each node what are the asset holdings for each type of investor and whether the borrowing constraint is binding or not. A “−” indicates closed credit markets (there is no borrowing or lending). In regimes 1 to 11, if credit markets are active, optimists always borrow and pessimists always lend and the contrary is true in regimes 12 and 13.

With complete markets, an increase in pessimists’ wealth would destroy contagion. With incomplete markets contagion holds regardless of the wealth gap (provided there is disagreement between agents). In fact, the degree of contagion increases (given a disagreement level) with the wealth gap, both measured as the gap between \( U \) and \( D \) or as the fall from 1 to \( D \) as shown in graphs 1 and 2.\(^{17}\) The reason for this is that in regimes in which pessimists are lenders, the wealth differential increases leverage and hence the optimist’s ability to buy more assets and have more extreme portfolio positions, all without moving interest rates.

### 4.3 Flight to Liquidity and Endogenous Margins

In this section we go back to our example in simulation 4 with emerging market assets of differing quality, but no collateral. Although leverage was

\(^{17}\) We just show the degree of contagion for two disagreement levels, more information is available upon request.
not crucial for contagion, now it will play the shining role.

Simulation 6 solves the equilibrium for the same parameters as in simulation 4, except that now both emerging market assets can be used as collateral. Without loss of generality we still assume that this is not the case for $H$. Tables 13 and 14 present the results.

The portfolio and consumption effects are still present, and hence so is contagion. However, simulation 6 exhibits a new thing: differential contagion. The price of $E_B$ falls more than the price of $E_G$ from $U$ to $D$ and from 1 to $D$.

The key is that different assets have different endogenous collateral capacities. These differences become important and lead to different collateral values when agents’ need for liquidity, measured by the liquidity preference, increases. At $D$ collateral capacities are very different and liquidity preference is very high, at $U$ collateral capacities are also very different but the liquidity preference is small, and at 1 collateral capacities are nearly the same. Thus the collateral values of $E_G$ and $E_B$ are very different at $D$ whereas at 1 and $U$ they are similar.

More precisely, at $D$ the collateral capacities of $E_G$ and $E_B$ per dollar of asset (given by 1 minus the margin requirement) are $1 - m_G = .26$ and $1 - m_B = .07$ respectively. The high liquidity preference, $LP = .2$, gives rise to different collateral values of $CV_G = .04$ and $CV_B = .02$. At $U$ the collateral capacities are also very different, however the liquidity preference is low ($LP = .03$) so that the collateral values are negligible and virtually the same, $CV_G = .006$ and $CV_B = .003$. At 1, the endogenous collateral capacities per dollar of assets are $1 - m_G = .88$ and $1 - m_B = .86$, which though big are very similar. Combined with a low liquidity preference ($LP = .03$) they lead to very similar, though not negligible, collateral values of $CV_G = .031$ and $CV_B = .03$.

This differentiated behavior in collateral values explains the differential fall in prices. The fall in payoff values for both assets is virtually the same. From $U$ to $D$ the fall in payoff values of 16.5% for both assets is cushioned by an increase in the collateral value of 3.66% for $E_G$ but only of 1.86% for $E_B$. From 1 to $D$ the difference is even more drastic, since the fall in payoffs
of 12% is cushioned by an increase in the collateral value of 0.9% for $E^G$ but exacerbated by a further decrease in the collateral value of 1.23% for $E^B$.\(^\text{18}\)

We say that there is Flight to Liquidity when: (1) liquidity preference is high, (2) margins are high and (3) the dispersion of margins between assets is high. These conditions generate an increase in the spread between assets through differential movements of collateral values. During a flight to liquidity, investors would rather buy those assets that enable them to borrow money more easily (lower margins). The other side of the coin, what we could call Flight from Illiquidity, is that investors who need to raise cash get more by selling those assets on which they did not borrow money (higher margins) because the sales revenues net of loan repayments are higher.

Traditionally, the price deterioration of low quality assets is explained in terms of “flight to quality” type of arguments: an increase in risk aversion lowers the payoff value of volatile assets. Flight to Liquidity emphasizes a different channel. The gap in prices is created entirely by movements in collateral values. Even in the absence of flight to quality behavior (associated with movements in payoff values), we may still observe a bigger price deterioration of bad quality assets due to time-varying liquidity preference and margin requirements.

Finally, the model also provides a testable forecasting result. As was mentioned before, information volatility is not stationary in our model. The volatility of expectations of the final payoffs increases from 0 at 1, to .56 at $U$ and $D$ for $E^G$, and from 0 at 1, to .67 at $U$ and $D$ for $E^B$. From this we might expect the margins for $E^G$ and $E^B$ to be 0 at 1, and higher for $E^G$, and even higher for $E^B$, at $U$ and $D$. This is indeed the case at $U$ and $D$. However, contagion at $D$ causes volatility in the prices of $E^G$ and $E^B$, and thus positive margins at 1. The flight to liquidity at $D$ causes more price volatility for $E^B$ than for $E^G$, and hence higher margins at 1 for $E^B$ than for $E^G$. Thus the margins during normal times at 1 can predict which asset will suffer more during future flight to liquidity at anxious times.

To sum up, different endogenous margin requirements create differential contagion and flight to liquidity which gives a rationale for Stylized Fact 2.\(^\text{18}\)

\(^{18}\)As before, it can be shown that the result is robust to different parameter specifications. We will save the reader from this discussion since there is nothing conceptually new from the analysis already presented.
Real world margins during normal times are 10% for high-rated emerging markets bonds and 20% or more for low-rated Emerging Markets bonds. Provided that the expected flow of future information across credit ratings is symmetric, these margins during normal times indicate that low-rated emerging market bonds will be the ones suffering during future flight to liquidity episodes.

5 Model II: Collateral General Equilibrium with Adverse Selection

In this section we will focus on the issuance problem. For this, we extend the first model. Instead of taking the supply of $E$ as fixed we explicitly model the issuance choice of emerging market assets.

5.1 Model

5.1.1 Emerging Countries

In each state, $s \in S$, each country $ks$ chooses to issue assets. To simplify our calculation we assume that each country has only one chance to issue assets and is not allowed to trade on secondary markets. We will also assume that countries consume only at the period of issuance and at the end.\(^\text{19}\) Each country $ks$ has Bernoulli utility $u^{ks}(x)$ for consumption of $x$ units of the consumption good in state $s$ and in states $t \in S_T(s)$, where $S_T(s)$ is the set of terminal nodes that follow $s$. Utilities satisfy the usual assumptions discussed before. Country $ks$ assigns subjective probability $q^{ks}_\alpha$ to the transition from any state $\alpha^*$ to $\alpha$. (Naturally $q^{ks}_1 = 1$). Letting $\bar{q}^{ks}_\alpha$ be the product of all $q^{ks}_\alpha$ along the path from 1 to $\alpha$, the von-Neumann-Morgenstern expected utility to country $ks$ is

\[
U^{ks} = \sum_{\alpha \in \{s\} \cup S_T(s)} \bar{q}^{ks}_\alpha \delta^{(s)}(\alpha) u^{ks}(x_\alpha) \tag{14}
\]

Each country $ks$ has an endowment of 1 unit of asset $E^k$ to sell to investors at its issuance date $s$. We denote the issuance at $s$ by $z^{ks}$. Countries are

\[^{19}\text{Adding intermediate consumption when countries are not allowed to trade or issue would not affect any of the results.}\]
endowed with the consumption good only at each terminal node $t \in S_T(s)$, hence they need to issue debt in order to consume at $s$.

5.1.2 Types and Asymmetric Information

In each state $s \in S$ there are two types of countries, “good”, $k = G$, and “bad”, $k = B$, issuing assets in the primary market. Assets issued by different types differ in their deliveries; the good type always pays at least as much as the bad type: $D_{\alpha G} \geq D_{\alpha B}, \forall \alpha \in S$. We assume that the deliveries of countries of the same type are the same (even if they were issued at different states). Thus, all assets known to be good (bad) at $s$ will trade for the same price $p_{sG}$ ($p_{sB}$), whether issued at $s$ and trading for the first on the primary market at $s$, or issued previously and trading on the secondary market at $s$. However, the prices $p_{sG}$ and $p_{sB}$ may or may not coincide. The key element in this extension is that there is asymmetric information: investors cannot perfectly observe a country’s type and hence the type of credit they are trading.

5.1.3 The Market as a Designer

At this point we face a problem: how can we make compatible the adverse selection problem arising from the asymmetric information with the perfect competition framework described in Model I? To attack this problem we follow the modeling strategy used in Dubey-Geanakoplos (2002) to study insurance in a different competitive framework. We apply their techniques to extend the Collateral General Equilibrium model of section 3 to encompass adverse selection and issuance rationing.

In each state $s \in S$, there are many different debt markets, each characterized by a quantity limit (which a seller in that market cannot exceed) and its associated market clearing price:

$$\overline{p}_s = \{(z_s, p_s(z_s)); z_s \in (0, 1], p_s \in R_+\}. \quad (15)$$

The issuance-price schedule $\overline{p}_s$ is taken as given and emerging countries and investors decide in which of these debt markets to participate. We assume exclusivity, i.e., countries can only issue (sell) in one debt market at any given
time. So they must choose a quantity $z_s$ to sell and then take as given the corresponding market clearing price $p_s(z_s)$.

Given the price schedule $\overline{p_s}$, country $k_s$ decides consumption and issuance in order to maximize utility (10) subject to the budget set defined as

$$B^{k_s}(\overline{p_s}) = \{(x, z) \in R_+^{1+S_T(s)} \times R_+: x_s \leq p_s(z)z, \quad z \leq 1 \quad \forall \alpha \in S_T(s) : x_\alpha = e^{k_s}_\alpha + (1 - z)D_{nk}\}$$

Consumption at $s$ has to be less or equal than the income from issuance of quantity $z$. Issuance at $s$ cannot exceed the total endowment of the asset $k$ of 1 unit. Finally, consumption at each terminal node that follows $s$ has to be less or equal than the endowment of the consumption good plus the deliveries on the remaining asset that was not sold at $s$.

Investors who buy assets in market $(z_s, p_s(z_s))$ get a pro rata share of the deliveries of all assets sold in that market. If the proportion of the sales at $z_s$ of the bad type exceeds the proportion of bad types in the economy, then the buyer at $z_s$ gets an adverse selection. Investors are assumed to be rational and to have the correct expectation of deliveries from each market $(z_s, p_s(z_s))$. Thus, if only one country type is choosing to sell at the quantity $z_s$, then it reveals its type, and from then on, its asset payoffs are known to be the corresponding type. With this interpretation there is room for signalling as well as adverse selection without destroying market anonymity. Countries may (falsely) signal more reliable deliveries by publicly committing to (small) quantity markets where the prices are high because the market expects only good types to sell there. The quantity limit characterizing each debt market is exogenous and the associated price is set endogenously as in any traditional competitive model. However, it may occur that in equilibrium only a few debt markets are active, even when all the markets are priced in equilibrium. In this sense, the quantities are set endogenously as well, without the need of any contract designer. Market clearing and optimizing behavior are enough.

5.1.4 Separating Collateral Equilibrium

A formal definition of equilibrium in this model is quite involved, because there are so many markets, and because the secondary market prices will
depend on what is revealed in the primary markets. However, there is a shortcut to this problem. We say that an equilibrium is pooling if at any state \( s \) two countries of different types decide to sell the same amount, and hence participate in the same market. In contrast, an equilibrium is separating when different types, \( Gs \) and \( Bs \), always issue different amounts in the same state. Dubey-Geanakoplos (2002) show that their model exhibits a unique refined separating equilibrium. Their techniques are still valid in the present model to show the existence of such an equilibrium. A formal definition of a separating equilibrium is simpler.

A **Separating Collateral Equilibrium** is defined as a standard **Collateral Equilibrium** \( ((\overline{p}, r), (x^i, y^i, \phi^i)_{i \in I}, (x_{Gs}, z_{Gs}, x_{Bs}, z_{Bs})_{s \in S}) \) with the following extra requirement: \( z_{Gs} < z_{Bs} \).

Finally, let us stress why it is so important that the model exhibits a separating equilibrium from a computational point of view. In general, equilibrium would have forced us to solve for prices for all possible quantity limits, and to distinguish assets sold later by how much of them were originally issued. This is an infinite dimensional problem. In a separating equilibrium we need only to keep track of good and bad asset prices, \( p_{Gs} \) and \( p_{Bs} \), and good and bad issuance levels, \( z_{Gs} \) and \( z_{Bs} \). Now we have reduced the problem to calculate a finite set of variables as we had before.

### 5.2 Adverse Selection, Margins and Issuance Rationing

Simulation 7 solves the equilibrium for the same parameters as before. The new parameters are the ones describing countries. Utilities are quadratic:

\[
U^{ks} = (x_s - \beta x_s^2) + \sum_{s' \in T(s)} q^k s \delta_{ks} (x_{s'} - \beta x_{s'})
\]

with \( \beta = 1/370 \). Endowments and beliefs are the same as the optimists investors, so \( c_{s' \in T(s)}^{ks} = 20 \), \( q^{ks} = .9 \). Tables 15 and 16 present the results. The price behavior described in simulation 6 is still present here: there is contagion and flight to liquidity. Portfolio and consumption effects are present, hence both emerging market asset prices fall from \( U \) to \( D \) and from 1 to \( D \). However, since \( E^B \) exhibits higher margins in equilibrium, its price falls more.

---

\(^{20}\)The definition of equilibrium requires prices \( p_s(z_s) \) for those markets \( z_s \) that are not active to be determined as well. For \( z_s < z_{Gs} \), \( p_s(z_s) \) is determined so that the good type is indifferent between issuing \( z_s \) and \( z_{Gs} \). For \( z_s \geq z_{Gs} \), \( p_s(z_s) \) is such that the bad type is indifferent between \( z_s \) and \( z_{Bs} \).
The new thing in this simulation comes from the supply side. At $D$ there is a drop in issuance, and more importantly a more severe drop for the good type. The bad type issuance goes from $z^B_1 = 1$ to $z^B_D = .75$ whereas the good type issuance goes from $z^G_1 = .8$ all the way to $z^G_D = .08$. The gap in issuance between $U$ and $D$ is also bigger for the good type than for the bad type. Now, adverse selection plays the leading role.\textsuperscript{21}

It is not surprising that with contagion and the corresponding fall in prices, equilibrium issuance falls as well. The interesting thing is that flight to liquidity combined with informational asymmetries generates issuance rationing; the fall in price of the good type is less yet its drop in issuance is much more. The greater the price difference between types the more drastic the drop in good quality issuance.

The explanation is that the bigger price spread between types requires a smaller good type issuance for a separating equilibrium to exist. Unless the good issuance levels become onerously low, bad types would be more tempted by the bigger price spread to mimic good types and sell at the high price $p_{Gs}$. There exists such a separating level $z_{Gs}$ since it is more costly for the bad type to rely on the payoff of its own asset for final consumption than it is for the good type.

In standard models of adverse selection incentive compatibility constraints play a central role. In the present model with adverse selection embedded in a general equilibrium framework, the presence of a price-issuance schedule and utility maximization subject to budget constraints are enough.

In a world with no informational noise, spillovers from other markets may even help good issuance relative to bad issuance. However, if to market incompleteness, investor disagreement and heterogeneous margin requirements, we add some degree of informational noise between countries and investors, we get that good quality issuance suffers more.\textsuperscript{22} This result solves our third problem and in particular rationalizes Stylized Fact 3: high rated issuance falls more than low-rated issuance during closures despite the fact that high-rated spreads increase less than low-rated spreads.

\textsuperscript{21}As in section 3, the simulation is robust to other choices of parameters.

\textsuperscript{22}One may wonder at the role of credit agencies as information revealing devices. To make our explanation consistent with the existence of rating agencies, we need to assume that credit agencies do not know anything more than can be inferred from price, and that in effect they just follow the market.
6 References.


7 Figures, tables and graphs.
Figure 1: Average Spreads around Closures (-20/+20)
Figure 2: Average Volatility around Closures (-10/+20)
Figure 3: Average percentage change in Emerging Market spreads for different credit ratings around Closures
Figure 4: Main Example

- $U$: $q$
- $D$: $(1-q)q$
- $D_D$: $(1-q)q$
- $D_D$: $(1-q)q$
- $D_D$: $(1-q)^2$

$B < G < 1, H < 1$
Figure 5: Robustness Analysis
<table>
<thead>
<tr>
<th>Closure</th>
<th>Year</th>
<th>Date</th>
<th>Duration (weeks)</th>
<th>Associated Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1997</td>
<td>03/17-04/06</td>
<td>3</td>
<td>Thailand turmoil</td>
</tr>
<tr>
<td>2</td>
<td>1997</td>
<td>08/18-09/07</td>
<td>3</td>
<td>Thailand devaluation</td>
</tr>
<tr>
<td>3</td>
<td>1997</td>
<td>10/27-12/07</td>
<td>6</td>
<td>Korea crisis</td>
</tr>
<tr>
<td>4</td>
<td>1998</td>
<td>08/03-10/26</td>
<td>12</td>
<td>Russia Default and LTCM</td>
</tr>
<tr>
<td>5</td>
<td>1999</td>
<td>01/01-01/31</td>
<td>4</td>
<td>Brazil devaluation</td>
</tr>
<tr>
<td>6</td>
<td>1999</td>
<td>07/12-08/02</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1999</td>
<td>08/16-09/05</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2000</td>
<td>04/03-05/01</td>
<td>4</td>
<td>US interest rate anxieties</td>
</tr>
<tr>
<td>9</td>
<td>2000</td>
<td>09/25-10/30</td>
<td>5</td>
<td>US stock market crash</td>
</tr>
<tr>
<td>10</td>
<td>2001</td>
<td>08/20-09/10</td>
<td>3</td>
<td>US recession concerns</td>
</tr>
<tr>
<td>11</td>
<td>2002</td>
<td>04/29-06/17</td>
<td>7</td>
<td>Brazil turmoil</td>
</tr>
<tr>
<td>12</td>
<td>2002</td>
<td>08/05-09/02</td>
<td>4</td>
<td>US stock market</td>
</tr>
<tr>
<td>13</td>
<td>2002</td>
<td>09/23-10/14</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
Table 2: Simulations 1 and 2.

Simulation 1: Representative Agent.

<table>
<thead>
<tr>
<th>Asset</th>
<th>$p_1$</th>
<th>$p_U$</th>
<th>$p_D$</th>
<th>$(p_U-p_D)/p_U$ %</th>
<th>$(p_1-p_D)/p_1$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>0.9082</td>
<td>0.9082</td>
<td>0.9083</td>
<td>-0.011</td>
<td>-0.011</td>
</tr>
<tr>
<td>H</td>
<td>0.9901</td>
<td>0.9981</td>
<td>0.9183</td>
<td>7.99</td>
<td>7.25</td>
</tr>
</tbody>
</table>

Simulation 2: Complete Markets and Heterogeneous Agents.

<table>
<thead>
<tr>
<th>Asset</th>
<th>$p_1$</th>
<th>$p_U$</th>
<th>$p_D$</th>
<th>$(p_U-p_D)/p_U$ %</th>
<th>$(p_1-p_D)/p_1$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>0.553</td>
<td>0.555</td>
<td>0.545</td>
<td>1.8</td>
<td>1.45</td>
</tr>
<tr>
<td>H</td>
<td>0.8</td>
<td>0.998</td>
<td>0.599</td>
<td>39.97</td>
<td>25.12</td>
</tr>
</tbody>
</table>
Table 3: Simulation 3, Incomplete Markets. Prices.

<table>
<thead>
<tr>
<th>Asset</th>
<th>1</th>
<th>U</th>
<th>D</th>
<th>(U-D)/U %</th>
<th>(1-D)/1 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>0.7954</td>
<td>0.863</td>
<td>0.7273</td>
<td>15.72</td>
<td>8.56</td>
</tr>
<tr>
<td>H</td>
<td>0.9097</td>
<td>0.9986</td>
<td>0.7364</td>
<td>26.25</td>
<td>19.05</td>
</tr>
</tbody>
</table>

Table 4: Simulation 3, Incomplete Markets. Portfolio.

<table>
<thead>
<tr>
<th>Asset</th>
<th>1</th>
<th>U</th>
<th>D</th>
<th>O</th>
<th>P</th>
<th>O</th>
<th>P</th>
<th>O</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>6624</td>
<td>1.3376</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Simulation 3, Incomplete Markets. Consumption.

<table>
<thead>
<tr>
<th>Cons.</th>
<th>1</th>
<th>U</th>
<th>D</th>
<th>UU</th>
<th>UD</th>
<th>DUU</th>
<th>DDU</th>
<th>DUD</th>
<th>DDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>x^0</td>
<td>19.3589</td>
<td>20.8428</td>
<td>19.0272</td>
<td>22</td>
<td>20.2</td>
<td>24</td>
<td>22.2</td>
<td>22.4</td>
<td>20.6</td>
</tr>
</tbody>
</table>
### Table 6: Simulation 4, Incomplete Markets with 3 assets. Prices

<table>
<thead>
<tr>
<th>Asset</th>
<th>1</th>
<th>U</th>
<th>D</th>
<th>(U-D)/U</th>
<th>(1-D)/1</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>0.7817</td>
<td>0.8378</td>
<td>0.7431</td>
<td>11.3</td>
<td>4.93</td>
</tr>
<tr>
<td>B</td>
<td>0.7679</td>
<td>0.823</td>
<td>0.7301</td>
<td>11.3</td>
<td>4.93</td>
</tr>
<tr>
<td>H</td>
<td>0.8477</td>
<td>0.9162</td>
<td>0.7485</td>
<td>18.3</td>
<td>11.7</td>
</tr>
</tbody>
</table>

### Table 7: Simulation 4, Incomplete Markets. Portfolio

<table>
<thead>
<tr>
<th>Asset</th>
<th>1</th>
<th>U</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>0.4669</td>
<td>0.5331</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>0.4675</td>
<td>0.5325</td>
<td>1</td>
</tr>
<tr>
<td>H</td>
<td>2</td>
<td>0</td>
<td>0.5219</td>
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</tbody>
</table>

### Table 8: Simulation 4, Incomplete Markets with 3 assets. Consumption

<table>
<thead>
<tr>
<th>Cons.</th>
<th>1</th>
<th>U</th>
<th>D</th>
<th>UU</th>
<th>UD</th>
<th>DUU</th>
<th>DDU</th>
<th>DUD</th>
<th>DDD</th>
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<tbody>
<tr>
<td>x^O</td>
<td>19.2</td>
<td>20.5</td>
<td>19.2</td>
<td>22.5</td>
<td>20.7</td>
<td>24</td>
<td>22.25</td>
<td>22.4</td>
<td>20.65</td>
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</table>
Table 9: Simulation 5, Incomplete Markets with Collateral. Prices

<table>
<thead>
<tr>
<th>Asset</th>
<th></th>
<th></th>
<th></th>
<th>(U-D)/U %</th>
<th>(1-D)/1 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>0.8511</td>
<td>0.8695</td>
<td>0.7416</td>
<td>14.7</td>
<td>12.86</td>
</tr>
<tr>
<td>H</td>
<td>0.9316</td>
<td>0.9985</td>
<td>0.7306</td>
<td>26.83</td>
<td>21.58</td>
</tr>
<tr>
<td>loan (rₜ)</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</table>

Table 10: Simulation 5, Incomplete Markets with Collateral
Price components, Liquidity Preference and Margins

<table>
<thead>
<tr>
<th>Price comp</th>
<th></th>
<th></th>
<th></th>
<th>(U-D)/p₁ %</th>
<th>(1-D)/p₁ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>0.0388</td>
<td>0.0398</td>
<td>0.2019</td>
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<td>Assets</td>
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<td></td>
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<td></td>
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<table>
<thead>
<tr>
<th>Price comp</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>E PV</td>
<td>0.8223</td>
<td>0.8655</td>
<td>0.7215</td>
<td>16.56</td>
<td>11.86</td>
</tr>
<tr>
<td>E CV</td>
<td>0.0287</td>
<td>0.004</td>
<td>0.0202</td>
<td>-1.86</td>
<td>1</td>
</tr>
<tr>
<td>E m</td>
<td>0.1286</td>
<td>0.8852</td>
<td>0.8651</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H PV</td>
<td>0.9316</td>
<td>0.9985</td>
<td>0.7306</td>
<td>26.83</td>
<td>21.58</td>
</tr>
<tr>
<td>H CV</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H m</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
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<td>18.7</td>
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<tr>
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<tr>
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<td>2*(.74)</td>
<td>2*(.1)</td>
<td>2*(.1)</td>
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<td>Node U</td>
<td>Node 1</td>
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<td>Pessimists</td>
<td>Borrowing Constraint</td>
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<td>Optimists</td>
<td>Pessimists</td>
<td>Borrowing Constraint</td>
</tr>
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<td>B</td>
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<td>H</td>
<td>E</td>
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</tr>
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</table>
Table 13: Simulation 6, Incomplete Markets with Collateral. 3 assets.

Prices.

<table>
<thead>
<tr>
<th>Asset</th>
<th>1</th>
<th>U</th>
<th>D</th>
<th>(U-D)/U %</th>
<th>(1-D)/1 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>0.8724</td>
<td>0.889</td>
<td>0.7747</td>
<td>12.86</td>
<td>11.2</td>
</tr>
<tr>
<td>B</td>
<td>0.8566</td>
<td>0.8753</td>
<td>0.7465</td>
<td>14.72</td>
<td>12.85</td>
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<tr>
<td>H</td>
<td>0.9305</td>
<td>0.9985</td>
<td>0.7358</td>
<td>26.3</td>
<td>20.92</td>
</tr>
<tr>
<td>Loan G ($r_s$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Loan B ($r_s$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 14: Simulation 6, Incomplete Markets with Collateral. 3 assets.

Price components, Liquidity Preference and Margins.

<table>
<thead>
<tr>
<th>LP</th>
<th>1</th>
<th>U</th>
<th>D</th>
<th>(U-D)/p_U %</th>
<th>(1-D)/p_1 %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0396</td>
<td>0.0364</td>
<td>0.199</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Assets</th>
<th>Price comp</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>PV</td>
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<tr>
<td></td>
<td>0.8411</td>
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<tr>
<td></td>
<td>CV</td>
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<td></td>
<td>0.0313</td>
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<tr>
<td></td>
<td>m</td>
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<tr>
<td></td>
<td>0.1119</td>
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<tr>
<td>B</td>
<td>PV</td>
</tr>
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<td>0.8264</td>
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<td></td>
<td>CV</td>
</tr>
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<td>0.0302</td>
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<tr>
<td></td>
<td>m</td>
</tr>
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<td></td>
<td>0.1371</td>
</tr>
</tbody>
</table>
Table 15: Simulation 7, Incomplete Markets with Collateral and Adverse Selection. Prices.

<table>
<thead>
<tr>
<th>Asset</th>
<th>1</th>
<th>U</th>
<th>D</th>
<th>(U-D)/U %</th>
<th>(1-D)/1 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>0.8149</td>
<td>0.8409</td>
<td>0.6957</td>
<td>17.3</td>
<td>14.6</td>
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<tr>
<td>B</td>
<td>0.7807</td>
<td>0.8117</td>
<td>0.6385</td>
<td>21.3</td>
<td>18.2</td>
</tr>
<tr>
<td>H</td>
<td>0.8849</td>
<td>0.9967</td>
<td>0.6326</td>
<td>36.5</td>
<td>28.5</td>
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<tr>
<td>loan G ($r_s$)</td>
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<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>loan B ($r_s$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
</tbody>
</table>

Table 16: Simulation 7, Incomplete Markets with Collateral and Adverse Selection. Prices. Issuance.

<table>
<thead>
<tr>
<th>Type</th>
<th>1</th>
<th>U</th>
<th>D</th>
<th>(U-D)/U %</th>
<th>(1-D)/1 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>0.8018</td>
<td>0.8524</td>
<td>0.0808</td>
<td>90</td>
<td>89.9</td>
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<td>B</td>
<td>1</td>
<td>1</td>
<td>0.75</td>
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</table>
Graph 1: Contagion for disagreement level .2

Graph 2: Contagion for disagreement level .4