An Estimation of Economic Models
with Recursive Preferences

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Introduction


- Two reasons recursive utility is of interest
  1. More flexibility as regards attitudes toward risk and intertemporal substitution.
  2. Preferences deliver an added risk factor for explaining asset returns.

- But, only a small amount of econometric work on recursive preferences → gap in the literature.
Introduction: Why Estimation is Difficult

- Recursive utility (Epstein, Zin (’89, ’91) & Weil (’89))

\[
V_t = [(1 - \beta) C_t^{1-\rho} + \beta R_t (V_{t+1})^{1-\rho}]^{\frac{1}{1-\rho}}
\]

\[
R_t (V_{t+1}) = (E [V_{t+1}^{1-\theta} | \mathcal{F}_t])^{\frac{1}{1-\theta}}
\]

- \(V_{t+1}\) is continuation value, \(\theta\) is RRA, \(1/\rho\) is EIS.

- Rescale utility function (Hansen, Heaton, Li ’05):

\[
\frac{V_t}{C_t} = \left[ (1 - \beta) + \beta R_t \left( \frac{V_{t+1}}{C_{t+1}} \frac{C_{t+1}}{C_t} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}
\]

- **Difficulty**: MRS a function of \(V/C\), in general unobservable, embeds \(R_t(\cdot)\).

- Special case: \(\rho = \theta \rightarrow\) CRRA separable utility

\[
V_t = \beta \frac{C_t^{1-\theta}}{1 - \theta}.
\]
Introduction: Why Estimation is Difficult

- The MRS is pricing kernel with added risk factor:

\[
M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \left( \frac{V_{t+1} C_{t+1}}{C_{t+1} C_t} \right)^{\rho-\theta} \left( \frac{R_t \left( \frac{V_{t+1} C_{t+1}}{C_{t+1} C_t} \right)}{R_{w,t+1}} \right)^{\theta-\rho}
\]

- Epstein-Zin exploit relation btw \( V/C \) and return to aggregate wealth, \( R_{w,t+1} \)

\[
M_{t+1} = \left\{ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \right\}^{\frac{1-\theta}{1-\rho}} \left\{ \frac{1}{R_{w,t+1}} \right\}^{\frac{\theta-\rho}{1-\rho}}
\]

- Unobservability of \( V/C \) can be overcome if \( R_w \) proxied (Epstein-Zin ’91).

- But, \( R_{w,t+1} \) is the return on a claim to future \( C_t \), may not be well proxied by observable asset market returns.
Introduction: Difficulties Overcome in Special Cases

1. If $\text{EIS}=1$, and $\Delta \log C_{t+1}$ follows a loglinear time-series process, $\log(V/C)$ has an analytical solution.

2. If returns, $C_t$ are jointly lognormal and homoskedastic, risk premia are approx. log-linear functions of COV between returns, and news about current and future $C_t$ growth.

• But....
  – $\text{EIS}=1 \rightarrow$ consumption-wealth ratio is constant, contradicting statistical evidence.
  – Joint lognormality strongly rejected in quarterly data.

• $===>$ May be desirable to allow for more general representations of EZW model.
Introduction: This Paper

- Employ semiparametric estimation technique to estimate EZW utility model \textit{without}
  1. Need to proxy $R_{w,t+1}$ with observable returns.
  2. Loglinearizing the model.
  3. Tight parametric restrictions on law of motion or joint dist. of $C_t$ and $R_{i,t}$, or on value of key preference parameters.

- Present estimates of $\beta$, RRA $\theta$, EIS $\rho^{-1}$

- Evaluate model’s ability to fit asset return data relative to competing specifications.

- Investigate implications for $R_{w,t+1}$ and return to human wealth.
**Introduction: Results Preview**

- Estimated RRA is high, from 17-60, higher values for representative agent version of the model (aggregate consumption) than for stockholder consumption.

- EIS is typically above one, from 1.1 - 2.

- Estimated aggregate wealth return weakly correlated with aggregate stock market return, and much less volatile.
  - Suggests return to human wealth is negatively correlated with the stock market return.

- EZW recursive model can explain a cross-section of equity returns better than time-separable CRRA, better than the scaled CCAPM model (Lettau-Ludvigson ’01), but not as well as Fama-French 3 factor model.
Empirical Model

- First order conditions for optimal consumption:

\[ E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \left( \frac{V_{t+1}}{R_t} \frac{C_{t+1}}{C_t} \right)^{\frac{\theta - \rho}{1 - \rho}} \right] R_{i,t+1} - 1 = 0 \]  \hspace{1cm} (1)

- Using \( \frac{V_t}{C_t} = \left[ (1 - \beta) + \beta R_t \frac{V_{t+1}}{C_t} \frac{C_{t+1}}{C_t} \right]^{\frac{1}{1 - \rho}} \) in (1):

\[ E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \left( \frac{1}{\beta} \left[ \frac{V_t}{C_t} \right]^{1 - \rho} - (1 - \beta) \right) \right]^{\frac{1}{1 - \rho}} R_{i,t+1} - 1 = 0 \hspace{1cm} i = 1, ..., N. \]  \hspace{1cm} (2)

- \( N \) test asset returns, \( \{R_{i,t+1}\}_{i=1}^N \). (2) is a cross-sectional asset pricing model.

- (2) Forms the basis of our empirical investigation.

- (2) embeds an unknown function \( V_t/C_t \). Estimate nonparametrically.

- Empirical model is semiparametric; Let \( \delta \equiv (\beta, \theta, \rho)' \).
Empirical Model

- Model $\frac{V_t}{C_t}$ as an unknown function $V: \mathbb{R}^2 \rightarrow \mathbb{R}$

$$
\frac{V_t}{C_t} = V \left( \frac{V_{t-1}}{C_{t-1}}, \frac{C_t}{C_{t-1}} \right)
$$

- Justified if, for example,
  - Assume $\Delta \log(C_{t+1})$ a possibly nonlinear function of a hidden first order Markov process $x_t$.
  - Under fairly general assumptions, information in $x_t$ is summarized by $V_{t-1}/C_{t-1}$ and $C_t/C_{t-1}$.

- With a nonlinear Markov process, $V(\cdot)$ can display nonmonotonicities in both arguments.
Empirical Implementation

• Let $\delta_o \equiv (\beta_o, \theta_o, \rho_o)'$, $V_o \equiv V_o(z_t, \delta_o)$ denote true parameters, $z_t$, data.

• Write moment conditions to be estimated as:

$$E \{ \gamma_i(z_{t+1}, \delta_o, V_o(\cdot, \delta_o))|w^*_t \} = 0 \quad i = 1, ..., N, \quad (3)$$

• $w^*_t$ denotes the agents information set at time $t$

• $z_{t+1}$ contains all observations at $t + 1$ and

$$\gamma_i(z_{t+1}, \delta, V) \equiv \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \left( \frac{V \left( \frac{V_t}{C_t}, \frac{C_{t+1}}{C_t} \right)}{\left\{ \frac{1}{\beta} \left[ V \left( \frac{V_{t-1}}{C_{t-1}}, \frac{C_t}{C_{t-1}} \right) \right]^{1-\rho} - (1 - \beta) \right\}^{1-\rho}} \right)^{\rho - \theta} R_{i,t+1} - 1$$

• Let $w_t$ denote an observable subset of $w^*_t$. Equation (3) implies:

$$E \{ \gamma_i(z_{t+1}, \delta_o, V_o(\cdot, \delta_o))|w_t \} = 0, \quad i = 1, ..., N.$$
Empirical Implementation

- The true parameter value $V_o(\cdot, \delta_o)$ is the solution to:

$$V_o(\cdot, \delta_o) = \arg \inf_{V \in V} E \left[ m(w_t, \delta_o, V)' m(w_t, \delta_o, V) \right]$$

- $m(w_t, \delta, V) = E\{\gamma(z_{t+1}, \delta, V)|w_t\}$

- $\gamma(z_{t+1}, \delta, V) = (\gamma_1(z_{t+1}, \delta, V), \ldots, \gamma_N(z_{t+1}, \delta, V))'$

- For any candidate $\delta \equiv (\beta, \rho, \theta)' \in \mathcal{D}$, define
  $V^* \equiv V^*(z_t, \delta) \equiv V^*(\cdot, \delta)$ as:

$$V^*(\cdot, \delta) = \arg \inf_{V \in V} E \left[ m(w_t, \delta, V)' m(w_t, \delta, V) \right]$$

- It is clear that $V_o(z_t, \delta_o) = V^*(z_t, \delta_o)$

- Estimate $\delta_o$ based on $Nd_x \times 1$ vector of unconditional moments:

$$E \left\{ \gamma_i(z_{t+1}, \delta_o, V^*(\cdot, \delta_o)) \otimes x_t \right\} = 0, \quad i = 1, \ldots, N.$$
Empirical Implementation: Two-step Estimator

1. First step: For any candidate $\delta \in \mathcal{D}$, an initial estimate of $V^*(\cdot, \delta)$ is obtained using the sieve minimum distance (SMD) estimator (Newey-Powell ’03, Ai-Chen ’03).
   - SMD procedure has two parts:
     (a) Replace the conditional expectation with a consistent, nonparametric estimator (specified later).
     (b) Approximate the unknown function $V_o$ by a sequence of finite dimensional unknown parameters (sieves) $V_{K_T}$.
        - Approximation error decreases as $K_T$ increases with $T$.

2. Consistent estimates of $\delta_o$ obtained by solving a sample minimum distance problem.
First Step SMD Estimator of $V^*$

- Recall $m(w_t, \delta_o, V_o) \equiv E \{ \gamma(z_{t+1}, \delta_o, V_o (\cdot, \delta_o))|w_t \} = 0$.

- First-step SMD estimate $\hat{V} (\cdot)$ for $V^* (\cdot)$ based on

$$\hat{V} (\cdot, \delta) = \arg \min_{V_{K_T}} \frac{1}{T} \sum_{t=1}^{T} \hat{m}(w_t, \delta, V_{K_T}(\cdot, \delta))^\prime \hat{m}(w_t, \delta, V_{K_T}(\cdot, \delta)),$$

- $\hat{m}(w_t, \delta, V_{K_T}(\cdot, \delta))$ any nonpara. estimator of $m(w_t, \delta_o, V_o)$.

- $V_{K_T}$ is a bivariate sieve used to approximate $V^* \left( \frac{V_{t-1}}{C_{t-1}}, \frac{C_t}{C_{t-1}}; \delta \right)$:

$$V^* \left( \frac{V_{t-1}}{C_{t-1}}, \frac{C_t}{C_{t-1}}; \delta \right) \approx V_{K_T} (\cdot, \delta) = a_0(\delta) + \sum_{j=1}^{K_T} a_j(\delta) B_j \left( \frac{V_{t-1}}{C_{t-1}}, \frac{C_t}{C_{t-1}} \right)$$

- Initial value for $\frac{V_t}{C_t}$ at time $t = 0$, denoted $\frac{V_0}{C_0}$, taken as a unknown scalar parameter to be estimated.
Second Step Estimator of $\delta_o$

- Estimate $\delta_o$ based on $Nd_x \times 1$ vector of unconditional moments:

$$E \{ \gamma_i(z_{t+1}, \delta_o, V^*(\cdot, \delta_o)) \otimes x_t \} = 0, \quad i = 1, \ldots, N.$$ 

- Sample analog denoted:

$$g_T(\delta, \hat{V}(\cdot, \delta); y^T) \equiv \frac{1}{T} \sum_{t=1}^T \gamma(z_{t+1}, \delta, \hat{V}(\cdot, \delta)) \otimes x_t$$

- Estimate $\delta_o$ by minimizing GMM objective:

$$\hat{\delta} = \arg \min_{\delta \in D} \left[ g(\delta, \hat{V}(\cdot, \delta); y^T) \right]' W \left[ g((\delta, \hat{V}(\cdot, \delta); y^T) \right]$$

- Two values $W = I$, $W = G_T$ where $(i,j)$th element of $G_T$ is

$$\frac{1}{T} \sum_{t=1}^T R_{i,t} R_{j,t}$$ for $i,j = 1, \ldots, N$.

- $\hat{V}(\cdot, \delta)$ not held fixed in this step: depends on $\delta$. 
Data


   - Consumption mimicking factor (Malloy, Moskowitz, Vissing-Jorgensen ’05): Regress $\bar{C}_t$ on aggregate variables, take fitted values. Why? Longer time-series, reduce measurement error.

3. Test asset returns $\{R_{i,t+1}\}_{i=1}^N$: 3-Mo T-bill, 6 size/book-market sorted equity returns (Fama and French). All stocks traded on NYSE, NASDAQ, AMEX.
Figure 1
Estimated Continuation Value-Consumption Ratio, Aggregate Consumption, $W=I$

Estimated Continuation Value-Consumption Ratio, Aggregate Consumption, $W=G_T$
Figure 2
Estimated Continuation Value-Consumption Ratio, Stockholder Consumption, $W=I$

Estimated Continuation Value-Consumption Ratio, Stockholder Consumption, $W=G_T$
<table>
<thead>
<tr>
<th>2nd Step Estimation</th>
<th>( \beta )</th>
<th>( \theta )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(95% CI)</td>
<td>(95% CI)</td>
<td>(95% CI)</td>
</tr>
<tr>
<td>Aggregate Consumption</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( W = I )</td>
<td>0.990</td>
<td>57.5</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>(.985, .996)</td>
<td>(27.5, 129)</td>
<td>(.24, .99)</td>
</tr>
<tr>
<td>( W = G_T^{-1} )</td>
<td>0.999</td>
<td>60</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>(.994, .9999)</td>
<td>(42,144)</td>
<td>(.20, .75)</td>
</tr>
<tr>
<td>Stockholder Consumption</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( W = I )</td>
<td>0.994</td>
<td>20.00</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>(.993, .9995)</td>
<td>(.25, 40)</td>
<td>(.38, 1.24)</td>
</tr>
<tr>
<td>( W = G_T^{-1} )</td>
<td>0.998</td>
<td>17.0</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>(.992, .9999)</td>
<td>(1, 43.3)</td>
<td>(.23, 1.01)</td>
</tr>
</tbody>
</table>
### Table 3
Specification Errors for Alternative Models: HJ Distance

<table>
<thead>
<tr>
<th>Model</th>
<th>HJ Dist (1)</th>
<th>HJ Dist (2)</th>
<th>HJ Dist (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aggregate Consumption</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recursive</td>
<td>0.451</td>
<td>0.591</td>
<td></td>
</tr>
<tr>
<td>CRRA Utility</td>
<td>0.514</td>
<td>0.627</td>
<td></td>
</tr>
<tr>
<td>Fama-French</td>
<td>0.363</td>
<td>0.515</td>
<td></td>
</tr>
<tr>
<td>Scaled CCAPM</td>
<td>0.456</td>
<td>0.625</td>
<td></td>
</tr>
<tr>
<td><strong>Stockholder Consumption</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recursive</td>
<td>0.463</td>
<td>0.605</td>
<td></td>
</tr>
<tr>
<td>CRRA Utility</td>
<td>0.517</td>
<td>0.627</td>
<td></td>
</tr>
<tr>
<td>Fama-French</td>
<td>0.363</td>
<td>0.515</td>
<td></td>
</tr>
<tr>
<td>Scaled CCAPM</td>
<td>0.490</td>
<td>0.620</td>
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</tr>
</tbody>
</table>
The Return to Aggregate Wealth and Human Wealth

- The unobservable $R_{w,t+1}$ can be inferred from our estimates of $V_t/C_t$:

$$\beta \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \left( \frac{V_{t+1} C_{t+1} C_t}{C_{t+1} C_t C_t} \right)^{\rho-\theta} = \left\{ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \right\} \left\{ \frac{1}{R_{w,t+1}} \right\}^{\frac{\theta-\rho}{1-\rho}}.$$

- Following Campbell (1996) assume that $R_w$ is portfolio weighted average of human wealth, $R_y$ and nonhuman (asset wealth), $R_a$:

$$R_{w,t+1} = (1 - \nu_t) R_{a,t+1} + \nu_t R_{y,t+1},$$

- Exercise similar in spirit to Lustig and Van Nieuwerburgh '06: investigate loglinearized EZW model, under assumption of joint lognormality and homoskedasticity.
**Table 6**

Summary Statistics for Return to Aggregate Wealth, Human Wealth, $W = I$

<table>
<thead>
<tr>
<th></th>
<th>Model-Implied Aggregate Wealth Return</th>
<th></th>
<th>Rep Stockholder</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Representative Agent</td>
<td>Rep Stockholder</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{w,t}$</td>
<td>$R_{CRSP,t}$</td>
<td>$R_{w,t}$</td>
<td>$R_{CRSP,t}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1.00$</td>
<td>$0.171$</td>
<td>$1.00$</td>
<td>$-0.049$</td>
</tr>
<tr>
<td>$R_{CRSP,t}$</td>
<td>$1.00$</td>
<td></td>
<td>$1.00$</td>
<td>$1.00$</td>
</tr>
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</table>

Panel A: Correlation Matrix

Panel B: Univariate Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>$R_{w,t}$</th>
<th>$R_{CRSP,t}$</th>
<th>$R_{w,t}$</th>
<th>$R_{CRSP,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.057</td>
<td>0.084</td>
<td>0.109</td>
<td>0.084</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.010</td>
<td>0.165</td>
<td>0.036</td>
<td>0.165</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.234</td>
<td>0.055</td>
<td>-0.08</td>
<td>0.055</td>
</tr>
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</table>
### Table 6, continued

#### Model-Implicit Human Wealth Return, $\nu = 0.333$

<table>
<thead>
<tr>
<th></th>
<th>Representative Agent $R_{y,t}$</th>
<th>$R_{CRSP,t}$</th>
<th>Rep Stockholder $R_{y,t}$</th>
<th>$R_{CRSP,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Correlation Matrix</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{y,t}$</td>
<td>1.00</td>
<td><strong>-0.996</strong></td>
<td>1.00</td>
<td><strong>-0.953</strong></td>
</tr>
<tr>
<td>$R_{CRSP,t}$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel B: Univariate Summary Statistics</strong></td>
<td>0.003</td>
<td>0.327</td>
<td>0.044</td>
</tr>
<tr>
<td>Mean</td>
<td>0.084</td>
<td>0.165</td>
<td>0.055</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.160</td>
<td>0.353</td>
<td>0.042</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.084</td>
<td>0.165</td>
<td>0.055</td>
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</table>

#### Model-Implicit Human Wealth Return, $\nu = 0.667$

<table>
<thead>
<tr>
<th></th>
<th>Representative Agent $R_{y,t}$</th>
<th>$R_{CRSP,t}$</th>
<th>Rep Stockholder $R_{y,t}$</th>
<th>$R_{CRSP,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Correlation Matrix</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{y,t}$</td>
<td>1.00</td>
<td><strong>-0.982</strong></td>
<td>1.00</td>
<td><strong>-0.847</strong></td>
</tr>
<tr>
<td>$R_{CRSP,t}$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel B: Univariate Summary Statistics</strong></td>
<td>0.043</td>
<td>0.082</td>
<td>0.036</td>
</tr>
<tr>
<td>Mean</td>
<td>0.084</td>
<td>0.165</td>
<td>0.055</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.121</td>
<td>0.101</td>
<td>0.016</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.084</td>
<td>0.165</td>
<td>0.055</td>
</tr>
</tbody>
</table>
Conclusion

• Purpose of this study is to help fill a gap in the literature, by providing a formal econometric evaluation of the EZW recursive utility model.

• We avoid proxying for the aggregate wealth return, linearizing the model, or placing tight parametric restrictions on law of motion or joint distribution of $C_t$ and returns, or on value of pref. params.

• Estimated RRA = 17 - 60 ≠ to inverse of EIS = 1.11 - 2.

• EZW model explains the data better than the standard CRRA model, better than the scaled CCAPM of Lettau-Ludvigson ’01.

• Aggregate wealth return is much less volatile than stock market return; Results suggest human wealth return is strongly negatively correlated with stock market return.