The Ultimatum Game: Interdependent Preferences in Experimental Setting

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Subject A (Proposer) chooses a number - \( p \) (demand, 1–offer) - between 0 and 1.

Subject B chooses “Yes” (accept) or “No” (reject).

Payments:
- If Responder rejects the payoffs are: \((0, 0)\)
- If Responder accepts the payoffs are: \((p, 1 - p)\)
Considerable variations between and within experiments and cultures:

- In most western societies modal and median offers are 40-50% of the pie.
- Offers below 20% of the pie are rejected about half of the time.
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- Higher demand results in higher probability of rejection.
- Expected revenue to Proposer (demand $\times$ probability of acceptance) is humped shaped, with offers slightly lower than 50% generating highest expected revenue.
- Intentions matter (e.g. Blount; Falk, Fehr, Fischbacher): it is not only the payment distribution that triggers rejection, but the fairness or information revealed as reflected in the offer.
Existing Interpretation of Experimental Results

Other Regarding Preferences: rejection of the selfish preferences assumption:
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- Interdependent Preferences: agent’s preferences may depend on her opponent’s type (e.g. altruistic, spiteful). The opponent’s action may affect both the material allocation and the inference the agent makes about the opponent type (Levine, Gul and Pesendorfer).
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- We should not limit ourselves to SPE but other NE that can be supported through an evolutionary process (Binmore, Samuelson, and others).
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- The set of preferences considered is large. We let the equilibrium select the subset of preferences that is consistent with the experimental data.
- Both “other-regarding” preferences and “rule rationality” can be recasted in this way.
- Test the equilibrium predictions of the model against existing models of inequity aversion and intention-based reciprocity. Results consistent with the equilibrium predictions of the proposed model.
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- This approach to modeling of the experiment enables to use the information from the experiment in mechanism design problems, where ultimatum game is involved.
Framework

- \( P \): a finite collection of \( n \) feasible demands by the proposer.
  \[
  0 = p_1 < p_2 < \ldots < p_n = 1.
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  Patience (rule rationality).

- $\alpha \in \{0, 1\}$ is the responder’s action ($\alpha = 1$ means “accept”, and $\alpha = 0$ means “reject”).
Proposer’s Payoff: Main Assumptions

\[ u_p(p, \alpha, s) \text{ function of demand, responder’s choice and proposer’s type} \]
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\( u_p(p, \alpha, s) \) function of demand, responder’s choice and proposer’s type

**Main Assumptions:**

- \( u_p(p, 1, s) - u_p(p, 0, s) \) is monotonically increasing in \( s \).
- Single crossing: if a proposer of type \( s \) weakly prefers a higher demand (with lower probability of acceptance), then a proposer of type \( s' < s \) would strictly prefer the higher demand.
Responder’s Payoff

\[ u_r (p, \alpha, s) \] function of demand, responder choice and proposer’s type.
\[ u_r (p, 0, s) - u_r (p, 1, s) \]: the responder’s marginal utility of rejecting a demand \( p \) if she knows the proposer’s type is \( s \).
Responder’s Payoff

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$u_r(p, 0, s) - u_r(p, 1, s)$: the responder’s marginal utility of rejecting a demand $p$ if she knows the proposer’s type is $s$.

Main Assumptions

The function $u_r(p, 0, s) - u_r(p, 1, s)$ is monotonically increasing and supermodular in $s$ and $p$. 
Negative Interdependence

High type proposers have a strong desire that responders accept offers. Responders who know this, on the other hand, are more inclined to reject. High type proposers would like to hide their type.
Possible Interpretations

Rule Rationality

A high type proposer might be a very impatient proposer against whom the responder could expect to do well in the continuation game.
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**Other-regarding: Greed**
A high type proposer may simply be a greedy proposer - responders want to reject offers by greedy proposers simply because they are greedy.
Other Possible Interpretation

Other-regarding: Spite

- The higher the proposer’s type, the more spiteful he is.
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**Other-regarding: Spite**

- The higher the proposer’s type, the more spiteful he is.
- The responder’s marginal utility of rejecting is increasing in the proposer’s spite.
- But the more spiteful the proposer is (as he knows that the responder’s payoff as a result of rejecting is increasing) - the more he would like the responder to accept.
Theorem

Let $p' > p$ be two demands made on the equilibrium path. The probability with which the demand $p$ is accepted is at least as large as the probability with which $p'$ is accepted.
Theorem

An ascending sequence of demands \((\pi_1, \ldots, \pi_K)\) can be supported as Perfect Bayesian Equilibrium demands if:

1. \(\pi_K = 1; \text{ and} \)

2. There exists a strictly descending sequence of \(K + 1\) types \((s_1, \ldots, s_K, s_{K+1})\) with \(s_1 = \underline{s}\) and \(s_{K+1} = \overline{s}\) satisfying

\[
\int_{s_{k+1}}^{s_k} \left\{ u_r(\pi_k, 0, s) - u_r(\pi_k, 1, s) \right\} dF(s) \leq 0
\]

with equality holding for all \(k\) except possibly for \(k = 1\).

3. Furthermore, every equilibrium in which all offers are accepted with positive probability satisfies (1) and (2).
Maximally Dispersed Equilibrium

\[
\int_{s_3}^{s_4} [u_r(1, 0, s) - u_r(1, 1, s)] dF(s) = 0
\]

Choose \( q_3 \) to make \( s_3 \) indifferent between \( l = \pi_3 \) and \( \pi_2 \)

\[
\int_{s_3}^{s_2} [u_r(\pi_2, 0, s) - u_r(\pi_2, 1, s)] dF(s) = 0
\]

Choose \( q_2 \) to make \( s_2 \) indifferent between \( \pi_2 \) and \( \pi_1 \)

\[
\int_{s_2}^{\bar{s}} [u_r(\pi_1, 0, s) - u_r(\pi_1, 1, s)] dF(s) \leq 0
\]

\( \bar{s} = s_1 \)

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Interdependent Preferences in Ultimatum Game
Theorem

Suppose that \( u_p(p, 1, s) = p\phi(s) \) for some strictly positive function \( \phi \) and that there is some proposer type \( s < \bar{s} \) such that \( u_p(0, 0, s) < 0 \). Then the function \( q_kp_k \) is decreasing when \( u_p(p, 0, s) \) is positive and increasing otherwise.
Experimental Investigation

- Experiment: setting an upper bound to demand (lower bound on offer)
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- Theoretical predictions:
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  - Inequity Aversion: concentration of demands around the upper bound. No change in demands beyond that. No change in acceptance probability.
Experimental Investigation

- Experiment: setting an upper bound to demand (lower bound on offer)
- Theoretical predictions:
  - Inequity Aversion: concentration of demands around the upper bound. No change in demands beyond that. No change in acceptance probability.
  - Intention-based reciprocity: perceived kindness diminishes. Acceptance rate decreases. Demand will weakly decrease.
Comparative Statics

Set $p \leq \pi_2$:

1. $1 = \pi_3$

$$\int_{\pi_2}^{\pi_3} \{u_i(\pi_2,0,s) - u_i(\pi_2,1,s)\}dF(s) < 0$$

$$\int_{\pi_2}^{\pi_1} \{u_i(\pi_2,0,s) - u_i(\pi_2,1,s)\}dF(s) = 0$$

2. Set $q'$ to make $s'$ indifferent between $\pi_2$ and $\pi_1$:

$$q'u_p(\pi_2,1,s') + (1-q')u_p(\pi_2,0,s') = u_p(\pi_1,1,s')$$

$$q_2u_p(\pi_2,1,s') + (1-q_2)u_p(\pi_2,0,s') < u_p(\pi_1,1,s')$$

Since $u_p(\pi_2,1,s') > u_p(\pi_2,0,s') \Rightarrow q' > q_2$

$$\int_{\tilde{s}}^{\pi_1} \{u_i(\pi_1,0,s) - u_i(\pi_1,1,s)\}dF(s) \leq 0$$
Equilibrium Predictions

- Lower offers (higher demands).
- Higher probability of acceptance of low offers.
Experimental Design

- Contest assignment to roles using “I Spy.”
- Understanding confirmed.
- Proposer-Experimenter-Responder anonymity.
- Recruitment: by e-mails to random sample of undergraduate students at UBC using Student Service Centre.
Procedure

- First round:
  - Each Proposer made offers to half of the Responders.
  - Each Responder received offers from half of the Proposers and decided whether to accept or reject each one of them.
  - Every Proposer got to see the responses to his offers.
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- **Second round:**
  - Each Proposer made offers to the other half of Responders.
  - Each Responder Received offers from the other half of the Proposers and decided whether to *accept* or *reject* each one.
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- Each Proposer made offers to the other half of Responders.
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Payments:
- $5 to Proposer independently of the game.
- Pie: $55, with no show-up fee.
- Payment determined by randomly choosing one offer to each Proposer and each Responder.
## Summary Results

<table>
<thead>
<tr>
<th></th>
<th>B-R1</th>
<th>B-R2</th>
<th>L-R1</th>
<th>L-R2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of proposers</td>
<td>12</td>
<td>12</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Average offer</td>
<td>19.07</td>
<td>21.21</td>
<td>15.31</td>
<td>15.00</td>
</tr>
<tr>
<td>Average acceptance rate</td>
<td>0.63</td>
<td>0.88</td>
<td>0.87</td>
<td>0.90</td>
</tr>
<tr>
<td>Within SD of offers</td>
<td>2.26</td>
<td>1.47</td>
<td>3.18</td>
<td>2.09</td>
</tr>
<tr>
<td>Total SD of offers</td>
<td>8.15</td>
<td>6.64</td>
<td>6.45</td>
<td>5.78</td>
</tr>
</tbody>
</table>
The Ultimatum Game Framework (Negative Interdependence)
Interpretation and Equilibrium
Experiment
Conclusion

Predictions
Experimental Design
Results

Distribution of Offers

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Acceptance Probability

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Expected Revenue to Proposer

![Graph showing Expected Revenue to Proposer](image_url)

- **X-axis (Offer)**: Represents the offer amount in increments of 5, starting from 0 to 35.
- **Y-axis (Expected Revenue)**: Represents the expected revenue for the proposer in increments of 5, starting from 0 to 45.

Legend:
- **Limit**
- **Base**

*Graph based on predictions and experimental design results by Yoram Halevy and Michael Peters.*
Concluding Comments

- We concentrate on the maximally dispersed equilibrium, that allows to extract maximum information from the Responder. As in other informed principal problems - other equilibria exist, which are less informative.
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- Dictator Games are excellent example how the experimenter may influence the outcomes (Hoffman, McCabe and Smith 1996; Cherry, Frykblom and Shogren 2002; Bradsley 2007; List 2007; Dana, Weber and Kuang 2007)
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- The insight of negative interdependence can be applied to mechanism design problems, labor contracts etc.
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Interdependent Preferences in Ultimatum Game