One or Two Monies?

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Motivation

- Micro-founded monetary theory consensus: Money acts as a substitute for missing record-keeping (Kocherlakota 1998a, 1998b)
- Most papers have a single money
- This paper: Money is divisible, concealable and in variable supply. A single money may or may not be a sufficient substitute. Two monies are a perfect substitute.
Approach

- Quasilinear environment as in Lagos and Wright (2005) w/o search friction
  Analytical tractability, no need to keep long history
- Mechanism design approach
  No market, solve a social planner’s problem subject to:
  - Resource constraints
  - Constraints imposed by frictions: lack of commitment and private info.
  - Constraints imposed by available memory technology:
    - Full record-keeping
    - One money
    - Two monies
Physical Environment

Figure 1: Environment

Type a: -z (Measure 1)

Type b: -z (Measure 1)

Day

Night

Type a: \( \delta u(c_1), \delta > 1 \)

Type b: \( u(c_1) \)

Type a: \( u(c_2) \)

Type b: \( \delta u(c_2), \delta > 1 \)

Location a

Location b

\( c_1 \): consumption of night good 1; \( c_2 \): consumption of night good 2;
\( z \): production (consumption if negative) of day good;
Shocks are i.i.d. across time and agents (of the same type).
Preferences

**Type a agent** $i \in (0, 1)$:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -z_t^a + \frac{1}{2} \left[ \delta u(c_{1,t}^a(i)) + u(c_{2,t}^a(i)) \right] \right\}.$$

$u(0) = 0$, $u' > 0$, $u'(0) = \infty$, $u'' < 0$.

**Type b agent** $j \in (0, 1)$:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -z_t^b(j) + \frac{1}{2} \left[ \delta u(c_{2,t}^b(j)) + u(c_{1,t}^b(j)) \right] \right\}.$$
∀ i and j ∈ (0, 1) and k ∈ \{a, b\},

Night
 ∀ t ≥ 0, \( c^a_{1,t}(i) = c^b_{2,t}(j) = c_h \), \( c^a_{2,t}(j) = c^b_{1,t}(i) = c_\ell \), \( c_h + c_\ell = 2y \)

Day
 \( z^a_0(i) = z^b_0(j) = 0; \)
\( \forall t \geq 1, z^k_t(\cdot) = \begin{cases} z_h, & \text{if consumed } c_h \text{ at } t - 1; \\ z_\ell, & \text{if consumed } c_\ell \text{ at } t - 1; \end{cases} \)
First-best Allocation

Night

\[ \delta u'(c^*_h) = u'(c^*_\ell) \begin{cases} \quad c^*_h + c^*_\ell = 2y \end{cases} \longrightarrow c^*_h > y > c^*_\ell; \]

Day

\[ z^a_0(i) = z^b_0(j) = 0; \]

\[ \forall t \geq 1, \text{ any } \{z_h, z_\ell\} \text{ that satisfies } z_h + z_\ell = 0. \]
Lack of Commitment – IR

Punishment: perpetual autarky \( W_0 = \frac{1}{2} \frac{1}{1-\beta} [\delta u(y) + u(y)] \)

Night stage

\[
\delta u(c_h) + \beta (-z_h + W) \geq \delta u(y) + \beta W_0; \quad \text{(NIRH)} \\
u(c_\ell) + \beta (-z_\ell + W) \geq u(y) + \beta W_0; \quad \text{(NIRL)}
\]

Day stage

\[
-z_h + W \geq W_0; \quad \text{(DIRH)} \\
-z_\ell + W \geq W_0. \quad \text{(DIRL)}
\]

\((\text{DIRH}) \rightarrow (\text{NIRH}); (\text{NIRL}) \rightarrow (\text{DIRL})\) if \( c_h \geq y \geq c_\ell \).
Private Information – IC

▶ Types, and thus valuation of night consumption are private information.

▶ Two ways to meet ICs

▶ Report type before pref. shocks – early sorting

\[
\frac{1}{1 - \beta} \left\{ \frac{\delta u(c_h) - \beta z_h}{2} + \frac{u(c_\ell) - \beta z_\ell}{2} \right\} \geq \frac{1}{1 - \beta} \left\{ \frac{\delta u(c_\ell) - \beta z_\ell}{2} + \frac{u(c_h) - \beta z_h}{2} \right\};
\]

Holds if \( c_h \geq c_\ell \).

▶ Report valuation after pref. shocks – late sorting

\[
\delta u(c_h) + \beta(-z_h + W) \geq \delta u(c_\ell) + \beta(-z_\ell + W) \quad \text{(ICH)}
\]

\[
u(c_\ell) + \beta(-z_\ell + W) \geq u(c_h) + \beta(-z_h + W) \quad \text{(ICL)}
\]
Allocation with Record-keeping

ICT satisfied at \((c_h^*, c_\ell^*)\);
DIRH and NIRL;
1st-best iff \(\beta \geq \frac{[u(y) - u(c_\ell^*)] / \{\delta[u(c_h^*) - u(y)]\}}{[u(c_h^*) - u(y)]} \equiv \beta_0\).
Single Money Mechanism

**Limited commitment**

- Reward participants with money and increase the amount of money required for future participation.
- Catch nonparticipants cast them into perpetual autarky.
- IR conditions same as with record-keeping.
Single Money Mechanism Cont’d

Private information

- Early sorting ineffective
- Incentives aligned through late sorting (ICH and ICL)
  - All agents enter the night stage with same amount of money;
  - High valuation consumers consume more at night and exit
    with less money balances, work during day.
- 1st-best iff
  \[ \beta \geq \frac{u(c^*_h) - u(c^*_\ell)}{(\delta + 1)[u(c^*_h) - u(y)]} \equiv \beta_1 > \beta_0. \]
Two-Money Mechanism

IR constraints same as before.

Private info.: induce types to hold different portfolios with *same total money balances*.

\[
(r_a + g_a = r_b + g_b, \ r_a > r_b)
\]

\[
\begin{align*}
\text{Location } a & : (r_a, g_a), c_h \\
\text{Location } b & : (r_b, g_b), c_l \\
\end{align*}
\]

→ Cannot change portfolios to mimic the other type.

→ Early sorting effective.

→ Achieve first-best iff \( \beta \geq \beta_0 \).
Two Monies as Perfect Substitute for Record-keeping

- If money divisible and non-concealable, 
  One-to-one correspondence between records and money balances 
  $\rightarrow$ One money perfect substitute.
- If money divisible and concealable, 
  - Higher balances concealed to imitate smaller balances 
    $\rightarrow$ One-to-one correspondence destroyed - one money imperfect substitute. 
  - One-to-one correspondence between records and monetary portfolios with the same total money balances 
    $\rightarrow$ Two monies perfect substitute.
Summary

- Limited commitment
  Rewarding participants with more money → one money works well

- Private information
  Two ways to meet IC: early and late sorting.
  One money: only late sorting effective
  Two monies: early sorting rehabilitated

- Agents are patient enough
  → late sorting powerful enough for ICH, ICL and IRs
  → a single money sufficient.

- Agents are impatient enough
  → second money essential by allowing for early sorting.

- Two monies are perfect substitute for record-keeping
  → no need to introduce a third money.

- Future research: dencentralization
  Market mechanism + fiscal and monetary policy
Comparison with existing literature

- **Kocherlakota (1999):**
  - Money indivisible - **more monies needed if there are > 2 types of agents.**
  - (Our model) Day stage with linear preferences - 2nd money not essential if agents patient enough.

- **Kocherlakota (2002):**
  - A second money necessary to deal with limited commitment. **Money in fixed supply**
  - With a single money, participating producers and consumers have different amounts of money → inconsistent with first-best
  - Introduce a second money to serve as receipts → all participants have the same total balances of money
Single Money Mechanism

At date 0 night stage, each agent gets $1.

At date 0 night stage, after shocks,

\[ \tau^{**} = y - c^*_\ell; \quad 0 < \rho_h < \rho_\ell < 1. \]

If show 1 $, \begin{cases} \text{receive } \tau^* \text{ good (1 or 2) and } \rho_h \text{ $; or} \\ \text{use } \tau^* \text{ good (1 or 2) to exchange for } \rho_\ell \text{ $}. \end{cases}
At date 1 day stage,
Show \((1 + \rho_h) \) $, use \(z_h \rightarrow (1 - \rho_h) \$; 
Show \((1 + \rho_\ell) \$), use \(z_\ell \rightarrow (1 - \rho_\ell) \$.

At date 1 night stage,
If show 2 $, \left\{ \begin{array}{l}
\text{receive } \tau^* \text{ good (1 or 2) and } \rho_h \$; \text{ or } \\
\text{use } \tau^* \text{ good (1 or 2) to exchange for } \rho_\ell \$.
\end{array} \right.$
At date $t \geq 2$ day stage, the choices are:

Show $(t + \rho_h) \$, use $z_h \rightarrow (1 - \rho_h) \$;$

Show $(t + \rho_\ell) \$, use $z_\ell \rightarrow (1 - \rho_\ell) \$;$

At date $t \geq 2$ night stage, the choices are:

If show $(t + 1) \$, then

\begin{align*}
\text{receive } & \tau^* \text{ good (1 or 2) and } \rho_h \$; \text{ or } \\
\text{use } & \tau^* \text{ good (1 or 2) to exchange for } \rho_\ell \$.
\end{align*}
Two-Money Mechanism

At date 0 day stage, 1 \( R \) or 1 \( G \)?

At date 0 night stage, after the shocks are realized, \( 0 < \rho < 1 \);

At location \( a \),
- If show 1 \( R \), receive \( \tau^* \) good 1 and \( \rho \ R \);
- If show 1 \( G \), use \( \tau^* \) good 1 to exchange for \( \rho \ R \);

At location \( b \),
- If show 1 \( G \), receive \( \tau^* \) good 2 and \( \rho \ G \);
- If show 1 \( R \), use \( \tau^* \) good 2 to exchange for \( \rho \ G \);
Two-Money Mechanism Cont’d

At date 1 day stage,

If show \((1 + \rho) R\), use \(z_h \rightarrow (1 - \rho) R\);
If show \((1 + \rho) G\), use \(z_h \rightarrow (1 - \rho) G\);
If show \(R + \rho G\), use \(z_\ell\) and \(\rho G \rightarrow 1 R\);
If show \(G + \rho R\), use \(z_\ell\) and \(\rho R \rightarrow 1 G\);

At date 1 night stage,

At location \(a\),
If show 2 \(R\), receive \(\tau^*\) good 1 and \(\rho R\);
If show 2 \(G\), use \(\tau^*\) good 1 to exchange for \(\rho R\);

At location \(b\),
If show 2 \(G\), receive \(\tau^*\) good 2 and \(\rho G\);
If show 2 \(R\), use \(\tau^*\) good 2 to exchange for \(\rho G\);
Two-Money Mechanism Cont’d

At date $t \geq 2$ day stage,

If show $(t + \rho) \, R$, use $z_h \rightarrow (1 - \rho) \, R$;
If show $(t + \rho) \, G$, use $z_h \rightarrow (1 - \rho) \, G$;
If show $t \, R + \rho \, G$, use $z_{\ell}$ and $\rho \, G \rightarrow 1 \, R$;
If show $t \, G + \rho \, R$, use $z_{\ell}$ and $\rho \, R \rightarrow 1 \, G$;

At date $t \geq 2$ night stage,

At location $a$,
If show $(t + 1) \, R$, receive $\tau^*$ good 1 and $\rho \, R$;
If show $(t + 1) \, G$, use $\tau^*$ good 1 to exchange for $\rho \, R$;

At location $b$,
If show $(t + 1) \, G$, receive $\tau^*$ good 2 and $\rho \, G$;
If show $(t + 1) \, R$, use $\tau^*$ good 2 to exchange for $\rho \, G$;
Kocherlakota (2002)

- Preference: \( E_0 \sum_{t=0}^{\infty} \beta^t U_t \) with \( U_t = \begin{cases} -y_t & \text{w.p. } 1/2; \\ u(c_t) & \text{w.p. } 1/2; \end{cases} \)

\( 0 < \beta < 1 \): discount factor; \( c_t \): consumption, \( y_t \): production. \( u'' < 0 < u', \ u(0) = -\infty \).

- Stationary first-best: \( u'(c^*) = 1; \ y^* = c^* \).

- No record-keeping

- Public information about technology shocks

- Limited commitment - participation constraint
Kocherlakota (2002) - Single-money Mechanism

Endow everybody with $1.

At period 0
1$, consumer, $0.1$ → $y^*$
1$, producer, $y^*$ → $0.1$
o/w, no trade.

At period 1
1.1$ or $0.9$, consumer, $0.01$ → $y^*$
1.1$ or $0.9$, producer, $y^*$ → $0.01$
o/w, no trade.

If money nonconcealable, everybody trades $[u(0) = -\infty]$.

If money concealable, producer can choose ‘no trade’ in period 0, and claim in period 1 that he was buyer by hiding $0.1$. 
Kocherlakota (2002) - Two-money Mechanism

Two monies: $R$ and $G$. Endow everybody with $(1R, 1G)$.

At period 0
$(1R, 1G)$, consumer, $0.1R \rightarrow (y^*, 0.1G)$
$(1R, 1G)$, producer, $(y^*, 0.1G) \rightarrow 0.1R$
o/w, no trade.

At period 1
$(1.1R, 0.9G)$ or $(0.9R, 1.1G)$, consumer, $0.01R \rightarrow (y^*, 0.01G)$
$(1.1R, 0.9G)$ or $(0.9R, 1.1G)$, producer, $(y^*, 0.01G) \rightarrow 0.01R$
o/w, no trade.

Even if money is concealable, nobody chooses ‘no trade’: cannot claim to have been a consumer or well-behaved producer.