Endogenous Valuation of Collateral and Ex Ante Trade in Market Fundamentals

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The possibility of trade in spot markets creates the externality, as spot prices and the bindingness of collateral constraints interact.

We allow for both “tranching” and “pyramiding”, e.g., collateralized mortgage obligations.

Allowing agents to buy the right to contract ex ante on market fundamentals, determining spot prices, restores efficiency of competitive equilibria.

In equilibrium, some agents are paying to trade in a price island, and others are being paid to trade in the same island.
Related Literature: Incomplete List

Timing, States, and Commodities

- Two periods: \( t = 0, 1 \).
- In period 1: the probability of state \( s \) is \( \pi_s \), for \( s = 1, 2, \ldots, S \) such that \( \sum_s \pi_s = 1 \).
Timing, States, and Commodities

- Two periods: $t = 0, 1$.
- In period 1: the probability of state $s$ is $\pi_s$, for $s = 1, 2, ..., S$ such that $\sum_s \pi_s = 1$.
- Two physical goods: good-1 and good-2.
  - Good-1 in not storable or is perishable, used as a numeraire good.
  - Good-2 is storable, and can be used as collateral, henceforth called collateral good.
There are $H$ types of continuum agents, $h = 1, \ldots, H$, each of which consists of $\alpha^h$ fraction of population with $\sum_h \alpha^h = 1$. 
Preferences, Endowment and Technology

- There are $H$ types of continuum agents, $h = 1, \ldots, H$, each of which consists of $\alpha^h$ fraction of population with $\sum_h \alpha^h = 1$.
- Endowment of agent $h$ in period-0: $e^h_0 = (e^h_{10}, e^h_{20})$.
- Endowment of agent $h$ in state $s$: $e^h_s = (e^h_{1s}, e^h_{2s})$, $s = 1, \ldots, S$. 
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- Endowment of agent $h$ in period-0: $e_0^h = (e_{10}^h, e_{20}^h)$.
- Endowment of agent $h$ in state $s$: $e_s^h = (e_{1s}^h, e_{2s}^h)$, $s = 1, \ldots, S$.
- Storage technology: One unit of good-2 will become $R_s$ units of good-2 in state $s$. Note: $R_s$ could be constant across states.
Preferences, Endowment and Technology

- **Expected utility:**

\[
U \left( c_{10}^h, c_{20}^h \right) + \beta \sum_{s=1}^{S} \pi_s U \left( c_{1s}^h, c_{2s}^h \right)
\]

- \( (c_{10}^h, c_{20}^h) \) - consumption of good-1 and good-2 in period-0,
- \( (c_{1s}^h, c_{2s}^h) \) - the consumption of good-1 and good-2 in state \( s \).
Preferences, Endowment and Technology

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- \( (c_{1s}^h, c_{2s}^h) \) - the consumption of good-1 and good-2 in state \( s \).
- \( U \left( c_{1}^h, c_{2}^h \right) \) is homothetic, continuous, strictly concave, strictly increasing in both arguments, and satisfies the usual Inada conditions.

- The preferences are identically homothetic; the ratio of good-1 to good-2 determines the marginal rate of substitution.
Market Fundamentals

- Market fundamental in state $s$, $z_s$, determines the spot price of good-2. It is a sufficient statistic for the spot price.
- With homothetic preferences, $z_s$ is the aggregate ratio of good-1 to good-2:

$$z_s = \frac{\sum_h \alpha^h e_{1s}^h}{\sum_h \alpha^h \left[ e_{2s}^h + R_s k^h \right]}$$

where $k^h$ is the collateral held by agent $h$ in period-0.
- With market fundamental $z_s$, spot-market-clearing price is $p(z_s)$. 
State-Contingent Collateralized Contracts

- The payoff of a contract paying one unit of good-1 in state $s$:
  1. **Promises** to pay $= \begin{cases} 1 & \text{if the state is } s \\ 0 & \text{otherwise} \end{cases}$
  2. **Collateral** requirement: $\hat{C}$ units of good-2 in period-0.
State-Contingent Collateralized Contracts

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  - **Promises** to pay $= \begin{cases} 1 & \text{if the state is } s \\ 0 & \text{otherwise} \end{cases}$
  - **Collateral** requirement: $\hat{C}$ units of good-2 in period-0.

- Due to limited commitment, the payoff of the contract in state $s$ (in units of good-1) is given by

  $$\hat{D} = \begin{cases} \min \left(1, p(z_s)R_s\hat{C}\right) & \text{if state is } s \\ 0 & \text{otherwise} \end{cases}$$

  No further penalty other than losing collateral.

- Lemma: No loss to consider only no-default contracts $\implies$ collateral constraints.
Commodity Space

- \( x^h(b) = x^h(c_0, k, \hat{\theta}, \theta, z, \Delta) \) is the probability or fraction of agent type \( h \) of receiving bundle \( (c_0, k, \hat{\theta}, \theta, z, \Delta) \):
  1. \( c_0 = (c_{10}, c_{20}) \) - consumption allocation in \( t = 0 \),
  2. \( k \) - collateral holding in \( t = 0 \),
  3. \( \hat{\theta} \equiv (\hat{\theta}_s)_{s=1}^S \) - contracts paying in \textit{good-1} traded within island-\( z_s \),
  4. \( \theta \equiv (\theta_s)_{s=1}^S \) - contracts paying in \textit{good-2} traded within island-\( z_s \),
  5. \( z = (z_s)_{s=1}^S \) - market fundamentals in \( t = 1 \),
  6. \( \Delta \equiv (\Delta_s)_{s=1}^S \) - “\textit{individual deviation from the fundamental}”:

\[
\Delta_s = z_s (e_{2s}^h + R_s k) - e_{1s}^h
\]  (1)

- \( \Delta_s \) is determined by \( z_s \) and \( k \), chosen by the agent.
Consumption Possibility Set

- Let $\mathcal{B}$ be the set of $n$ feasible bundles, and let $b = \left(c_0, k, \hat{\theta}, \theta, z, \Delta\right)$ be a typical bundle.
- Lotteries: $x^h \equiv \left[x^h(b)\right]_{b \in \mathcal{B}} \in \mathbb{R}^n$.
- Collateral constraints: the net-value of collateral allocation, $k$, and contracts (regardless of collateral units), $\left(\hat{\theta}_s, \theta_s\right)$, must be larger than zero.

$$p(z_s) R_s k + \hat{\theta}_s + p(z_s) \theta_s \geq 0, \ \forall s \quad (2)$$

- The consumption possibility set for $h$:

**Definition (Consumption Possibility Set)**

$$X^h = \left\{ x^h \in \mathbb{R}^n_+ : \sum_{b \in \mathcal{B}} x^h(b) = 1, \text{ and } x^h(b) \text{ satisfies (1) – (2)} \right\}$$

- An allocation $x \equiv \left(x^h\right)_{h=1}^{H} \in X$ where $X \equiv X^1 \times \ldots \times X^H$. 

p(z_s) R_s k + \hat{\theta}_s + p(z_s) \theta_s \geq 0, \ \forall s \quad (2)
The expected utility of an agent type $h$, holding a lottery $x^h$, is given by

$$
\sum_{(c_0,k,\hat{\theta},\theta,z,\Delta)} x^h \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) \left\{ U(c_{10}, c_{20}) + \beta V^h \left( k, \hat{\theta}, \theta, z \right) \right\}
$$

where

$$
V^h \left( k, \hat{\theta}, \theta, z \right) = \sum_s \pi_s U \left( e_{1s}^h + \hat{\theta}_s, e_{2s}^h + R_s k + \theta_s \right)
$$

$V^h \left( k, \hat{\theta}, \theta, z \right)$ depends also on agent $h$’s endowment profile.
Definition (Pareto Program 2)

\[
\max_{x \in \mathcal{X}} \sum_h \lambda^h \alpha^h \sum x^h \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) \left\{ U(c_{10}, c_{20}) + \beta V^h \left( k, \hat{\theta}, \theta, z \right) \right\}
\]

subject to

\[
\sum_h \sum \alpha^h x^h \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) c_{10} \leq \sum_h \alpha^h e^h_{10}
\]

\[
\sum_h \sum \alpha^h x^h \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) \left( c_{20} + k \right) \leq \sum_h \alpha^h e^h_{20}
\]

\[
\sum_h \sum \alpha^h x^h \left( c_0, k, \hat{\theta}, \theta, z_{-s}, z_s, \Delta \right) \hat{\theta}_s = 0, \forall s, z_s
\]

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\sum_h \sum \alpha^h x^h \left( c_0, k, \hat{\theta}, \theta, z_{-s}, z_s, \Delta \right) \theta_s = 0, \forall s, z_s
\]

\[
\sum_h \sum \alpha^h x^h \left( c_0, k, \hat{\theta}, \theta, z_{-s}, z_s, \Delta \right) \Delta_s = 0, \forall s, z_s
\]

where \( z_{-s} = [z_1, \ldots, z_{s-1}, z_{s+1}, \ldots, z_S] \).
Decentralized Trade in Market Fundamentals: Consumers

Definition (Consumer’s Problem)

Each agent $h$, taking prices $\left( P_{20}, P \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) \right)$ as given, chooses $x^h$ in period $t = 0$ to maximize its expected utility:

$$\max_{x^h \in X^h} \sum_{(c_0,k,\hat{\theta},\theta,z,\Delta)} x^h \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) \left\{ U(c_{10},c_{20}) + \beta V^h \left( k, \hat{\theta}, \theta, z \right) \right\}$$

subject to

$$e_{10}^h + P_{20} e_{20}^h - \sum_{(c_0,k,\hat{\theta},\theta,z,\Delta)} P \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) x^h \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) \geq 0$$
Decentralized Trade: Market-Makers

- Market-Makers: with constant returns to scale, assume that there is a single representative market-maker.
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- The market-maker issues $y \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right)$ units of each bundle $b \in B$, at the unit price $P \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right)$ (in unit of good-1 in $t = 0$).
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- In order to deliver the promises, the market-maker buys \( C_1 \) units of good-1 and \( C_2 + I \) units of good-2 at price \( P_{20} \), in \( t = 0 \).
Definition (Market-Maker’s Problem)

\[
\max_{(y,C_1,C_2,I)} \sum y \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) P \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) - \left[ C_1 + P_{20} C_2 + P_{20} I \right]
\]

subject to

\[
\sum y \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) c_{10} = C_1
\]

\[
\sum y \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) c_{20} = C_2
\]

\[
\sum y \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) k = I
\]

\[
\sum y \left( c_0, k, \hat{\theta}, \theta, z_{-s}, z_s, \Delta \right) \hat{\theta}_s = 0, \ \forall s, z_s
\]

\[
\sum y \left( c_0, k, \hat{\theta}, \theta, z_{-s}, z_s, \Delta \right) \theta_s = 0, \ \forall s, z_s
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Decentralized Trade: Market-Makers

The market-maker’s optimal condition is

\[ P \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) \leq c_{10} + P_{20} c_{20} + P_{20} k + \sum_{s=1}^{S} \hat{P}_a(z_s, s)\hat{\theta}_s + \sum_{s=1}^{S} P_a(z_s, s)\theta_s + \sum_{s=1}^{S} P_\Delta(z_s, s)\Delta_s \]

The term \( P_\Delta(z_s, s)\Delta_s \in \mathbb{R} \) is the net-price of the right to trade in each island.

It depends on the “individual deviation from the fundamental”, \( \Delta_s \in \mathbb{R} \).

Some agents are paying to trade in an island-\( z_s \), and others are being paid to trade in the same island.
Decentralized Trade: Market-Clearing Conditions

Definition (Market-Clearing Conditions)

Markets for good-1 and good-2 in $t = 0$ clear:

$$
\sum_{(c_0, k, \hat{\theta}, \theta, z, \Delta)} y \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) c_{10} = \sum_{h} \alpha^h e_{10}^h \\
\sum_{(c_0, k, \hat{\theta}, \theta, z, \Delta)} y \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) [c_{20} + k] = \sum_{h} \alpha^h e_{20}^h
$$

and the markets for lotteries in $t = 0$ clear:

$$
\sum_{h} \alpha^h x^h \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right) = y \left( c_0, k, \hat{\theta}, \theta, z, \Delta \right)
$$

for every $\left( c_0, k, \hat{\theta}, \theta, z, \Delta \right)$. 
Theorems

Theorem

With local nonsatiation of preferences, a competitive equilibrium allocation is (collateral constrained) Pareto optimal.

Theorem

Any Pareto optimal allocation corresponding with strictly positive Pareto weights $\lambda > 0$ can be supported as a competitive equilibrium with transfers.

Theorem

For any given positive distribution of endowments, a competitive equilibrium exists.
An Example with Intertemporal Transfers

- No uncertainty $S = 1$. Discount factor $\beta = 1$.
- Two types of agents $H = 2$, with $\alpha^h = \frac{1}{2}$ for all $h$. 
An Example with Intertemporal Transfers

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An Example with Intertemporal Transfers

- No uncertainty $S = 1$. Discount factor $\beta = 1$.
- Two types of agents $H = 2$, with $\alpha^h = \frac{1}{2}$ for all $h$.
- Storage technology is constant with $R = 1$.
- Utility function:

$$U(c_1, c_2) = \frac{c_1^{1-\gamma}}{1-\gamma} + \frac{c_2^{1-\gamma}}{1-\gamma}$$

where $\gamma = 2$.

- Endowments: agent $h = 1$ is relatively well-endowed in period $t = 0$, and vice versa.

<table>
<thead>
<tr>
<th>Type of Agent</th>
<th>$t = 0$</th>
<th>$t = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e_{10}^h$</td>
<td>$e_{20}^h$</td>
</tr>
<tr>
<td>$h = 1$</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$h = 2$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
First-Best Allocation

- With $\beta = 1$, no saving, i.e. $k^h = 0$ for all $h$.
- $P_{20}^{fb} = 1$
- $z = 1$, and hence $p(z) = 1$.
- Perfect consumption smoothing: $c_{it}^h = 2$, for all $h = 1, 2$, $i = 1, 2$ and $t = 0, 1$. 
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- First-Best allocation:

<table>
<thead>
<tr>
<th>Type of Agent</th>
<th>$k^h$</th>
<th>$\hat{\theta}^h$</th>
<th>$\theta^h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h = 1$</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$h = 2$</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>

- Agents type 2 will borrow (negative $\hat{\theta}^h$ and $\theta^h$).
- Her collateral constraint is violated:

$$p(z)Rk^h + \hat{\theta}^h + p(z)\theta^h = 0 - 1 - 1 = -2 < 0$$

- This FB-allocation is not collateral-constrained feasible.
Collateral Equilibrium without Lotteries

**Definition (Collateral Equilibrium)**

A collateral equilibrium is a specification of the prices \( \left( P_{20}, \hat{P}_a, P_a, p(z_s) \right) \), and an allocation \( \left( c^h_0, k^h, \hat{\theta}^h, \theta^h \right)_h \) such that:

- taking prices \( \left( P_{20}, \hat{P}_a, P_a, p(z_s) \right) \) as given, for any \( h \), \( \left( c^h_0, k^h, \hat{\theta}^h, \theta^h \right) \) solves

\[
\max_{c_0^h, k^h, \hat{\theta}^h, \theta^h} U \left( c_0^{h, 0}, c_20^{h, 0} \right) + \beta \sum_s \pi_s U \left( e_1^h + \hat{\theta}^h_s, e_2^h + R_s k^h + \theta^h_s \right)
\]

subject to

\[
c_{10}^h - e_{10}^h + P_{20} \left[ c_{20}^h + k^h - e_{20}^h \right] + \hat{P}_a \cdot \hat{\theta}^h + P_a \cdot \theta^h \leq 0
\]

\[
p(z_s) R_s k^h + \hat{\theta}^h_s + p(z_s) \theta^h_s \geq 0, \ \forall s
\]
Collateral Equilibrium without Lotteries

Definition (Collateral Equilibrium Continued)

- all markets clear:

\[
\sum_{h} \alpha^h \left[ c^h_{10} - e^h_{10} \right] = 0
\]

\[
\sum_{h} \alpha^h \left[ c^h_{20} + k^h - e^h_{20} \right] = 0
\]

\[
\sum_{h} \alpha^h \hat{\theta}^h_s = 0, \quad \forall s
\]

\[
\sum_{h} \alpha^h \theta^h_s = 0, \quad \forall s
\]

- the market fundamental in state \( s \) is

\[
z_s = \frac{\sum_{h} \alpha^h e^h_{1s}}{\sum_{h} \alpha^h \left( e^h_{2s} + R_s k^h \right)}
\]
Collateral Equilibrium Allocation (without Lotteries)

- Collateral allocation: \( k^1 = 1.3595 \) and \( k^2 = 0 \).
- An agent type 2 (poor agent) would like to borrow but holds no collateral: cannot borrow.
- In equilibrium, with high price of collateral, holding collateral and borrowing against it leads to zero net-transaction: a general equilibrium phenomenon.
- \( P_{20}^{cc} = 2.2948 > P_{20}^{fb} \): the distortion (need for collateralization) induces higher price collateral.
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- \( P_{20}^{cc} = 2.2948 > P_{20}^{fb} \): the distortion (need for collateralization) induces higher price collateral.
- \( z = 0.7463 \), and hence \( p(z) = 0.5570 \): the distortion generates a reduction of the spot price.
- Imperfect consumption smoothing:

<table>
<thead>
<tr>
<th>Agent</th>
<th>( c_{10}^h )</th>
<th>( c_{20}^h )</th>
<th>( c_{11}^h )</th>
<th>( c_{21}^h )</th>
<th>( \mathcal{U}_{cc}^h )</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h = 1 )</td>
<td>2.6899</td>
<td>1.7756</td>
<td>1.3252</td>
<td>1.7756</td>
<td>-2.2527</td>
<td>NB</td>
</tr>
<tr>
<td>( h = 2 )</td>
<td>1.3101</td>
<td>0.8649</td>
<td>2.6748</td>
<td>3.5839</td>
<td>-2.5724</td>
<td>B</td>
</tr>
</tbody>
</table>
Competitive Equilibrium with Lotteries

- All agents are assigned to a single island with market fundamental $z = 0.7729$.
- Collateral allocation: $k_1 = 1.1750$ and $k_2 = 0$.
- $P_{20}^{\text{lott}} = 2.0115 < P_{20}^{\text{cc}} = 2.2948$: the distortion (externality) induces higher price collateral.
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<tr>
<th>Agent</th>
<th>$c^h_{10}$</th>
<th>$c^h_{20}$</th>
<th>$c^h_{11}$</th>
<th>$c^h_{21}$</th>
<th>$U^h_{lott}$</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h = 1$</td>
<td>2.6133</td>
<td>1.8450</td>
<td>1.2970</td>
<td>1.6779</td>
<td>-2.2821</td>
<td>NB</td>
</tr>
<tr>
<td>$h = 2$</td>
<td>1.3867</td>
<td>0.9800</td>
<td>2.7030</td>
<td>3.4971</td>
<td>-2.4329</td>
<td>B</td>
</tr>
</tbody>
</table>

- Agents type 1 is worse off, $U^h_{lott} = -2.2821 < U^h_{cc} = -2.2527$, but agents type 2 is better off, $U^h_{lott} = -2.4329 > U^h_{cc} = -2.5724$: inequality is reduced in the lottery equilibrium relative to the collateral equilibrium.
Competitive Equilibrium with Lotteries

- Equilibrium prices:

<table>
<thead>
<tr>
<th>Prices</th>
<th>$z = 0.7479$</th>
<th>$z = 0.7729$</th>
<th>$z = 0.7979$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(z)$</td>
<td>0.5594</td>
<td>0.5974</td>
<td>0.6366</td>
</tr>
<tr>
<td>$\tilde{P}(z)$</td>
<td>4.2308</td>
<td>4.0621</td>
<td>3.9257</td>
</tr>
<tr>
<td>$P(z)$</td>
<td>2.3668</td>
<td>2.4269</td>
<td>2.4996</td>
</tr>
<tr>
<td>$P_\Delta(z)$</td>
<td>0.4639</td>
<td>0.5375</td>
<td>0.6118</td>
</tr>
</tbody>
</table>

- $P_\Delta(z_s, s)$ is increasing in $z_s$:
  1. if $\Delta_s > 0$ (endowed with more of good-2): it is better to be in an island with high $z_s$. Hence, need to pay to be in an island with high $z_s$.
  2. if $\Delta_s < 0$ (endowed with more of good-1): it is better to be in an island with low $z_s$. Hence, need to be paid to be in an island with high $z_s$. 
Conclusion

- The possibility of trade in spot markets creates the externality, as spot prices and the bindingness of collateral constraints interact.

- Allowing agents to buy the right to contract ex ante on market fundamentals, determining spot prices, restores efficiency of competitive equilibria: this could be normative.

- In equilibrium, some agents are paying to trade in a price island, and others are being paid to trade in the same island.

- With “tranching” and “pyramiding”, we should be able to use this model to study “liquidity crisis” and “liquidity” in general?