Asset trading and valuation with uncertain exposure

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June 21, 2008
...As I have mentioned, financial markets continue to be under considerable stress. Heightened investor concerns about the credit quality of mortgages,... triggered the financial turmoil. However, other factors, including ... uncertainties about the exposures of major financial institutions to credit losses have also roiled the financial markets in recent months...

Chairman Ben S. Bernanke.
February 27, 2008.
OBJECTIVE

• Study how the presence of agents who are affected asymmetrically in different states (heterogeneous exposure) may affect asset prices in an economy with asymmetric information.

• We consider a canonical Arrow-Debreu economy in which agents have partial information about
  – the probability distribution over future state realizations,
  – the distribution of individual exposure levels.

• What are the biases introduced when our setup with full information is used to account for data generated by the model with asymmetric information?
**Findings**

- Agents tend to be more optimistic about the state to which they are relatively more exposed.

- The presence of asymmetric information magnifies the sensitivity of prices to aggregate exposure but not the sensitivity of prices to expected payoffs.

- Identify cases in which an outside observer who reads data through the lens of the model without informational frictions may (incorrectly) infer that agents are overly pessimistic or optimistic.

- Model may help to generate a stronger negative relationship between stock prices and excess returns (compared to the benchmark with full information).
THE MODEL

- Two-period endowment economy with uncertainty.
- Agents consume in the second period.
- There is an aggregate event $\tau$ that is realized at date 2, $\tau \in \{1, 2\}$.
- Event $\tau$ determines the allocation of endowments at date 2.
- In the first period agents trade two Arrow-Debreu securities that pay contingent on $\tau$. In the second period agents consume.
- Positive mass of agents. Agents can be of different types. Finite number of types.
- The agent’s type determines his endowment, preferences and information.
INDIVIDUAL PROBLEM OF TYPE $\theta$

- Let $\hat{\delta} = \text{Prob}(\tau = 1)$.

\[
\max_{(c_1,c_2)} \left\{ \hat{\delta} u(c_1; \theta) + (1 - \hat{\delta}) u(c_2; \theta) \right\}
\]

s.t. $pc_1 + (1 - p)c_2 = p\omega_1(\theta) + (1 - p)\omega_2(\theta)$, and

$\hat{\delta} = E[\delta \mid \mathcal{I}(\theta)]$,

where $\mathcal{I}(\theta)$ denotes the information set of type $\theta$, and $p$ denotes the price of a contingent claim paying when $\tau = 1$.

- Agents use Bayes’ rule to update their beliefs about $\delta$. 

Definition of exposure

- The exposure of an agent of type $\theta$ to the event $\tau = 1$ relative to the event $\tau = 2$ is denoted as $e_1(\theta)$ and defined as

$$e_1(\theta) = \log \left( \frac{u'(\omega_2(\theta); \theta)}{u'(\omega_1(\theta); \theta)} \right).$$

- If $e_1 = 0$, the agent has zero exposure, e.g., his endowment of goods is independent of $\tau$.

- If $e_1 < 0$, we say that the agent has negative exposure. At the initial endowment allocation, the agent is worse off when the event $\tau = 1$ is realized.
Distribution of exposure levels

- Individual exposure may take one of two values.

- A fraction $\varepsilon$ of agents have exposure $\bar{e}$.

- A fraction $1 - \varepsilon$ have exposure $\underline{e}$, with $\bar{e} > \underline{e}$.

- The distribution of exposure levels across agents is random.

- The fraction $\varepsilon$ may take one of two values: $\{\varepsilon^l, \varepsilon^h\}$, with $\varepsilon^l < \varepsilon^h$.

- Nature draws the value of $\varepsilon$ at the beginning of the first period.
Private signals I

• At the beginning of the first period Nature draws the values of $\delta$ and $\varepsilon$ from a joint probability distribution $F$ with support $[0, 1] \times \{\varepsilon^l, \varepsilon^h\}$.

• The distribution $F$ is common knowledge.

• It is important for non-revelation that $\delta$ is drawn from a continuum.

• Agents do not observe the realizations of $\delta$ and $\varepsilon$.

• Instead, they receive private signals.
  - A fraction $\delta$ receive a signal $s_1$.
  - A fraction $1 - \delta$ receive a signal $s_2$. 


Order of events at date 1

Nature draws $\delta$ and $\varepsilon$. Agents observe their type: endowment, preferences and private signals. Agents observe prices, update beliefs and trade.
Order of events at date 2

Uncertainty about $\tau$ is resolved.

Agents transfer resources according to the contracts signed at date 1.

Agents consume.
EQUILIBRIUM

A rational expectations equilibrium consists of a price function $P : [0, 1] \times \{\varepsilon^l, \varepsilon^h\} \rightarrow [0, 1]$ and individual demand functions $\{c_1 (p; \theta), c_2 (p; \theta)\}$ such that:

1. Consumers maximize utility, given beliefs consistent with $P$.

2. The markets for claims on states $\tau = 1$ and $\tau = 2$ clear.
EQUILIBRIUM PROPERTIES

- A equilibrium with fully revealing prices does not exist.

- If it exists, the equilibrium is non-revealing.

- Define $\delta^i(p)$ as the value of $\delta$ that satisfies $P(\delta^i(p), \epsilon^i) = p$

- Function $\delta^i(p)$ is well defined as long as the price function $P$ is invertible at every $\epsilon$. 
HIGH EXPOSURE DEPRESS PRICE $p$

- Proposition: when preferences display a constant coefficient of absolute risk tolerance, the price of the contingent claim that pays in the event $\tau = 1$ decreases with the fraction of agents with high exposure to that event: $P(\delta, \varepsilon^l) > P(\delta, \varepsilon^h)$.

  - This property is also observed when agents have full information.

  - The decrease in $p$ with $\varepsilon$ is reinforced in the equilibrium with partially revealing prices. Why?
Proposition:
(a) Agents with better signal about the event $\tau = 1$ believe that event to be more likely: $\delta(s_1, e, p) > \delta(s_2, e, p) \quad \forall e, p$.

(b) Agents with higher exposure to the event $\tau = 1$ believe that event to be more likely: $\delta(s, \bar{e}, p) > \delta(s, e, p) \quad \forall s, p$.

• Why (b)?
Partially revealing prices

\[ P(\delta, \varepsilon^i) \]

\[ \delta^l (p) \quad \delta^h (p) \]

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Prices more responsive to $\varepsilon$

Proposition: In the equilibrium with asymmetric information, prices are relatively more sensitive to $\varepsilon = \varepsilon^h$ than in the equilibrium with full information:

$$P(\delta, \varepsilon^l) > P_{FI}(\delta, \varepsilon^l) > P_{FI}(\delta, \varepsilon^h) > P(\delta, \varepsilon^h).$$

- When there is asymmetric information, changes in $\varepsilon$ have an additional role: they affect the distribution of beliefs across agents.
Biased beliefs I

• Assume an economist who has an infinitely long sample of prices and the distribution of endowment across agents.

• He observes individual preferences but does not know the information structure.

• He assumes that in a given period, agents have the same information and thus the same beliefs. This enables him to use a representative agent model to fit the data.

• What does he conclude about agents’ beliefs?
Biased beliefs II

Proposition: When \( u(c) = \log(c) \) and high exposure types receive a larger endowment of goods in all states, the economist would (mistakenly) conclude that agents are too optimistic about the event \( \tau = 1 \).

- Intuition:
  1) agents with more resources have more influence on prices.
  2) agents with high exposure are more optimistic about the probability of event \( \tau = 1 \).

- Application: if event \( \tau = 1 \) corresponds to low dividends, an economist using a representative agent setup would infer that the equity premium is larger than what can be explained by a representative agent model—with unbiased beliefs.
Excess returns and prices I

• Puzzle: why do high prices predict low excess returns?

• This is observed in many markets, e.g. high price dividend ratio on stocks tend to predict low future stock returns.

• We show an example where the presence of asymmetric information tends to accentuate the negative co-movement between prices and excess returns.

• In the model, the wealth distribution changes with prices, and so does the average belief.
  – High exposure investors are pessimistic about stocks.
  – High exposure investors are relatively more wealthy when stock prices are low ⇒ depress stock prices even more.
Excess returns = \frac{E[d \mid q]}{q} - 1,
• Share of wealth of high exposure type.
• Expected excess returns:

\[
\frac{\hat{\delta}d_1 + \left(1 - \hat{\delta}\right) d_2}{pd_1 + (1 - p) d_2} - 1
\]

• Can interpret \( p \) as risk-neutral probabilities.

• Consider the full information case: \( \hat{\delta} = \delta \).

• Consider case with \( d_1 < d_2 \).

• At the corners there is no uncertainty \( \Rightarrow \) risk-neutral probabilities = actual probabilities.

• For all other prices, \( p > \delta \). Stocks offer poor hedging properties against the aggregate state with low dividends (\( \tau = 1 \)).
• Agents are more concerned about ending up in state $\tau = 1$ when that event is more likely to occur (low stock prices), and tend to require a higher premium for holding stocks.

• Asymmetric information may accentuate negative relationship between stock prices and excess returns.

• Why?
  – Beliefs of more pessimistic agents have a higher weight when stock prices are low.
  – Agents with high exposure to event $\tau = 1$ are more pessimistic: they assign more probability to the event $\tau = 1$ (low dividends) compared to other agents.
  – Share of wealth of high exposure type decrease with the price of stocks.
Conclusions

- We study a framework in which agents are heterogeneous along two dimensions: exposure and the value of the private signal.

- Agents face a non-trivial signal extraction problem. They start with a common prior but end up with heterogeneous beliefs.

- We show how asymmetric information may help account for some puzzling behavior of asset prices:
  - Equity premium: excess pessimism may be (mistakenly) inferred from a representative agent model.
  - Higher stock returns in bad times.