Fiscal Reaction Functions in an Overlapping Generations Setting:
Why “Unfair” Fiscal Reforms Are Most Effective

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Starting Point: Standard Budget Analysis

1. Budget equation: \( \text{Debt} = (1+r) \times (\text{Previous debt}) + \text{Primary Deficit} \)
   - Interest rate “r” is typically exogenous. Often scaled by GDP (growth g).

2. Sustainability condition: Positive response of primary surplus to debt (Bohn 1998)
   \[
   \frac{\text{Primary Surplus}}{\text{GDP}} = a + b \times \frac{\text{Debt}}{\text{GDP}} + \text{OtherVariables}
   \]
   - Response \( b > 0 \) sufficient for IBC; \( b > r - g \) implies stationary \( \frac{\text{Debt}}{\text{GDP}} \).
   - If \( r < g \) => No surplus needed. Dynamic inefficiency.
   - With \( r > g \) or uncertainty: Stability requires a positive response coefficient.

• Questions for this paper:
  - What if interest rates are endogenous and increasing with debt?
  - Are all elements of the primary surplus equally effective for stabilizing debt?

• Setting: Overlapping generations model.
Basic Findings & Intuition

1. Endogenous interest rates are destabilizing.
   - Higher debt => Crowding out: Lower capital => Higher interest => More debt.
     => Need stronger fiscal stabilization. (Higher “b”. Need b>0 even if r<g.)
   - Complications with wage income taxes: Effects on the tax base and on labor supply.

2. Fiscal responses that encourage savings are more effective than others.
   - Intuition: Higher savings => Higher K/L => Lower r => Lower debt service.
     - Example in the OG model: If debt triggers reduced retirement benefits.
   - Fiscal responses that reduce private savings can be counterproductive.
     - Example: If debt triggers taxes on retirement savers (middle cohort).

3. Welfare issue: Who should bear the burden of debt-stabilization—rich or poor?
   - Observation: Savers tend to have relatively high income.
   - Suggests “unfair” reforms are most effective. May explain why we see them.
Model Design: Overlapping Generations

• Generations live for three periods: Young, middle-age, retired.

• Liquidity-constrained young (High time preference) => Diamond-type dynamics.

• Three different versions in the conference draft
  1. Basic model with focus on savings. Exogenous labor supply.
     • Demonstrates the interaction of debt & capital. Provides most intuition.
  2. General Model with fully endogenous labor supply. (work in progress)
     • Technically more complicated. Shows how the basic intuition generalizes.
  3. Intermediate model with elastic young-age & fixed middle-age labor supply.
     • Counterintuitive point: Taxing elastic labor supply may enhance stabilization.
     • Reasoning: Less labor => Higher capital/labor ratio => Lower interest rates.
The Basic Model (1)

- Preferences: 
  \[ U_t = \frac{u(c^1_t)}{\beta_0} + u(c^2_{t+1}) + \beta u(c^3_{t+2}) \]

- Demographics: Cohort size \( N_t \). Population growth factor \( n = N_t / N_{t-1} \geq 1 \).
  
  Labor force in period \( t \): \( L_t = e \cdot N_t + N_{t-1} \). \([\text{Age-earnings factor } e]\)

- Production: 
  \[ Y_t = F(K_t, L_t) \] \([\text{CRS. Old capital subsumed into } F]\)
  \[ R_t = F_K(K_t/L_t, 1) \equiv R(\kappa_t) \quad \text{where } \kappa_t = K_t/L_t \]
  \[ w_t = F_L(K_t/L_t, 1) = w(\kappa_t), \]

- Government: 
  \[ D_{t+1} = G_t + R_t D_t - N_t \theta^1_t - N_{t-1} \theta^2_t + N_{t-2} \cdot \gamma_t, \]
  
  Taxes per-person \((\theta^1_t, \theta^2_t)\), total spending \( G_t \), retirement transfers \( \gamma_t \).

  Debt at the start of period \( t \): \( D_t \). Next period starts with \( D_{t+1} \).

- Per-capita: 
  \[ \sigma^1 \theta^1_t + \sigma^2 \theta^2_t + n \cdot d_{t+1} = g_t + R_t d_t + \sigma^3 \gamma_t \]
  
  with population shares \((\sigma^1, \sigma^2, \sigma^3)\), \( R=1+r \), \( d=D/\text{Population} \).
The Basic Model (II)

• Individual problem: maximize utility

  - Assume $\beta_0$ small enough that the young consume $c_t^1 = e \cdot w_t - \theta_t^1$, no savings.
  
  - Middle-aged maximize $U$ s.t. $c_t^2 = w_t - \theta_t^2 - s_t$ and $c_{t+1}^3 = R_{t+1} \cdot s_t + \gamma_{t+1}$.

  $\Rightarrow$ Optimal savings function

  $$s_t = s(w_t - \theta_t^2 + \frac{\gamma_{t+1}}{R_{t+1}} R_{t+1}) - \frac{\gamma_{t+1}}{R_{t+1}}$$

• Capital market equilibrium: $K_{t+1} + D_{t+1} = N_{t-1} s_t$.

• Shocks to government spending and wages $(\hat{g}_t, \hat{w}_t)$: $g_t = \tilde{g}_t + \hat{g}_t, \ w_t = w(\kappa_t) + \hat{w}_t$.

  [Only to motivate fluctuations, obtain clean DSGE framework. Focus is on propagation.]

• Primary surplus $\pi_t = \tilde{\pi}_t + \hat{g}_t$ with controlled part $\tilde{\pi}_t = \sigma^1 \cdot \theta_t^1 + \sigma^2 \cdot \theta_t^2 - \sigma^3 \cdot \gamma_t - \tilde{g}_t$

• Dynamics: Markov process with state vector $(d_t, \kappa_t, \hat{w}_t, \hat{g}_t)$. Two key equations:

  $$d_{t+1} = \frac{1}{n} [R(\kappa_t) \cdot d_t + \hat{g}_t - \tilde{\pi}_t]$$
  
  $$\lambda \cdot \kappa_{t+1} + d_{t+1} = \frac{\sigma^2}{n} s_t = \sigma^3 \cdot [s(w_t - \theta_t^2 + \frac{\gamma_{t+1}}{R(\kappa_{t+1})} R(\kappa_{t+1})) - \frac{\gamma_{t+1}}{R(\kappa_{t+1})}]$$
Points of Reference: Two Special Cases

Case 1: Linear Production ⇒ Standard dynamics of debt with fixed R and w.

• Stochastic process for debt has root \( \mu_d = \frac{\partial d_{t+1}}{\partial d_t} = \frac{R}{n} - \frac{1}{n} \tilde{\pi}_d \), where

\[
\tilde{\pi}_d = \sigma^1 \cdot \theta^1_d + \sigma^2 \cdot \theta^2_d - \sigma^3 \cdot \gamma_d - \tilde{g}_d
\]

collects tax and spending responses to debt.

• Stability requires \( \mu_d < 1 \iff \tilde{\pi}_d > R - n \).

• Definition 1: Policy tools are called equally effective (for some purpose) if their relative impact is proportional to their contribution to the primary surplus. A policy tool is called more effective than another if the policy tool’s relative impact exceeds the relative weights in the primary surplus.

• Definition 2: Public debt is called self-stabilizing, if the economy is locally stable … even if the components of the primary surplus do not respond to the economy’s state variables.

• Benchmark results:

1. Debt-stabilization requires \( \tilde{\pi}_d > R - n \).

2. Debt is self-stabilizing if and only if \( R < n \).

3. If debt-stabilization is needed, all policy instruments are equally effective.
Case 2: Zero debt => Private sector OG dynamics as in Diamond (1965)

- Objective: Define private sector stability conditions – as benchmark.

- Linearize \( \kappa_{t+1} = \frac{\sigma^3}{\lambda} \cdot \left[ s \left( w(\kappa_t) + \hat{w}_t - \theta_t^2 + \frac{\gamma_{t+1}}{R(\kappa_{t+1})}, R(\kappa_{t+1}) \right) - \frac{\gamma_{t+1}}{R(\kappa_{t+1})} \right] \)

- Characteristic root is \( \mu_{0,\kappa} = \partial \kappa_{t+1} / \partial \kappa_t = \omega_0 / \psi_0 \), where

\[
\omega_0 = \sigma^3 s_W \cdot w'(\kappa) > 0 \quad \text{and} \quad \psi_0 = \lambda + \sigma^3 (s_R + (1 - s_W) \frac{\gamma}{R^2}) \cdot \{-R'(\kappa)\}.
\]

- Assumption 1: \( 0 \leq \omega_0 < \psi_0 \) => Ensures stability.


General Analysis: Debt and Neoclassical Production

- Linearized dynamics described by \( \Psi \cdot x_{t+1} = \Omega \cdot x_t + Z \cdot (\hat{g}_t, \hat{w}_t)' \)

where \( x_t = (d_t - d, \kappa_t - \kappa) \) and \( (\Psi, \Omega, Z) \) are 2x2 matrices [Elements: \( \psi, \omega, \)]

=> Stability depends on the characteristic roots.

- Impose Assumption 1 to focus on policy-related instability.
Stability Analysis (1)

- Intuition from linearized dynamics without policy responses:

$$\begin{pmatrix} 1 & 0 \\ \psi_0 & \kappa_{t+1} - \kappa \end{pmatrix} \begin{pmatrix} d_{t+1} - d \\ \kappa_t - \kappa \end{pmatrix} = \begin{pmatrix} R/n & R'({\kappa})d/n \\ 0 & \omega_0 \end{pmatrix} \begin{pmatrix} d_t - d \\ \kappa_t - \kappa \end{pmatrix} + \begin{pmatrix} 1/n & 0 \\ 0 & \sigma^3 s_W \end{pmatrix} \begin{pmatrix} \hat{g}_t \\ \hat{w}_t \end{pmatrix}$$

where $0 \leq \omega_0 < \psi_0$ from Assumption 1.

- Notation:

  \[ \psi_{dd} = 1 \quad \psi_{d\kappa} = 0 \quad \text{and} \quad \omega_{dd} = R/n > 0 \quad \omega_{d\kappa} = R'({\kappa})d/n < 0 \]

  \[ \psi_{kd} = 1 \quad \psi_{\kappa\kappa} = \psi_0 \quad \omega_{kd} = 0 \quad \omega_{\kappa\kappa} = \omega_0 \]

- Interaction between debt and capital through two off-diagonal terms:

  1. Unit coefficient $\psi_{kd} = 1$ captures one-for-one crowding out.

  2. Negative coefficient $\omega_{d\kappa} = R'({\kappa})d/n < 0$ captures endogenous interest rates.

  Intuition: Interaction creates instability: Crowding out $\Rightarrow$ K down $\Rightarrow$ R up.

- Proposition 1: Dynamic inefficiency does not ensure a self-stabilizing debt.

  Debt is self-stabilizing if and only if $R < n - \Lambda_\theta$, where $\Lambda_\theta = \frac{-R'({\kappa})}{\psi_0 - \omega_0} \cdot d > 0$.

  [Paper also derives a modified version with wage income taxes – omitted here.]
Stability Analysis (II): With Policy Responses

• Policy responses imply modified coefficients, notably

\[ \psi_{kd} = 1 + \frac{1 - s_w}{R} \sigma^3 \gamma_d \neq 1 \quad \text{and} \quad \omega_{kd} = -s_w \sigma^3 \theta_d^2 \neq 0. \]

• Insight: Retiree transfers and middle-age taxes enter not only through the budget, but also through their impact on savings => More/less effective for stabilization.

• Stability condition with policy responses can be written as

\[ \sigma^1 \theta_d^1 + \sigma^2 \cdot \Phi^2 \cdot \theta_d^2 - \sigma^3 \cdot \Phi^3 \cdot \gamma_d - \tilde{g}_d > R - n + \Lambda \]

where the weights

\[ \Phi^2 = 1 - \Lambda_i \cdot \frac{\sigma^2}{n} s_w < 1 \quad \text{and} \quad \Phi^3 = 1 + \Lambda_i \cdot \frac{1 - s_w}{R} > 1 \]

quantify relative effectiveness.

• Proposition 2:
  a. Reduced transfers are most effective for stabilizing the public debt.
  b. Spending & taxes on young are less effective than transfers (equally).
  c. Higher taxes on middle-aged are least effective, or counterproductive (if \( \Phi^2 < 0 \)).
The General Model (1)

- Preferences: 
  \[ U_t = v_1(c_{i,t}, 1 - l_{i,t})/\beta_0 + v_2(c_{i+1,t}, 1 - l_{i+1,t}) + \beta_0 u(c_{i+2,t}) \]

- Labor supply per-capita 
  \[ \lambda_t = \sigma^1 e \cdot l_{i,t}^1 + \sigma^2 \cdot l_{i,t}^2 \]

- Tax revenues with labor income taxes 
  \[ \theta_t^1 = ew_t l_{i,t}^1 \tau_t \] and \[ \theta_t^2 = w_t l_{i,t}^2 \tau_t^2 \].

- Young: Static FOC implies supply function
  \[ l_{i,t}^1 = l_{i,t}^1[e \cdot w_t (1 - \tau_t^1)] \]

- Middle: Euler equations
  \[ (w(\kappa_t) + \hat{w}_t)(1 - \tau_t^2) \cdot v_{2,c}(c_{i,t}^2, 1 - l_{i,t}^2) = v_{2,1-i}(c_{i,t}^2, 1 - l_{i,t}^2) \]
  \[ v_{2,c}(c_{i,t}^2, 1 - l_{i,t}^2) = \beta E_t[u'(c_{i+1,t})R(\kappa_{t+1})] \]

- Uncertainty is secondary => Consider perturbations around deterministic path.

  \[ \Rightarrow \] Labor supply 
  \[ l_{i,t}^2 = l_{i,t}^2[w_t (1 - \tau_t^2), R_{t+1}, \gamma_{t+1}]. \]

  Savings 
  \[ s_t = s\left(w_t (1 - \tau_t^2) \cdot l_{i,t}^2 + \frac{\gamma_{t+1}}{R_{t+1}}, R_{t+1}\right) - \frac{\gamma_{t+1}}{R_{t+1}} \]
The General Model (II)

• Capital market equilibrium:

\[ \kappa_{t+1} \cdot \lambda_{t+1} + d_{t+1} = \sigma^3 \cdot [s\left( w(\kappa_{t+1}) + \hat{w}_t \right)(1 - \tau_t^2) \cdot l_t^2 + \frac{\gamma_{t+1}}{R(\kappa_{t+1})},R(\kappa_{t+1})] - \frac{\gamma_{t+1}}{R(\kappa_{t+1})} ] \]

• Dynamics: Markov process with state vector \((d_t, \kappa_t, \lambda_t, \hat{w}_t, \hat{g}_t)\).
  - Linearized dynamics described by \( \Psi \cdot x_{t+1} = \Omega \cdot x_t + Z \cdot (\hat{g}_t, \hat{w}_t)' \)
    where \( x_t = (d_t - d, \kappa_t - \kappa, \lambda_t - \lambda) \) and \((\Psi, \Omega, Z)\) are now 3x3 matrices.
  - Intuition: Labor supply depends on \( R(\kappa_{t+1}) \), which depends on future labor supply => Must solve for the rational expectations solution.
  - Saddle-path stability requires TWO roots inside \([-1,1]\) and ONE root outside.

• Line of Argument:

  1. **Linear production case**: Labor supply distortions imply reduced effectiveness of labor income tax responses. Transfer responses more effective.
The General Model (III)

2. No debt case: Markov dynamics with \((\kappa_t, \lambda_t, \hat{w}_t, \hat{g}_t)\).

- Assume saddle-path stability: Characteristic polynomial \(P_0(\mu)\) must have ONE root inside \([-1,1]\) and ONE root outside.

- Several cases arise. Focus on \(\partial l_2 / \partial R > 0\). Then \(0 \leq \mu_{01} < 1 < \mu_{02}\).

3. General Analysis: Characteristic polynomial \(P(\mu) = |\Omega - \mu \Psi|\) is cubic.

- Write \(P(\mu) = (\mu - \mu_d) \cdot P_0(\mu) + P^*(\mu)\). Denote roots \((\mu_1, \mu_2, \mu_3)\).

  If \(P^*(\mu) = 0\), \(P(\mu)\) would have roots \((\mu_{01}, \mu_{02}, \mu_d)\). Stable iff \(\mu_d < 1\).

- Find (with conditions): \(P^*(\mu) < 0\) on \(\mu \in (0,1]\) \(\Rightarrow \mu_2 > \min(\mu_d,1)\).

  Increased second root is destabilizing (less interesting: \(0 < \mu_1 < \mu_{01}\) and \(\mu_3 > 1\))

  [Conditions: “Not too large” interest rate effects on middle-age labor.]

- Policy effects from the basic model still valid (with conditions).

- New: Stability may be enhanced by elastic labor supply.
Intuition: Elastic Young-Age Labor Only
(Intermediate Model)

- **Preferences:**
  \[ U_t = v_1(c^1_t, 1 - l^1_t)/\beta_0 + u(c^2_{t+1}) + \beta u(c^3_{t+2}) \]

- **Labor supply:**
  \[ \lambda_t = \sigma^1 e \cdot l^1_t + \sigma^2 \text{ with } l^1_t = l^*(k_t, 1 - \tau^1_t) \]

- Express the young taxes and labor supply in terms of revenues \( \theta^1_t \).

- Write \( \kappa_t = \kappa(k_t, \theta^1_t) \) with \( \partial \kappa_t / \partial \theta^1_t = \kappa_\theta \) iff labor supply is elastic.

- **Linearized dynamics described by**
  \[ \Psi \cdot x_{t+1} = \Omega \cdot x_t + Z \cdot (\hat{g}_t, \hat{w}_t)' \]
  where \( x_t = (d_t - d, k_t - k) \) and \( (\Psi, \Omega, Z) \) are 2x2 matrices.

- **Stability condition with policy responses can be written as**
  \[ \Phi^1 \cdot \sigma^1 \cdot \theta^1_d + \Phi^2 \cdot \sigma^2 \cdot \theta^2_d - \Phi^3 \cdot \sigma^3 \cdot \gamma_d - \tilde{g}_d > R - n + \frac{(-R')d \cdot \kappa_k}{\psi_0 - \omega_0} \text{ with weights} \]
  \[ \Phi^1 = 1 + \frac{1}{\sigma^1 \left( \frac{(-R')d \cdot \kappa_k}{\psi_0 - \omega_0} \right)}\kappa_\theta, \quad \Phi^2 = 1 - \frac{1}{\psi_0 - \omega_0}s_W < 1, \quad \text{and} \quad \Phi^3 = 1 + \frac{1}{\psi_0 - \omega_0}\frac{1-s_W}{R} > 1. \]

Analogous to the basic model, but \( \Phi^1 > 1 \) iff \( \kappa_\theta > 0 \).
Elastic Young-Age Labor (II)

- **Proposition 3**: With a variable labor supply of young agents:
  
a. The effectiveness of taxes on the young for stabilizing debt is increasing in the labor supply elasticity.

b. If young-age labor supply elasticity has positive elasticity (negative), taxes on the young are more (less) effective than spending cuts.

c. If the labor supply elasticity is high enough, then taxes on the young are more effective than cuts in retiree transfers ($\Phi^1 > \Phi^3 > 1$).

d. If the labor supply elasticity is low enough, then taxes on the young are less effective than taxes on the middle aged ($\Phi^1 < \Phi^2 < 1$).
Conclude

1. Endogenous interest rates are destabilizing.
   • Higher debt => Crowding out: Lower capital => Higher interest => More debt.

2. Fiscal responses that encourage savings are relatively more effective.
   • Intuition: Higher savings => Higher K/L => Lower r => Lower debt service.
   • Basic ranking for stabilization, from most to least effective:
     Reduced transfers - Lower spending - Higher taxes on savers (middle-aged).

3. Fiscal responses that discourage labor supply can be effective.
   • Reduced labor supply reduces interest rates (lower K/L).
   • However: with wage income taxes, also a negative impact on the tax base.
     => Ranking of wage taxes depends on labor supply considerations.