Unawareness and Strategic Announcements in Games with Uncertainty

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Motivation

How do decision makers form beliefs when they learn a contingency that did not cross their mind before?

How does the strategic nature of the information source affect this belief formation?
Motivation

Aware Broker vs. Unaware Investor (mutual fund)

- Investor: which mutual fund to buy

  Return of the fund depends on the realization of the contingencies

- Investor may be *unaware* of some of the contingencies

- Broker: *aware* of all the contingencies and how likely they are

- Aware Broker: Strategic Announcements

- Investor: (Belief Formation) *How to assign probability to the newly announced contingencies?*
Motivation

- Gambling
- Insurance
- Parents
- Government
- Commercials
- ...

...
Roadmap

- Model
- Solution Concept
- Refinement
- Examples: *We should not always everything to our children.*
- Discussion: *Zero probability vs. Unawareness*
Model

**Two players**: Announcer and Decision Maker (DM), indexed by 1 and 2

**Ω**: Finite set of moves of a chance player (contingencies)

**π**: True distribution of contingencies, $\pi(\omega) \neq 0$ for any $\omega \in \Omega$. 
Model

Two players: Announcer and Decision Maker (DM), indexed by 1 and 2

Ω: Finite set of moves of a chance player (contingencies)

\( \pi \): True distribution of contingencies, \( \pi(\omega) \neq 0 \) for any \( \omega \in \Omega \).

Awareness Structure

- Announcer is aware of \( \Omega \) and believes \( \pi \).
- DM is aware of \( \Omega_o \subseteq \Omega \) and believes \( \pi(.|\Omega_o) \).
- Announcer is aware of DM’s limited awareness,
- DM is unaware of announcer’s superior awareness.

DM perceives that the announcer is only aware of \( \Omega_o \), and believes \( \pi(.|\Omega_o) \).
**STRATEGIES:**

The announcer does not observe the realization of nature before the announcement.

\[ \mathcal{M} := 2^\Omega \setminus \Omega_o \] is the set of all strategies of the announcer

After an announcement, \( M \), the DM extends her awareness to \( \Omega_o \cup M \)

\( A \): Finite set of actions of the DM (\( A \) is the same set independent of the announcement).

Strategy of the DM: \( d : \mathcal{M} \rightarrow A \)

**PAYOFFS:**

payoff of player \( i \) is \( u_i : \Omega \times A \rightarrow \mathbb{R} \) for \( i = 1, 2 \).
Example 1

Let $\Omega = \{\omega_1, \omega_2\}$, $\Omega_o = \{\omega_1\}$, and $\pi(\omega_1) = \pi(\omega_2) = 0.5$.

The DM has two actions: left, and right. The payoffs are as follows:

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<thead>
<tr>
<th>Contingencies</th>
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</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td>1, 0</td>
<td>0, 1</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>1, 1</td>
<td>−2, −3</td>
</tr>
</tbody>
</table>
Example 1: cont.

Game that is initially understood by the DM.

\[ \begin{array}{c}
\omega_1 \\
A \\
\varnothing \\
DM \\
L \\
R \\
1, 0 \\
0, 1
\end{array} \]
Example 1: cont.

Game that will be understood by the DM when \( \{\omega_2\} \) is announced.
Solution Concept

A solution concept imposes some conditions on an assessment, 
\((M, d, F)\), which is a triplet containing strategies, \(M\) and \(d\), of each player and a belief function, \(F\).

**Definition**

A **belief function**, \(F\), assigns to each announcement, \(M\), a probability distribution, \(F_M\), on the union of sets of contingencies that are announced and that are in the initial awareness of the DM, \(M \cup \Omega_o\).
Both the announcer and the DM evaluate any strategy by calculating their expected utilities. For any action, $a \in A$, and announcement, $M \in \mathcal{M}$, the expected utility of the announcer is defined as:

$$EU_1(a) := \sum_{\omega \in \Omega} u_1(\omega, a) \pi(\omega)$$

and the expected utility of the DM with respect to the probability distribution, $F_M$, is defined as

$$EU_2(M, a|F_M) := \sum_{\omega \in M \cup \Omega_0} u_2(\omega, a) F_M(\omega)$$
Definition

An assessment \((M^*, d^*, F)\) is \textbf{rational} if

\[
M^* \in \arg \max_{M \in \mathcal{M}} EU_1 (d^*(M));
\]

\[
d^*(M) \in \arg \max_{a \in A} EU_2 (M, a \mid F_M), \text{ for any } M \in \mathcal{M}.
\]
Definition

An assessment, \((M^*, d^*, F)\), is \textbf{justifiable} if \(\forall M \subseteq M^*\),

\[
\sum_{\omega \in M^* \cup \Omega_o} u_1(\omega, d^*(M^*)) F_{M^*}(\omega) \geq \sum_{\omega \in M^* \cup \Omega_o} u_1(\omega, d^*(M)) F_{M^*}(\omega)
\]
Definition

A belief function, $F$, has **full support** if $\forall M \in \mathcal{M}$, and $\forall \omega \in M \cup \Omega_o$, $F_M(\omega) \neq 0$. 
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A belief function, $F$, has **full support** if $\forall M \in \mathcal{M}$, and $\forall \omega \in M \cup \Omega_o$, $F_M(\omega) \neq 0$.

**Definition**

A belief function, $F$, **respects to the initial belief** if $\forall M \in \mathcal{M}$, and $\forall \omega \in \Omega_o$, $F_M(\omega|\Omega_o) = \pi(\omega|\Omega_o)$. 
<table>
<thead>
<tr>
<th>Definition</th>
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<tbody>
<tr>
<td>An assessment, $(M^<em>, d^</em>, F^<em>)$ is <strong>awareness equilibrium</strong> if it is rational, justifiable, and $F^</em>$ has full support and respects to the initial belief.</td>
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Theorem 1

Awareness equilibrium always exists.
Proof of Theorem 1

Under no announcement, the DM holds her initial belief, $\pi(\cdot|\Omega_o)$, and set $d^*(\emptyset)$ as a maximal action for the announcer among the ones that maximize expected utility of the DM.

Define the belief function, $F$, such that for every announcement, $M \in \mathcal{M}$, $F_M$ respects the initial belief and assigns a very small but non-zero probability to all contingencies in the announcement, $M$, to guarantee that the best response of the DM, $d^*(M)$, is one of the actions that maximizes expected utility of the DM under no announcement.

Since for any $M \in \mathcal{M}$, $d^*(\emptyset)$ is a maximal among all $d^*(M)$ for the announcer, $M^* = \emptyset$. The assessment, $(\emptyset, d^*, F)$, is justifiable since $M^* = \emptyset$. ■
Is there a need for a refinement?

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<tr>
<td>$\omega_1$</td>
<td>1, 1</td>
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<tr>
<td>$\omega_2$</td>
<td>0, 0</td>
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Contingencies
Refinement

A probability distribution $P$ on $M \cup \Omega_o$ is called a *reasonable probability distribution* that supports the action $a \in A$, after the announcement $M \in \mathcal{M}$, and $d^*: M \to A$ if

(i) $P(\omega) \neq 0$ for any $\omega \in M \cup \Omega_o$

(ii) $P(\omega) = \pi(\omega | \Omega_o)$ for any $\omega \in \Omega_o$

(iii) $\sum_{\omega \in M \cup \Omega_o} u_1(\omega, a)P(\omega) \geq \sum_{\omega \in M \cup \Omega_o} u_1(\omega, d^*(M'))P(\omega)$ for any $M' \subset M$

(iv) $a \in \arg\max_{a' \in A} EU_2(M, a'|P)$

Let $\varphi(M, a|d^*)$ is the collection of $P$ satisfying (i) – (iv)
Definition

An awareness equilibrium, \((M^*, d^*, F^*)\) satisfies **reasoning refinement** if

for any non-empty \(M \in \mathcal{M}\) and \(a \in A \setminus \bigcup_{M' \subset M} \{d^*(M')\}\) such that \(\varphi(M, a|d^*) \neq \emptyset, F_M^* \in \varphi(M, d^*(M)|d^*)\).
Theorem 2

Awareness equilibrium that satisfies reasoning refinement always exists.
Proof of Theorem 2

Under no announcement, the DM holds her initial belief, $\pi(.|\Omega_o)$ and set $d^*(\emptyset)$, be a maximal action for the announcer among the ones that maximize expected utility of the DM.

Let $F$ be the belief function constructed in the proof of Theorem 1. Construct $F^*$ and $d^*$ inductively:

For $n \in \mathbb{N}$, let for any $M' \in \mathcal{M}$ s.t. $|M'| < n$, $d^*(M')$ and $F^*_{M'}$ be constructed. Construct $F^*_M$ and $d^*(M)$ for $|M| = n$:

If there is an action $a \in A$, that would not have been played after any announcement that is a proper subset of $M$ and if there is a reasonable probability distribution, $P$, on $M \cup \Omega_o$, supporting the action $a$ after the announcement $M$, then set $F^*_M = P$ and $d^*(M) = a$.

Otherwise, set $F^*_M = F_M$, and $d^*(M)$ as one of the actions that maximizes expected utility of the DM under no announcement (same as in the proof of Theorem 1).

Given the decision function, $d^*$, the announcer announces $M^*$ that maximizes his expected utility. ■
In order to change one’s action, shall we tell everything?

Example 2
Let $\Omega = \{\omega_1, \omega_2, \omega_3\}$ with $\pi(\omega_1) = 0.1$, $\pi(\omega_2) = 0.8$, $\pi(\omega_3) = 0.1$ and $\Omega_o = \{\omega_1\}$.

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<tr>
<td>$\omega_1$</td>
<td>3, 3</td>
<td>0, 0</td>
<td>2, 2</td>
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| Contingencies | $\omega_2$ | 0, 0 | 7, 7 | 2, 2 |

| $\omega_3$ | 2, 2 | 0, 0 | 2, 2 |

“middle” is the best.

$d(\emptyset) = left$
$d(\omega_2) = middle$
$d(\omega_3) = left$
$d(\omega) = right$
Discussion

Cheap Talk with Zero Probabilities

Alternative Model: No unawareness but assigning zero probability

DM initially believes $P(\Omega \setminus \Omega_o) = 0$

Announcer knows the true distribution and Cheap Talk (Crawford and Sobel, 1982)

But degenerate belief so no updating! Talk is not informative.
Let $\Omega = \{\omega_1, \omega_2\}$, $\Omega_o = \{\omega_1\}$, and $\pi(\omega_1) = \pi(\omega_2) = 0.5$.

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Some Final Thoughts

  *In US, condom sales rose from 240 million annually in 1986 to 299 million in 1988. The greatest increase occurred in 1987 after the Surgeon General’s report on AIDS was released.*

- Insurance Markets (Filiz Ozbay 2008)

- Financial Markets