Joint Commitment

Implementation Without Public Randomization

Francesco Nava

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Games & Commitments

Q: How does the ability to rule out actions affect behavior in a game?

Institutions used to rule out actions differ on several dimensions:

(1) agreement necessary to commit [individual or collective]

(2) the type of commitments that can be undertaken

(3) number of instances in which to commit

Q: What outcomes are implemented by different institutions?
Today Goals

Assume:

(A1) All players must agree for an action to be ruled out

(A2) Only deterministic commitments can be undertaken

To show that:

(R1) If players have a single instance to commit there are games in which no efficient payoff can be implemented

(R2) If players have sufficiently many instances to commit any individually rational payoff can be implemented
A Non-convex Prisoner’s Dilemma

Consider a the game:

\[
\begin{array}{c|ccc}
1 & 2 & n & c \\
\hline
n & 1,1 & 6,0 \\
\hline
c & 0,6 & 2,2 \\
\end{array}
\]

The independent payoff hull is not convex

(1, 1) is the only Nash payoff

Players would like to sign a contract that forces both to cooperate
A Non-convex Prisoner’s Dilemma

- A first instance:
  
  \[
  \begin{array}{c|ccc}
  1 \lor 2 & c, c & \ldots \\
  \hline 
  c, c & 2, 2 & 1, 1 \\
  \ldots & 1, 1 & \ldots \\
  \end{array}
  \]

- Such contract yields a payoff of (2, 2)

- (2, 2) is still inefficient

- But no other deterministic contract can improve upon it
If there are two instances to commitment

- A second instance:

<table>
<thead>
<tr>
<th>1 \ 2</th>
<th>c, n</th>
<th>c, cn</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>c, n</td>
<td>0, 6</td>
<td>2, 2</td>
<td>1, 1</td>
</tr>
<tr>
<td>c, cn</td>
<td>2, 2</td>
<td>0, 6</td>
<td>1, 1</td>
</tr>
<tr>
<td>...</td>
<td>1, 1</td>
<td>1, 1</td>
<td>...</td>
</tr>
</tbody>
</table>

- If players have two rounds to commit more payoffs can be implemented

- \((1, 4)\) is an equilibrium

- Beliefs of future concessions lead to further concessions
Outline

One Round Commitment
   Commitment Structures
   Unanimous Implementation

Multiple Rounds and Efficiency
   Structures with Multiple Rounds
   Unanimous Implementation

Summary and Conclusion
A Commitment Extension

- \[ \{ L, \{ A_i, u_i \}_L \} = \langle A, u \rangle \] a strategic form game
- \[ \{ M_i, k_i \}_L \] a commitment structure
- \[ M_i \] the message spaces
- \[ k_i : M \rightarrow 2^{A_i \setminus \emptyset} \] the commitment maps

- Time Line:

  - \[ t=0 \]
  - \[ t=1 \]
  - \[ t=2 \]

  \[ M \; k(m) \subseteq A \; U \]

- Perfect monitoring on messages
Unanimity and Subgame Perfection

▶ A commitment structure is *unanimous* if:

1. \( k(m) \neq A \Rightarrow k(m'_i, m_{-i}) = A \) for \( \forall m'_i \neq m_i \) and \( i \in L \)

2. \( k(m) = A \) for some \( m_i \in M_i \) and \( \forall m_{-i} \in M_{-i} \) and \( i \in L \)

▶ \( \mathcal{N} \langle B, u \rangle \) .........................Nash payoff set of the game \( \langle B, u \rangle \)

▶ \( \mathcal{V} = \bigcup_{B \in \times_i 2^{A_i} \setminus \emptyset} \mathcal{N} \langle B, u \rangle \) ..............subgame equilibrium payoffs

▶ \( \mathcal{N} = \{ u | u_i \geq \min_{v \in \mathcal{N}} v_i, \ \forall i \in L \} \) ...............the Nash rational hull
Remark
Any allocation in $\mathcal{V} \cap \mathcal{N}$ is a SPE of some unanimous commitment extension.

Remark
Only an allocation in $\mathcal{N}$ can be SPE of a unanimous commitment extension.

Remark
Adding unanimous commitment devices cannot reduce the SPE payoff set.
**Proposition**

*Any payoff in $\text{co}(\mathcal{N})$ is approximately implemented by a unanimous commitment structure.*

**Proposition**

*If no payoff in $\mathcal{V} \cap \mathcal{N}$ is efficient then no SPE of any unanimous commitment structure is efficient.*

**Proposition**

*If so, it is possible to bound the set of unanimously implementable allocations with a convex set that does not intersect the Pareto frontier.*
Multiple Rounds of Commitment

- Consider a sequence of $t + 1$ commitment structures:
  \[ k^t = \{k_t, \ldots, k_0\} \]

- An extension has $t + 1$ rounds of commitment if for $s \in \{0, \ldots, t\}$:
  \[ k_s : M^s \to \times_i [2^A_i \setminus \emptyset] \quad \text{and} \quad k_s(m^s) \subseteq k_{s-1}(m^{s-1}) \]

- Time Line:
  \[
  \begin{align*}
  0 & \quad 1 \quad \ldots \quad t \quad t+1 \\
  M_0 & \quad k_0(m_0) \subseteq A \quad \ldots \quad k_t(m^t) \subseteq A \quad U
  \end{align*}
  \]

- If an action is ever ruled out it cannot be ruled in thereafter
Multiple Round Implementation

Theorem

If \( \text{vert}(\text{co}(\mathcal{V}) \cap \text{aff}(\mathcal{N})) \subseteq \mathcal{V} \) and

if \( \exists w \in \text{co}(\mathcal{V} \cap \mathcal{N}), v \in \mathcal{V}, \alpha \in [0, 1] \) and \( j \in L \):

\[
\alpha w_i + (1 - \alpha) v_i > \min_{\mathcal{N}} u_i \quad \text{for } \forall i \in L \setminus j
\]

\[
\alpha w_i + (1 - \alpha) v_i \geq \min_{\mathcal{N}} u_i \quad \text{for } \forall i \in L
\]

then as the number of rounds increases any allocation in \( \mathcal{N} \) is approximately implemented.

For \( L = \{ i \in L | \exists v \in \mathcal{N} \text{ s.t. } v_i > \min_{\mathcal{N}} u_i \} \)

If those who may gain stand to lose efficiency attains
A Non-convex Prisoner’s Dilemma

For the game:

<table>
<thead>
<tr>
<th>1 \ 2</th>
<th>n</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>1, 1</td>
<td>6, 0</td>
</tr>
<tr>
<td>c</td>
<td>0, 6</td>
<td>2, 2</td>
</tr>
</tbody>
</table>

Recall that:
- (1, 1) ∈ \( \mathcal{N} \)
- (2, 2) ∈ \( \mathcal{K}_U^0 \)
- (1, 4) ∈ \( \mathcal{K}_U^1 \)
Third round of commitment

- A third instance:

<table>
<thead>
<tr>
<th>1 \ 2</th>
<th>m1</th>
<th>m2</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1</td>
<td>6, 0</td>
<td>1, 4</td>
<td>...</td>
</tr>
<tr>
<td>m2</td>
<td>1, 4</td>
<td>6, 0</td>
<td>...</td>
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<td>...</td>
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</tbody>
</table>

- A 3 stage game since 
  \((1, 4) \in K^1_U\)

- Implements \((3.5, 2) \in K^2_U\)
A fourth round of commitment

- The a fourth instance:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>m1</th>
<th>m2</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>m1</td>
<td>6,0</td>
<td>3.5, 2</td>
</tr>
<tr>
<td>m2</td>
<td>3.5, 2</td>
<td>6,0</td>
<td>...</td>
<td></td>
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<td>...</td>
<td>...</td>
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<td>...</td>
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</tr>
</tbody>
</table>

- A 4 stage game since 

$$(3.5, 2) \in K^2_U$$

- Implements $(4.75, 1) \in K^3_U$
A fifth round of commitment

- A fifth instance:

<table>
<thead>
<tr>
<th></th>
<th>m1</th>
<th>m2</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m1</td>
<td>0, 6</td>
<td>4.75, 1</td>
<td>...</td>
</tr>
<tr>
<td>m2</td>
<td>4.75, 1</td>
<td>0, 6</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
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<td>...</td>
</tr>
</tbody>
</table>

- A 5 stage game since

\[(4.75, 1) \in \mathcal{K}_U^3\]

- Implements

\[(2.375, 3.5) \in \mathcal{K}_U^4\]
Conclusion

- Unanimous commitments lead to mutually beneficial agreements
- Such agreements are not necessarily be efficient
- Unless agents have sufficiently many instances in which to commit
- It takes “time” to reach efficient agreements

Related Projects:
- Multiple Rounds of Independent Commitment
- Incomplete Information
More Time?

- When the Implementation Fails

- Duopoly and Extra Examples

- Independent Commitment

- Commitment Layers
Thank you for time and attention!
Technical Details

- $U$ ....................... the set of unanimous commitment structures

- $\mathcal{K}(k)$ .......................... SPE payoffs of $\{M_i, k_i\}_L$

- $\mathcal{K}_U = \bigcup_{k \in U} \mathcal{K}(k)$ ............................ Unanimous SPE payoffs

- $\mathcal{N} \supseteq \mathcal{K}_U \supseteq \mathcal{V} \cap \mathcal{N}$

- If $\overline{M} \supseteq M$ and $\overline{k}(m) = k(m)$ for $\forall m \in M$ then $\mathcal{K}(\overline{k}) \supseteq \mathcal{K}(k)$

- $\co(\mathcal{N}) \subseteq \cl(\mathcal{K}_U)$
Proposition

If \(|\mathcal{N}| = 1\), then for any \(u \in \mathcal{K}_U\), \(\exists v \in \mathcal{V} \cap \mathcal{N}\) such that \(v \geq u\)

Proposition

\(\mathcal{K}_U \subseteq \cap_{i \in \mathcal{N}} \text{co}(\mathcal{N}_i \cap C)\)

Where:

- \(\text{cc}(X, Y) = \{\alpha x + (1 - \alpha)y | x \in X, y \in Y \& \alpha \in [0, 1]\}\)
- \(C = \text{cc}(\mathcal{V}, \text{co}(\mathcal{N}))\)
- \(\mathcal{N}_i = \{u \in \mathcal{U} | u_i \geq \min_{\mathcal{N}} u_i\}\)
Affine Hull & Vertices

- For \( x, y \in \mathbb{R}^n \) let \( L(x, y) \equiv \{(1 - \lambda)x + \lambda y | \lambda \in \mathbb{R}\} \), the line

- Then, a set \( H \) is a flat if \( x, y \in H \) implies \( L(x, y) \subseteq H \)

- The affine hull of \( X \), denoted \( \text{aff}(X) \), consists of the intersection of all flats that contain \( X \)

- If \( X \) is a convex polytope, then \( x \in \text{vert}(X) \) if \( y, z \in X, \lambda \in (0, 1) \) and \( x = (1 - \lambda)z + \lambda y \) implies \( x = y = z \)
Proof of Multi-round Implementation Theorem

- $k^t \in U^t$ ......................... $k_s$ unanimous for $\forall s \leq t$

- $\mathcal{K}(k^t)$ ......................... SPE payoffs of extension $k^t$

- $\mathcal{K}^t_U = \bigcup_{k^t \in U^t} \mathcal{K}(k^t)$ .................. Unanimous SPE payoffs

Corollary

The set of implementable payoffs weakly increases with the number of rounds. $\mathcal{N} \supseteq \mathcal{K}^{t+1}_U \supseteq \mathcal{K}^t_U$

Lemma

Any payoff in $\text{co}(\mathcal{K}^t_U)$ is approximately implemented with an additional round of commitment. $\text{cl}(\mathcal{K}^{t+1}_U) \supseteq \text{co}(\mathcal{K}^t_U)$
Proof of Multi-round Implementation Theorem

Notice that the equilibrium set can be expanded in all directions:

**Lemma**

For \( u_t \in \mathcal{K}_U^t, a \in A \) and \( \alpha \in \{1/q^{N-1} | q \in \mathbb{N}_+\} \):

\[
\alpha u(a) + (1 - \alpha)u_t \in \mathcal{N} \Rightarrow \alpha u(a) + (1 - \alpha)u_t \in \mathcal{K}_U^{t+1}
\]

Assumptions in theorem guarantee that \( \mathcal{K}_U^t \cap \text{ri}(\mathcal{N}) \neq \emptyset \) some \( t < \infty \)

From such starting point and expanding in every direction at even rounds and convexifying at odd rounds guarantees that the equilibrium payoff set converges to \( \mathcal{N} \)
**No Unanimous Implementation**

- **The game:**

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,2</td>
<td>1,1</td>
<td>1,1</td>
</tr>
<tr>
<td>2</td>
<td>1,1</td>
<td>0,0</td>
<td>6,0</td>
</tr>
<tr>
<td></td>
<td>1,1</td>
<td>0,6</td>
<td>0,0</td>
</tr>
</tbody>
</table>

- $N = (2,2)$

- $K_{U}^{t} = (2,2)$ for any $t$
No Unanimous Implementation

The game:

\[
\begin{array}{c|ccc}
1 & 2 & | & a & b & c \\
\hline
a & 2,2 & | & 1,1 & 1,1 \\
b & 1,1 & | & 0,0 & 6,0 \\
c & 1,1 & | & 0,6 & 0,0 \\
\end{array}
\]

\[K_U^t = (2,2) = u\]

Since for no \( v \in V \)
\[\alpha u + (1 - \alpha)v > u\]
The game:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>2,2</td>
<td>1,1</td>
<td>1,1</td>
</tr>
<tr>
<td>b</td>
<td>1,1</td>
<td>0,0</td>
<td>6,0</td>
</tr>
<tr>
<td>c</td>
<td>1,1</td>
<td>0,6</td>
<td>0,0</td>
</tr>
</tbody>
</table>

With multiple mixed layers all Nash rational payoffs are implementable.

\[ \mathcal{K}_L^1 \supseteq \mathcal{I} \langle v, M \rangle \cap \mathcal{N} \supseteq \mathcal{N} \]
A Duopoly Example

- Costs for $q \in [0, 1]$:
  $$c(q) = 6q - 3q^2$$

- Demand price:
  $$p^d = 10 - 3(q_1 + q_2)$$

- Profits of $i$:
  $$u_i = q_i(4 - 3q_i)$$

- NE $q_1 = q_2 = 1$

- For any mixed strategy:
  $$u_2 \leq \frac{16}{3} + u_1 - 8\sqrt{u_1/3} \text{ if } u_1 \in \left[\frac{1}{3}, 3\right]$$
A Duopoly Example

- NE $q_1 = q_2 = 1$
- Unilateral pledges cannot help since $q_i < 1$ is SD
- Joint pledge $q_1, q_2 \leq 2/3$
  implements $u_1 = u_2 = 4/3$
- Pledge $q_1 \leq .58, q_2 \leq .75$
  implements $u_1 = 1, u_2 = 1.7$
A Duopoly Example

▶ An additional round:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>m1</th>
<th>m2</th>
<th>m3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>m1</td>
<td>4, 0</td>
<td>1, 1.7</td>
</tr>
<tr>
<td>m2</td>
<td>1, 1.7</td>
<td>4, 0</td>
<td>1, 1.7</td>
<td></td>
</tr>
<tr>
<td>m3</td>
<td>1, 1.7</td>
<td>1, 1.7</td>
<td>4, 0</td>
<td></td>
</tr>
</tbody>
</table>

▶ Implements payoff (2, 1.1)

▶ Beliefs of future concessions lead to further concessions
When 3 rounds of commitment suffice

- The game:

\[
\begin{array}{c|cc}
1 & 2 & n & c \\
\hline
n & 0, 0 & .5, 1.5 \\
c & 0, 1 & 1, 0 \\
\end{array}
\]

- \( \mathcal{I} \) is not convex

- \( \mathcal{N} = (0, 1) \)

- \( \mathcal{K}_U^0 = (0, 1) \cup (.5, 1.5) \)
When 3 rounds of commitment suffice

The game:

<table>
<thead>
<tr>
<th></th>
<th>m1</th>
<th>m2</th>
<th>m3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1, 0</td>
<td>.5, 1.5</td>
<td>.5, 1.5</td>
</tr>
<tr>
<td>2</td>
<td>.5, 1.5</td>
<td>1, 0</td>
<td>.5, 1.5</td>
</tr>
<tr>
<td>m1</td>
<td>.5, 1.5</td>
<td>.5, 1.5</td>
<td>1, 0</td>
</tr>
</tbody>
</table>

(.5, 1.5) ∈ 𝐾^0_U

Thus (2/3, 1) ∈ 𝐾^1_U
When 3 rounds of commitment suffice

- The game:

\[
\begin{array}{c|ccc}
1 & 2 & n & c \\
\hline
n & 0, 0 & .5, 1.5 \\
c & 0, 1 & 1, 0 \\
\end{array}
\]

- Since \( \mathcal{N} = \text{co}(\mathcal{K}^1_U) \)

- \( \mathcal{K}^2_U = \mathcal{N} \)
A sixth round of commitment

- A sixth instance:

<table>
<thead>
<tr>
<th>1\2</th>
<th>m1</th>
<th>m2</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1</td>
<td>0, 6</td>
<td>u₁, u₂</td>
<td>...</td>
</tr>
<tr>
<td>m2</td>
<td>u₁, u₂</td>
<td>0, 6</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

- For \((u_1, u_2) = (2.375, 3.5)\)

- Implements \((1.1875, 4.75)\)
A seventh round of commitment

- A seventh instance:

<table>
<thead>
<tr>
<th>1 \ 2</th>
<th>m1</th>
<th>m2</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1</td>
<td>6, 0</td>
<td>u_1, u_2</td>
<td>...</td>
</tr>
<tr>
<td>m2</td>
<td>u_1, u_2</td>
<td>6, 0</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
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</tr>
</tbody>
</table>

- For \((u_1, u_2) = (1.1875, 4.75)\)

- Implements \((3.59375, 2.375)\)
A different seventh round

- The seventh instance:

<table>
<thead>
<tr>
<th>1\2</th>
<th>m1</th>
<th>...</th>
<th>m7</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1</td>
<td>0, 6</td>
<td>u1, u2</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>u1, u2</td>
<td>0, 6</td>
<td>u1, u2</td>
</tr>
<tr>
<td>m7</td>
<td>...</td>
<td>u1, u2</td>
<td>0, 6</td>
</tr>
</tbody>
</table>

- For \((u_1, u_2) = (1.1875, 4.75)\)

- Implements \((1.018, 4.929)\)
An eighth commitment round

- An eighth instance:

\[
\begin{array}{c|ccc}
1 \backslash 2 & m1 & m2 & \ldots \\
\hline
m1 & 6, 0 & u_1, u_2 & \ldots \\
m2 & u_1, u_2 & 6, 0 & \ldots \\
\vdots & \vdots & \vdots & \ddots \\
\end{array}
\]

- For \((u_1, u_2) = (3.59375, 2.375)\)

- Implements \((4.796875, 1.1875)\)
A Layered Extension

- A commitment extension with \( t + 1 \) layers is a sequence:

\[
k^t = \{k_t, \ldots, k_0\}
\]

- That satisfies \( \forall i \in \mathbb{N} \) and \( r \in \{1, \ldots, t\} \):

\[
k_{i,r} : M_r \Rightarrow \Delta(M_{i,r-1})
\]

\[
k_{i,0} : M_0 \Rightarrow \Delta_i
\]

- Time Line:

\[-t \quad -t+1 \quad \ldots \quad 1 \quad 2\]

\[
M_t \quad k_t(m_t) \subseteq M_{t-1} \quad \ldots \quad k_0(m_0) \subseteq A \quad U
\]
Results for the Layered Unanimous Structures

D. SPE payoffs for layered structure \( k^t \) are \( \mathcal{K}(k^t) \)

D. \( L^t = \{ k^t \mid k_s \text{ layered unanimous for } \forall s \leq t \} \)

D. \( \mathcal{K}_L^t = \bigcup_{k^t \in L^t} \mathcal{K}(k^t) \) As in the multiple round case:

Corollary
\[ \mathcal{N} \supseteq \mathcal{K}_L^{t+1} \supseteq \mathcal{K}_L^t \supseteq \mathcal{I} \cap \mathcal{N} \text{ for } \forall t \geq 0 \]

Corollary
\[ \text{cl}(\mathcal{K}_L^{t+1}) \supseteq \text{co}(\mathcal{K}_L^t) \text{ for } \forall t \geq 0 \]
Main Efficiency Result

**Theorem**
\[ \text{cl}(\lim_{t \to \infty} \text{cl}(K_L^t)) = N \]

**P.** Proof uses \( K_L^t \cap \text{ri}(N) \neq \emptyset \) some \( t < \infty \)

**D.** Beliefs about future favorable concessions motivate each player to concede in some events
Independent Implementation

D. A commitment structure is *independent* if $\forall i \in L$:

1. $k_i(m) = k_i(m_i)$ and
2. $k_i(m_i) = A_i$ for some $m_i \in M_i$

**Remark**

There exists $k \in I$, $v \in V(k)$ & $m \in M$ such that $v(m) \in K(k)$ if and only if $v(m) \in E \langle k(m), u \rangle$ & $v_i(m) \geq w_i(m_{-i}|k)$ for $\forall i \in N$

D. For $w_i(m_{-i}|k) = \max_{m_i \in M_i} \min_{\sigma \in E \langle u, k(m) \rangle} u_i(\sigma)$

**Remark**

If $k \in I$ and $u \in K(k)$, then $u \in R$. Hence, $R \supseteq K_I$

D. $R = \{ v \in U | v_i \geq \min_{\sigma_{-i} \in \Delta_{-i}} \max_{\sigma_i \in \Delta_i} u(\sigma), \forall i \in N \}$
Independent commitments may never be efficient

**Proposition**

If \( I \cap R \cap P = \emptyset \) then \( K_I \cap P = \emptyset \)

**P.** The probability of getting a payoff in \( R \cap P \) cannot be 1

**N.** But independent commitments can lead to efficiency gains. A tradeoff between commitment and flexibility can arise.
Flexibility vs Commitment

The game:

<table>
<thead>
<tr>
<th>1\2</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>3,3</td>
<td>1,2</td>
<td>0,4</td>
</tr>
<tr>
<td>b</td>
<td>2,1</td>
<td>0,0</td>
<td>0,2</td>
</tr>
<tr>
<td>c</td>
<td>4,0</td>
<td>2,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

For \((M_i, k_i) \in I\) such that:

\[ M_i = 2^{A_i} \& k_i(m_i) = m_i \]

Pledges \(m_i = \{a, b\}\) implement profile \((3, 3)\)