Social Security, Differential Fertility and the Dynamics of the Earnings Distribution

Kai (Jackie) Zhao

1Department of Economics
University of Western Ontario

NASM, 2008
Motivation

1. Fertility differs by income (differential fertility) (Jones and Tertilt (2007)).

2. Differential fertility is important for understanding both
   - income inequality (de la Croix and Doepke 2003).
   - wealth inequality (Knowles 1999).

3. The fertility of the poor declines proportionally more than the rich in the last several decades (see Table 1).

4. Social security creates an incentive for people to rear fewer children. (e.g. Boldrin, De Nardi and Jones (2005), Ehrlich and Kim (2007))
Figure 1: differential fertility in the U.S. (from JT (2007))

Kai (Jackie) Zhao
Social Security and Differential Fertility
Table 1: the poor’s fertility declines proportionally more

<table>
<thead>
<tr>
<th>Two cohorts</th>
<th>1896-1900</th>
<th>1951-1955</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTFR</td>
<td>2.82</td>
<td>2.05</td>
</tr>
<tr>
<td>Bottom half</td>
<td>3.29</td>
<td>2.14</td>
</tr>
<tr>
<td>Top half</td>
<td>2.36</td>
<td>1.95</td>
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**Bottom-top fertility differential**

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<td>Income elas. of fertility</td>
<td>0.93</td>
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</tr>
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</table>

**decline (#)**

|                | 0.77 | 1.14 | 0.41 | 0.73 | -0.33 |

**decline (%)**

|                | 27%  | 35%  | 17%  | 79%  | 66%   |

**Income elasticity of fertility:** the regression coefficient of the log fertility on the log income.

**Bottom half:** the average fertility rate of women in the bottom half of the income distribution;

**Top half:** the average fertility rate of women in the top half of the income distribution;
1. How does social security change differential fertility by earnings?

2. Does social security further affect the dynamics of the earnings distribution through its effects on differential fertility?
Set up a three-period OLG model in which

- Children are investment goods for parent’s old age.
- Social security reduces fertility by crowding children out of parents’ old-age portfolios;
- Given its progressivity, social security reduces the fertility of the poor by more than the rich, since its payments are a larger portion of the poor’s old-age portfolios.
Main findings

1. Social security reduces the fertility differential between the poor and the rich;

2. This smaller differential fertility generates a new stationary distribution with a smaller portion of poor people.

3. In the model, social security explains over 37% of the DF decline over the last 60 years in the U.S.
Two assumptions:  

1. Cost of child-rearing is only parental time.  
   - *The poor: low cost.*  
   - *The rich: high cost.*

2. Earnings are mean-reverting over generations.  
   - *The poor: relatively high return.*  
   - *The rich: relatively low return.*
The Environment

- An OLG model with heterogeneous agents.
- Three periods: childhood, middle age, and old age.
- One unit of time is endowed in middle age. (either work or rear child)
- Time cost of child-rearing: $b$.
- Productivity shock at the beginning of middle age: $\epsilon$, which follows a log-normal AR(1) process

$$\ln \epsilon_{t+1} = \rho \ln \epsilon_t + u_{t+1}, u_{t+1} \sim N(0, \sigma_u^2)$$

with $\rho < 1.0$ (intergenerational correlation).
A middle-age agent is altruistic to his old-age parent.

After $\epsilon$ is realized, a middle-age agent jointly decides:
1. saving for his own old age: $s$;
2. fertility choice: $n$;
3. middle-age consumption: $c^m$
4. transfer to his old-age parent: $d$;

Old age: consumption ($c^o$). Childhood: no decision.

The motive of fertility: old-age security.
The middle-age agent $i$ with $(\epsilon_t^i, n_{t-1})$:

$$\max_{s,n,d} u(c_t^m) + \gamma u(\sum_{j \neq i, j=1}^{n_{t-1}} d_t^j + T + d_t^i) + \beta E[u(c_{t+1}^o)|\epsilon_t^i]$$

subject to

$$d_t^i + s_t + c_t^m = W_t \epsilon_t^i (1 - \tau)(1 - bn_t),$$
$$c_{t+1}^o = R_{t+1} s_t + T_{t+1} + n_t D_{t+1}(n_t, \epsilon_{t+1}),$$

Assumption: $\epsilon_t^i = \epsilon_t^j$, for all $i, j \in \{1, 2, ... n_{t-1}\}$
The firm’s problem

A representative firm:

\[
\max Y_t - W_t L_t - (R_t - 1 + \delta)K_t
\]

with \( Y_t = K_t^\alpha (AL_t)^{1-\alpha} \) and \( \delta \) is the deprecation rate.
The distribution and aggregate population

\( \phi_t(n_{t-1}, \epsilon_t) \): the distribution of the middle-age generation.

\( N_t \): the aggregate population of the middle-aged generation.

\[
N_{t+1} = N_t \sum_{i=1}^{m} \int_{0}^{\hat{n}} \phi_t(n, \epsilon_i) G_t(n, \epsilon_i) dn,
\]

(2)

The law of motion for \( \phi_t(n_{t-1}, \epsilon_t) \):

\[
\phi_{t+1}(n_t, \epsilon_{t+1} = \epsilon_i) = \frac{\sum_{j=1}^{m} \pi(j, i) \int_{0}^{\hat{n}} \phi(n, \epsilon_j) G(n, \epsilon_j) I(G(n, \epsilon_j) = n_t) dn}{\sum_{j=1}^{m} \int_{0}^{\hat{n}} \phi_t(n, \epsilon_j) G(n, \epsilon_j) dn}.
\]

(3)

Here, \( I(.) \) is the indicator function.
Markets clear

\[ K_{t+1} = N_t \sum_{i=1}^{m} \int_{0}^{\hat{n}} \phi_t(n, \epsilon_i) H_t(n, \epsilon_i) dn. \] (4)

\[ L_{t+1} = N_{t+1} \sum_{i=1}^{m} \int_{0}^{\hat{n}} \phi_{t+1}(n, \epsilon_i) \epsilon_i (1 - bG_{t+1}(n, \epsilon_i)) dn. \] (5)

Government’s budget constraint

\[ N_{t-1} T_t = W_t L_t T_t. \] (6)
Social security is characterized by a pair of parameters $\{\tau, T\}$.

- pay-as-you-go
- financed via payroll tax, $\tau$ is the tax rate.
- lump-sum payment to the old age, denoted by $T$
**Definition 1**  
**a competitive equilibrium:** Given an initial distribution of a middle-age generation $\phi_0(n_{-1}, \epsilon_0)$, an initial stock of physical capital $K_0$, and an initial population of the middle-age generation $N_0$, a competitive equilibrium consists of sequences of prices $\{W_t, R_t\}$, aggregate quantities $\{L_t, K_{t+1}, N_{t+1}\}$, distributions $\phi_{t+1}(n_t, \epsilon_{t+1})$ and policy functions $\{D_t(.,.), G_t(.,.), H_t(.,.)\}$ such that:

1. the policy functions $\{D(.,.), G(.,.), H(.,.)\}$ solve the agent’s problem (P1);
2. the firm’s choices $L_t$ and $K_t$ maximize profits;
3. the prices $W_t$ and $R_t$ are such that markets clear, i.e. conditions (8) and (9) are satisfied;
4. the distribution evolves according to (7); and population, $N_t$, evolves according to (6); and
5. the government budget constraint (10) is satisfied.
**Definition 2** a *stationary equilibrium* is a competitive equilibrium where the density function, $\phi(\ldots)$, prices $R$ and $W$, social security payment, $T$, are all constant over time, and given the same prices and government parameters, agents of different generations share the same policy function for the transfer to old-age parents $D(\ldots)$. Thus it implies that population grows at a constant rate.
### Table 2: Benchmark model calibration

<table>
<thead>
<tr>
<th>Fixed Para.</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP: $A$</td>
<td>1</td>
<td>normalization</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>capital share</td>
</tr>
<tr>
<td>Depr. rate: $\delta$</td>
<td>0.08</td>
<td>BDJ (2005)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.0 (log utility)</td>
<td>BDJ (2005)</td>
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<tr>
<td>Discount factor: $\beta$</td>
<td>0.99 (yearly)</td>
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<tr>
<td>$\sigma^2_\mu$</td>
<td>0.37</td>
<td>De Nardi (2004)</td>
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<tr>
<td>Time cost : $b$</td>
<td>0.03</td>
<td>Juster and Stafford (1991)</td>
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<td>Intergen. corr.: $\rho$</td>
<td>0.62</td>
<td>income elas. of fertility: -0.17</td>
</tr>
<tr>
<td>SS tax: $\tau$</td>
<td>0.10</td>
<td>SS rev. as % GDP(1980-2000)</td>
</tr>
<tr>
<td>Altruism weight: $\gamma$</td>
<td>0.75</td>
<td>CTFR: 2.05</td>
</tr>
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**Objective:** the cohort of women born between 1951-1955 in the U.S.
Figure 2: Differential fertility by earnings
Figure 3: Differential fertility and social security
Figure 4: Earnings distribution and the SS tax
Table 1 (again)

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*Bottom half:* the average fertility rate of women in the bottom half of the income distribution;

*Top half:* the average fertility rate of women in the top half of the income distribution;
### Table 3: Data-model comparisons

<table>
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<tr>
<th>Cohort year/SS tax rate</th>
<th>1896-1900/0%</th>
<th>1951-1955/10%</th>
<th>The changes b/w two cohorts</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
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<td>Income elasticity of fertility</td>
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<td>-0.28</td>
<td>-0.17</td>
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<td>The bottom-top gap</td>
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<td>0.76</td>
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<tr>
<td>Cohort Total Fertility Rate</td>
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*Bottom half:* the average fertility rate of women in the bottom half of the income distribution;

*Top half:* the average fertility rate of women in the top half of the income distribution;

*The bottom-top gap:* the fertility difference between the bottom half and the top half;
Social security reduces the differential fertility between the poor and the rich.

This smaller differential fertility generates a new stationary distribution with fewer poor people, and raises the average income level.

This effect is quantitatively important. In the model, social security change explains over 37% of the DF decline over the last 60 years in the U.S.