Learning and Complementarities: Implications for Speculative Attacks

Itay Goldstein
University of Pennsylvania
Wharton School
itäyg@wharton.upenn.edu

Emre Ozdenoren
University of Michigan
Economics Department
emre0@umich.edu

Kathy Yuan
University of Michigan
Ross School of Business
kyuan@umich.edu

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Basic Idea

- Decision makers learn from the aggregate action of informed agents, and take an action that affects the agents’ payoffs.
  - **Examples:** Firm managers or providers of capital learn from the stock market, the central bank learns from speculators, policy makers learn from lobbying activities.
  - **Why?** Markets provide useful information about fundamentals by aggregating dispersed information: Roll (1984).

- **Problem:** Knowing that the aggregate action of speculators provides a signal that affects the decision, strategic complementarities may emerge:
  - Speculators take an action because they think others are taking that action: **Informational Complementarities**.
  - Aggregate action reflects **information** and **coordination** at the same time.
Speculative Attacks

- Central Bank decides whether to maintain or abandon a fixed exchange rate regime.

- Speculators ‘attack’ the regime by short-selling the currency.
  - Attack is costly; they benefit if the CB abandons the regime.

- Previous literature (Krugman (1979), Obstfeld (1996), Morris and Shin (1998)): CB knows fundamentals perfectly but attack is costly to defend – for example, depleting CB’s reserves.
Speculative Attacks

► **Here:** Attack aggregates information.

► **Three advantages:**
  
  - The importance of reserves decreased over time; CB has many defensive tools.
  - The idea that the CB is not fully informed and that he can gain additional information from the market seems very natural and empirically supported (see below).
  - Can also explain runs on under-valued currencies (e.g., Yuan).

► **Source of complementarities:** CB is likely to abandon when more speculators attack (due to the inference about the fundamentals).

► Need some **commonality** in speculators’ signals to enable coordination.
Information Revealed in Currency Markets

What can CB learn from the currency market?

- Speculators’ aggregate expectation of the various aspects of the economy
- Burnside, Eichenbaum, and Rebelo (2002) show that the prospective deficits associated with implicit bailout guarantees to failing banks were the reason for the Asian currency crises.
- Relating this to our paper, currency market aggregates market participants’ heterogeneous expectations of the severity of the impact of implicit bailout guarantees for the economy – which is useful to CB’s decision-making.
Related Empirical Evidence

- Piazzesi (2005): In the US, the Federal Reserve Bank is affected by market developments when setting monetary policy.
Contributions

- Literature on **feedback effect** from the financial market to the real economy:
  - Show that learning by a decision maker in the real side of the economy leads to complementarities and equilibria where they act as if they coordinate to manipulate the beliefs of the decision maker.

- Literature on **speculative attacks**:
  - Introduce the channel of CB learning.

- Literature on **strategic complementarities** and **global games**:
  - Show how strategic complementarities emerge endogenously.
Other Potential Applications

- Lobbying and political activism (Grossman and Helpman (2001), Battaglini and Benabou (2003))

- Multiple biased experts give opinion to a manager on $\theta$
Model: Actions and Payoffs

► Central Bank

- The value of defending the peg is given by $\theta \in \mathbb{R}$
- Two possible currency regimes: $\delta = 1$ means defend the status quo; $\delta = 0$ means abandon the status quo.
- Payoff: $U = \delta \theta$, and chooses $\delta$ to maximize $U$
- If CB perfectly informed: set $\delta = 1$ (defend) iff $\theta \geq 0$.

► Speculators: a continuum of measure one

- Decide whether to attack (by short-selling one unit of currency) and pay the attack cost of $c$ or not
- The payoff to attack is $1 - c$ if status quo is abandoned and $-c$ otherwise. The payoff of not attacking is normalized to $0$. 
Model: Timing and Information Structure

► Timing

• Both CB and speculators receive information regarding $\theta$
• Speculators independently and simultaneously decide whether to attack or not
• CB decides whether to abandon the status quo or not after observing the attack size

► Information structure

• CB and speculators have a common (improper) prior about $\theta$ uniform over $\mathbb{R}$
• CB receives private signal $s_c = \theta + \sigma_c \epsilon_c$, $\epsilon_c$ is std. normal
• CB also observes aggregate attack $\Phi^{-1}(A)$
• Speculator $i$ receives private signal $s_i = \theta + \sigma_s \epsilon_i$, $\epsilon_i$ is std. normal
• Speculators receive a common noisy signal $s_p = \theta + \sigma_p \epsilon_p$, $\epsilon_p$ is std. normal
Alternative Information Structure

Main results would go through if instead of endowing speculators with a common noisy signal, we assume one of these alternative information structures.

- All agents (including CB) receive a heterogenous signal about $\theta$, which has a common noise component and an idiosyncratic noise component.

- Or, all agents (including CB) observe the common signal, $s_p$, with probability less than 1. Since there is a continuum of speculators, a fixed proportion of the speculators always observes the common signal. Moreover, speculators know that with a positive probability CB misses this signal.
Equilibrium

An equilibrium consists of a strategy for the CB, $\delta(T, s_c)$, a symmetric strategy for the speculators, $g(s, s_p)$, probability measures, $\nu(\cdot|T, s_c)$ and $\mu(\cdot|s, s_p)$, such that:

$$
\delta(T, s_c) \in \text{argmax}_{\delta \in \{0, 1\}} \int_{-\infty}^{\infty} \delta \theta d\nu(\theta | T, s_c)
$$

$$
g(s_i, s_p) \in \text{argmax}_{a \in \{0, 1\}} a \cdot \left[ \int \int \mathbf{1}_{\delta(T, s_p)\theta + \sigma_c \epsilon_c = 0} d\mu(\theta | s_i, s_p) d\Phi(\epsilon_c) - c \right]
$$

$$
T(\theta, s_p) = \Phi^{-1} \left( \int_{-\infty}^{\infty} g(\theta + \sigma_s \epsilon, s_p) \phi(\epsilon) d\epsilon \right)
$$

$\nu(\theta|T, s_c)$ and $\mu(\theta|s, s_p)$ are obtained using Bayes’ rule.
Linear Equilibrium

**Proposition** There is a unique linear equilibrium where the speculators threshold strategy is:

\[
g(s_i, s_p) = \begin{cases} 
1 & \text{if } s_i \leq \hat{s}(s_p) \\
0 & \text{if } s_i > \hat{s}(s_p)
\end{cases},
\]

and the central bank’s strategy is

\[
\delta(T, s_c) = \begin{cases} 
1 & \text{if } T \leq \hat{T}(s_c) \\
0 & \text{if } T > \hat{T}(s_c)
\end{cases},
\]

where

\[
\hat{s}(s_p) = \hat{s}(0) - ks_p,
\]

$k > 0$ is the unique real root of the cubic equation:

\[
-\frac{\tau_c \tau_s}{\tau_p} k^3 + (\tau_c + \tau_p) k^2 + 2\tau_p k + \tau_p = 0,
\]
\( \hat{s}(0) \) satisfies

\[
c = \Phi \left( \frac{-\left( (1 + k) \frac{\tau_c}{\tau_T} + 1 \right) \frac{\tau_s}{\tau_s + \tau_p} \hat{s}(0)}{\sqrt{\left( (1 + k) \frac{\tau_c}{\tau_T} + 1 \right)^2 \frac{1}{\tau_s + \tau_p} + (1 + k)^2 \frac{\tau_c}{\tau_T^2}}} \right),
\]

and

\[
\hat{T}(s_c) = \frac{1}{\sigma_s} \left[ \hat{s}(0) + (1 + k) \frac{\tau_c}{\tau_T} s_c \right],
\]

where

\[
\tau_T = \tau_p \left( 1 + \frac{1}{k} \right)^2
\]

which is the precision of the attack as a signal of the fundamental.
Some Comments

- For the CB the attack provides an additional signal with precision $\tau_T$
  - precision of attack is endogenous and decreases in $k$

- CB abandons the exchange regime if and only if aggregate attack exceeds threshold that increases in its private signal

- Speculator who receives a signal, $s_i$, attacks if and only if $s_i$ falls below a threshold value, $\hat{s}(s_p)$ decreasing in the common noisy signal $s_p$.

- The coefficient $k$ is a measure of equilibrium level of informational complementarities
Proof

- Suppose an agent attacks if and only if $s_i + ks_p \leq \hat{s}(0)$ where $k > 0$.
  - That is, $\theta + \sigma_s \epsilon_s + ks_p < \hat{s}_0$ or, $\epsilon_s < \frac{\hat{s}(0) - \theta - ks_p}{\sigma_s}$.

- The size of the attack from speculators given $\theta$ and $s_p$ is $A(\theta, s_p) = \Phi \left( \frac{\hat{s}(0) - ks_p - \theta}{\sigma_s} \right)$.

- The CB observes
  $$T = \frac{\hat{s}(0) - ks_p - \theta}{\sigma_s},$$

- Rewrite as
  $$\frac{\hat{s}(0) - \sigma_s T}{1 + k} = \theta + \frac{k \sigma_p}{1 + k} \epsilon_p.$$
The precision of the attack as a signal of the fundamental is

\[ \tau_T = \frac{\tau_p (1 + k)^2}{k^2}, \]

This implies the status quo is abandoned if and only if

\[ T \geq \frac{\hat{s}(0)}{\sigma_s} + \frac{1 + k}{\sigma_s} \frac{\tau_c}{\tau_T} s_c = \hat{T}(s_c), \]
For a speculator, $\theta$ is distributed with mean $\frac{\tau_s}{\tau_s + \tau_p} s_i + \frac{\tau_p}{\tau_s + \tau_p} s_p$ and precision $\tau_s + \tau_p$.

The posterior belief of the regime change for a speculator with signal $s$ and $s_p$ is expressed as follows:

\[
\Pr \left( T \geq \frac{\hat{s}(0)}{\sigma_s} + \frac{1 + k}{\sigma_s} \frac{\tau_c}{\tau_T} s_c | s_i, s_p \right)
\]

\[
= \Pr \left( \left( (1 + k) \frac{\tau_c}{\tau_T} + 1 \right) \theta + (1 + k) \frac{\tau_c}{\tau_T} \sigma_c \epsilon_c \leq -k s_p | s_i, s_p \right)
\]

\[
= \Phi \left( \frac{-\left( (1 + k) \frac{\tau_c}{\tau_T} + 1 \right) \frac{\tau_s}{\tau_s + \tau_p} s_i - \left( \left( (1 + k) \frac{\tau_c}{\tau_T} + 1 \right) \frac{\tau_p}{\tau_s + \tau_p} + k \right) s_p}{\sqrt{\left( (1 + k) \frac{\tau_c}{\tau_T} + 1 \right)^2 \frac{1}{\tau_s + \tau_p} + \left( (1 + k) \frac{\tau_c}{\tau_T} \sigma_c \right)^2}} \right)
\]
In equilibrium the threshold strategy must satisfy the following equation for all $s_p$:

$$c = \Phi \left( -\left( (1 + k) \frac{\tau_c}{\tau_T} + 1 \right) \frac{\tau_s(0)}{\tau_s + \tau_p} - \left( (1 + k) \frac{\tau_c}{\tau_T} + 1 \right) \left( \frac{\tau_p}{\tau_s + \tau_p} - \frac{\tau_s}{\tau_s + \tau_p} k \right) + k \right) \frac{s_p}{\sqrt{\left( (1 + k) \frac{\tau_c}{\tau_T} + 1 \right)^2 \frac{1}{\tau_s + \tau_p} + (1 + k) \frac{\tau_c}{\tau_T} \sigma_c}}.$$

For a linear equilibrium to exist the coefficient of $s_p$ must be zero:

$$-k \frac{\tau_s}{\tau_s + \tau_p} + \frac{\tau_p}{\tau_s + \tau_p} + \frac{k}{\left( 1 + (1 + k) \frac{\tau_c}{\tau_T} \right)} = 0.$$

Substituting for $\tau_T$ and rearranging the above equation we obtain:

$$-\frac{\tau_c \tau_s}{\tau_p} k^3 + (\tau_c + \tau_p) k^2 + 2\tau_p k + \tau_p = 0.$$

The above cubic equation has a single strictly positive real root.
Informational Complementarities

► As a benchmark, suppose CB uses only its private signal to make policy decisions

► Speculators have no incentive to coordinate and the weight on the common signal is purely based on the signal’s information about $\theta$
  
  • Solve:  
  
  $\Pr (\theta + \sigma_c \epsilon_c < 0 | \hat{s}_{BM}(s_p), s_p) = \Phi$  
  
  where  
  
  $\hat{s}_{BM}(0) - k_{BM} s_p$

  • $k_{BM} = \tau_p/\tau_s$

► $k > k_{BM}$
  
  • Speculators put a larger weight on $s_p$ because it reveals not just the fundamentals but, more importantly, other speculators’ actions
  
  • They coordinate on attacking more (less) aggressively when $s_p$ is low (high)
  
  • Consequently, $\tau_T$ is lower and CB cannot tell the attack is motivated by coordination or information
Comparative Statics

Proposition The equilibrium value of $k$ decreases in $\tau_c$ and $\tau_s$, and increases in $\tau_p$.

- If CB’s or speculators’ private info is less precise, or the common signal is more precise, in equilibrium speculators coordinate better.
  - If CB holds a precise signal, it relies less on the information revealed in the aggregate actions of the speculators and their incentive to coordinate gets diminished.
  - If each speculator holds a very sharp private signal, they rely mostly on the private signal rather than the noisy common signal, hence there is less incentive to coordinate.
  - Conversely when the common signal is very precise, speculators rely more on the common signal and this gives them more incentive to coordinate.
Let $\tau_p$ Approach 0: Heterogenous Information

- Previously,
  - The attack reveals information about the fundamentals by aggregating dispersed private information.
  - Yet its informational value is dampened by the coordination among speculators, which is measured by $k$.

- As $\tau_p$ approaches zero, common signal becomes too noisy and the speculators can no longer coordinate, i.e., $k \to 0$ as $\tau_p \to 0$.

- The fundamental becomes fully revealing to the central bank.
  - A speculator attacks if and only if $s \leq \hat{s}(0)$.
  - CB observes $T(\theta) = \Phi^{-1}(A)$ or $T = (\hat{s}(0) - \theta) / \sigma_s$.
  - Hence, in equilibrium, CB infers $\theta$ perfectly: $\theta = \hat{s} - \sigma_s T$. 

Policy Efficiency

- We measure policy efficiency using probability of abandoning (maintaining) status quo given \( \theta > 0 \) \( (\theta < 0) \) before any signal is revealed.
  - If \( \theta > 0 \) \( (\theta < 0) \), abandoning (maintaining) the status quo is a policy error.
  - We consider the ex ante probability of making mistakes, so that our measure does not depend on particular signal realization.
Informational Complementarities and Policy Efficiency

**Proposition** The ex ante probability of abandoning the status quo for a given $\theta$ is

$$\Phi \left( -\sqrt{\tau_c + \tau_T \theta} \right).$$

Hence, when $\theta > 0$, the probability of making a policy mistake is $\Phi \left( -\sqrt{\tau_c + \tau_T \theta} \right)$, while when $\theta < 0$, it is $1 - \Phi \left( -\sqrt{\tau_c + \tau_T \theta} \right)$. 
More Informed Speculators Reveal Less Information

Proposition Probability of making a policy mistake decreases in $\tau_c$ and $\tau_s$, and increases in $\tau_p$ if $0 < \tau_p < \tau_c \frac{\sqrt{1+16\frac{\tau_s}{\tau_c}} - 1}{8}$ and decreases (increases) in $\tau_p$ if $\tau_p > \tau_c \frac{\sqrt{1+16\frac{\tau_s}{\tau_c}} - 1}{8}$.

- Competing effects as $\tau_p$ increases:
  - more precise information but better coordination
  - if $\tau_p$ is small then the coordination effect dominates
  - Once $\tau_p$ becomes large enough information effect takes over
Can the Central Bank do Better?

- CB should always pay attention to the attack and not ignore it.
  - Probability of devaluation for a given $\theta$ when the central bank ignores the attack is $\Phi(-\sqrt{\tau_c}\theta)$ and when it learns from the attack is $\Phi\left(-\sqrt{\tau_c + \tau T}\theta\right)$.
  - Latter probability is smaller when $\theta > 0$ and larger when $\theta < 0$. 
Commit to Strategically Ignore

Could CB improve his policy decision by putting slightly more than the equilibrium weight on his private signal?

There are two competing effects to consider.

- By putting less weight on the attack as a signal, the CB makes it less valuable for the speculators to coordinate on the common signal, and this improves the information quality of the attack.
- Yet CB now puts less than optimal weight on valuable information.

In equilibrium, CB can always decrease the probability of following incorrect devaluation policy by putting slightly less weight on the attack.

CB should be able to commit, as this policy is not time consistent.
Transparency

Could CB improve his policy decision by releasing a signal of his belief?

No

• Speculators now overweight this common signal as well as their own common signal: attacking more (less) aggressively when these common signals are low (high)
• CB inadvertently facilitates coordination

Could CB improve his policy decision by releasing a signal that allows individual interpretation?

Yes

• Precision of individual signal is improved and speculators rely less on the common signal to make attack decisions
Other Policy Implications

- The level of the opportunity cost, $c$ (as well as the wealth level of speculators) affects the size of the attack but does not affect the information content of the attack.

- Thus the probability of devaluation occurring for a given $\theta$ does not change.

- Result can explain why some speculative attacks are defended by the central banks while some, that are not as strong, are not.
Conclusion

► We analyze the aggregation and usage of information in the process of a speculative attack.

► Learning from the attack is a double-edged sword:
  • Information enhances efficiency of policy decision.
  • But, the fact that the central bank learns gives rise to endogenous complementarities that send a misleading message.
  • For complementarities to arise, some commonality has to exist in speculators’ information.

► We study comparative statics and policy implications.

► We contribute to three different literatures.