Inflation and Variety

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1 Introduction

1.1 Motivation

- Experiences in high inflation economies: quantities and varieties

Germany (1920s):

"Shops remained empty and their suppliers, unable for this reason to get rid of their wares, reduced production." – Guttmann and Meehan, 1975

"In that case few transactions are realized, markets become thinner, and some markets are thinned out of existence." – Heymann and Leijonhufvud, 1995

Russia (1990s):

"The crippling high inflation accompanies a loss of control over the money supply. And it will mean a conspicuous shortage, in both number and variety, of basic consumer goods, food, and housing." – Guardiano, 1993
"As the payment system becomes less efficient, there is a significant reduction in the variety of goods offered for sale." – Shevchenko, 2004

Zimbabwe (2007):

"Many consumer items have disappeared altogether, forcing supermarkets to fill out their shelves with empty packaging behind the few goods on display." – International Herald Tribune, 2007

Argentina (2002):

McKenzie and Schargrodsky (2003): product variety measured by the number of Stock Keeping Units offered in supermarkets fell by 14.3% between 2001 and 2002.

Midrigan (2007): the rate of net product creation was as high as 19% in the year ending with June 2001 and the rate dropped to -8% in the year ending with June 2002.

• Is variety important for welfare?
1.2 Purposes of this Paper

In a monetary model with explicit microfoundations:

- endogenize the choice of variety
- study the effect of inflation on variety and quantity
- assess the welfare cost of inflation due to changes of variety and quantity
2 Environment

- Time is discrete. A centralized market (CM) and a decentralized market (DM) open sequentially in each period.

- Two types of agents: households \((h \in \mathcal{H})\) and firms \((f \in \mathcal{F})\), each with measure 1. Firms are owned by households.

- One general good in the CM and a set of special goods in the DM. All goods are nonstorable.

- In the CM, firms distribute their profits to households, hire labor, produce the general good and make capital investment \(b\) and variety investment \(k(n)\) for the following DM production, with \(k' > 0, k'' \geq 0\). Households supply labor and consume the general good, with instantaneous utility \(v(x) - y\).
Environment-continued

- In the DM, a constant return to scale matching function: \( \mathcal{M} : \mathbb{R}^2_+ \to [0, 1] \). Let \( \alpha = \mathcal{M} \) denote the probability that a household/firm meets a firm/household.

- In the DM, a firm \( f \) can potentially produce a unique set of special goods \( \Phi_f \) with measure \( N \). The actual product variety that the firm can offer is \( n \).

  Once matched, the probability that a firm can produce the good that a household values is \( \sigma(n) \), with \( \sigma'' \leq 0 < \sigma' \). E.g. \( \sigma(n) = \frac{n}{N} \).

  Producing \( q \) units of any special good requires \( c(q) \) units of capital.

  Firms can transform the rest of the capital into the general good in the next CM. The technology is such that 1 unit of capital can be transformed into \( 1 + r \) units of the general good. Note the technology is very special, \( 1 + r = \frac{1}{\beta} \).

  Households get utility \( u(q) \) from consuming \( q \) units of the ideal variety.
A Benchmark Allocation

- A planner’s objective:

\[ E_0 \sum_{t=0}^{\infty} \beta^t [v(x_t) - y_t + \alpha \sigma(n_t)u(q_t)]. \]

The resource constraints for the general good and capital are:

\[ x_t + k(n_t) + b_t \leq y_t + (1 + r)[1 - \alpha \sigma(n_{t-1})]b_{t-1}; \]
\[ c(q_t) \leq b_t. \]

- Solution is characterized by:

\[ v'(x^*) = 1; \]
\[ u'(q^*) = c'(q^*); \]
\[ k'(n^*) = \alpha \sigma'(n^*)[u(q^*) - c(q^*)]. \]
4 Monetary Equilibrium with Bargaining

• Suppose that in the DM, agents are anonymous and lack of commitment. Money is essential. Aggregate money supply grows at $\gamma: M_+ = \gamma M$. Monetary authority injects new money only to households at the beginning of the CM and $\tau = (\gamma - 1)M$.

• In terms of the pricing mechanisms in the DM, I start with bargaining.

• As an alternative, I will consider price posting with directed search.
- Households in the CM:

\[ W(m) = \max_{x,y,\hat{m}} \{ v(x) - y + V(\hat{m}) \} \]

s.t. : \( \phi(\hat{m} - m - \tau) + x = y + \Pi_\tau \).

Note that \( W(m) \) is linear in \( m \). In particular, \( W(m) = W(0) + \phi m \).

- Households in the DM:

\[ V(\hat{m}) = \alpha \sigma(n)[u(q) + \beta W_+(\hat{m} - d)] + [1 - \alpha \sigma(n)]\beta W_+(\hat{m}). \]

- Firms face a sequence of static problems:

\[ \pi = \max_{b,n} \left\{ -b - k(n) + \alpha \sigma(n)[b - c(q) + \frac{1}{1 + r}\phi_d] + [1 - \alpha \sigma(n)]b \right\} \]

s.t. : \( c(q) \leq b \).
The terms of trade \((q, d)\) is determined by generalized Nash bargaining.

\[
\max_{q, d} \left[ u(q) + \beta W_+ (\hat{m} - d) - \beta W_+ (\hat{m}) \right]^{\theta} \left[ -c(q) + \frac{1}{1 + r} \phi + d \right]^{1-\theta}
\]

s.t. : \(d \leq \hat{m}\)
\(c(q) \leq b\).

**Proposition:** A bargaining monetary equilibrium exists if

\[
\max_q \left\{ -i g(q) + \alpha \sigma(n) [u(q) - g(q)] \right\} > 0.
\]

**Proposition:** Focusing on the solution that has the highest \((q, n)\), \(\frac{\partial q}{\partial \gamma} < 0\) and \(\frac{\partial n}{\partial \gamma} < 0\).

**Proposition:** \(q < q^*\) and \(n < n^*\). Friedman rule gives the best allocation, but the efficient allocation cannot be achieved.
5 Monetary Equilibrium with Price Posting

- In the DM, a set of submarkets $\Omega$ open. For each $\omega \in \Omega$, market makers announce $(q, d, n)$ at the beginning of each period. Households and firms who direct their search to the same $(q, d, n)$ form a submarket. Market tightness $Q(\omega) = \frac{H(\omega)}{F(\omega)}$ should be consistent with rational expectations.

- Households in the DM:

$$V(\hat{m}) = \max_{\omega \in \Omega} \{ \alpha_h(Q)\sigma(n)[u(q) + \beta W_+(\hat{m} - d)] + [1 - \alpha_h(Q)\sigma(n)]\beta W_+(\hat{m}) \}.$$ 

- Firms:

$$\pi = \max_{\omega \in \Omega} \left\{ -b - k(n) + \frac{(1 + r)b - (1 + r)\alpha_f(Q)\sigma(n)c(q) + \alpha_f(Q)\sigma(n)\phi + d}{1 + r} \right\}.$$
Market makers choose \((q, d, n, Q)\) to maximize \(\pi\) such that buyers get the equilibrium expected utility \(\bar{U}\).

Households choose \(\hat{m} = d\).

Market makers’ problem:

\[
\max_{q,d,Q,n} \left\{ \alpha_f(Q)\sigma(n)[-c(q) + \beta\phi_d] - k(n) \right\}
\]

s.t.: \(-[i + \alpha_h(Q)\sigma(n)]\beta\phi_d + \alpha_h(Q)\sigma(n)u(q) = \hat{U}\).

where \(\hat{U} = \bar{U} - \beta W_+(0) - \phi(m + \tau) - \Pi_\cdot\)
Price Posting Equilibrium–continued

- Note that in $\omega \in \Omega$, $\bar{U}$ is the "price" of households, which is endogenously determined by demand for households and supply of households.

- **Proposition**: A price posting monetary equilibrium exists if
  \[ \hat{U} = -i\beta \phi + d + \alpha_h(Q)\sigma(n)[u(q) - \beta \phi d] > 0. \]
  Moreover, the equilibrium is unique.

- Price posting equilibrium is $Q = 1$ and $(q, n)$ solves:
  \[
  \frac{u'(q)}{c'(q)} - 1 - \frac{i}{\alpha \sigma(n)} = 0;
  \]
  \[
  \alpha\sigma'(n)\frac{c'(q)[u(q) - c(q)]}{\eta u'(q) + (1 - \eta)c'(q)} - k'(n) = 0.
  \]
  where the elasticity $\eta$ is households’ contribution to firms’ probability of matching.
Price Posting Equilibrium–continued

• $d$ is given by

$$
\beta \phi_+ d = \frac{\eta u'(q)c(q) + (1 - \eta)c'(q)u(q)}{\eta u'(q) + (1 - \eta)c'(q)} \equiv g(q).
$$

• Proposition: $\frac{\partial q}{\partial \gamma} < 0$ and $\frac{\partial n}{\partial \gamma} < 0$.

• Proposition: $n \leq n^*$ and $q \leq q^*$. Friedman rule achieves $(q^*, n^*)$. 
Price Posting Equilibrium–continued

Figure 1: The effect of an increase in inflation
6 Quantitative Analysis

- Set $v(x) = a \log x$, $u(q) = \frac{q^{1-\rho}}{1-\rho}$, $c(q) = Aq$, $\sigma(n) = \frac{n}{n+1}$, $k(n) = \kappa n^2$ and $\alpha = 1 - e^{-1}$.

- Model's "money demand": $L(i) \equiv \frac{M}{PY} = \frac{1}{a + \frac{1}{(1+i)g(q)} + \alpha \sigma(n)}$


- Nonlinear least square: estimate $(a, \rho, A, \kappa)$.

- Hard to identify all parameters: estimate $(a, \rho)$.
- Welfare cost of 10% inflation in bargaining equilibrium:

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- Welfare cost of 10% inflation in price posting equilibrium:

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<tbody>
<tr>
<td></td>
<td>0.015</td>
<td>0.015</td>
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7 Conclusion

- The model predicts that inflation reduces variety, which is consistent with high inflation economy’s experiences.

- By endogenizing variety, the total welfare cost of inflation and variety induced welfare cost can be large, but it depends on the pricing mechanism and the bargaining power parameter.

- An extension is to endogenize entry decision.

- Deflationary episodes might also be associated with less product variety, but the current model is not suited to address this issue.