Idiosyncratic and Aggregate Risk in a Frictional Labor Market

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Business cycles in unemployment, vacancies, wages
Incomplete markets

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Business cycles in unemployment, vacancies, wages

Qualitatively and quantitatively
MODEL BASICS: AGENTS

- Workers $E_t \int_0^\infty e^{-\rho s} \frac{(t+s)^{1-\gamma}}{1-\gamma} ds$
- Entrepreneurs $E_t \int_0^\infty e^{-\rho s} \frac{(t+s)^{1-\gamma}}{1-\gamma} ds$
Production technologies:

- Market production:
  - flow output $z(t)$
  - match specific shocks $\delta$

- Workers’ home production: flow output $b$ with $0 < b < z(t)$
**Model Basics: Technologies**

**Production technologies:**
- Market production:
  - flow output $z(t)$
  - match specific shocks $\delta$
- Workers’ home production: flow output $b$ with $0 < b < z(t)$

**Search technology:**
- Wage contract $\sigma(\tau)$ specifies state contingent wage payments $w(\tau, \tau + s)$, $s \geq 0$
- Entrepreneurs post wage contracts at flow cost $\kappa$
- Workers apply for one
- Market for contract: Matching function – $\theta$ vu-ratio
Entrepreneurs face complete asset markets:
normalized prices $p(t)$
Workers excluded
ENTREPRENEURS’ PROBLEM

Given prices $p(t)$ and beliefs $\Theta(., t)$, choose $c^i(t), v^i(t), \sigma^i(t)$ to

$$\max E_0 \int_0^\infty e^{-\rho t} u(c^i(t)) \, dt$$

s.t.

$$E_0 \int_0^\infty p(t) \left[ \int_{-\infty}^t n^i(\tau, t)[z(t) - w^i(\tau, t)] \, d\tau - \kappa v^i(t) - c^i(t) \right] \, dt + W^i_0 = 0$$

with $W^i_0, n^i(\tau, 0), \sigma^i(\tau)$ given for $\tau < 0$
**Entrepreneurs’ Problem**

Given prices $p(t)$ and beliefs $\Theta(.,t)$, choose $c^i(t), v^i(t), \sigma^i(t)$ to

$$\max E_0 \int_0^\infty e^{-\rho t} u(c^i(t)) dt$$ \hspace{1cm} (P)

s.t.

$$E_0 \int_0^\infty p(t) \left[ \int_{-\infty}^t n^i(\tau, t)[z(t) - w^i(\tau, t)]d\tau - \kappa v^i(t) - c^i(t) \right] dt + W^i_0 = 0$$

with $W^i_0, n^i(\tau, 0), \sigma^i(\tau)$ given for $\tau < 0$

**Issue:** State space!
Prices for AD-claims $p(t)$,
Entrepreneurs’ beliefs $\Theta(., t)$, consumption $c^i(t)$, contracts $\sigma^i(t)$, associated market tightnesses $\theta^i = \Theta(\sigma^i(t), t)$, and vacancies $v^i(t)$, for all $i$

1. **Entrepreneurs maximize utility** in consuming, choice of contract, vacancy creation
2. Unemployed **workers maximize utility** in choice of contract, determining $\Theta()$
3. **Goods market clears**
4. **Labor market clears**
EQUILIBRIUM PROPERTIES

If equilibrium exists

- Can aggregate entrepreneurs
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- Unique contract
If equilibrium exists

- Can aggregate entrepreneurs
- Unique contract
- Contracts:

\[ e^{-\rho s \left( \frac{w(\tau, \tau+s)}{w(\tau, \tau)} \right) - \gamma} = \frac{p(\tau+s)}{p(\tau)} \]
EQUILIBRIUM PROPERTIES

If equilibrium exists

- Can aggregate entrepreneurs
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\[
e^{-\rho s \left( \frac{w(\tau, \tau+s)}{w(\tau, \tau)} \right)} - \gamma = \frac{p(\tau+s)}{p(\tau)} = e^{-\rho s \left( \frac{c(\tau+s)}{c(\tau)} \right)} - \gamma
\]
**EQUILIBRIUM PROPERTIES**

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- Can aggregate entrepreneurs
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\[ e^{-\rho_s \left( \frac{w(\tau, \tau + s)}{w(\tau, \tau)} \right) - \gamma} = \frac{p(\tau + s)}{p(\tau)} = e^{-\rho_s \left( \frac{c(\tau + s)}{c(\tau)} \right) - \gamma} \]

\[ \Rightarrow \quad w(\tau, \tau + s) = a(\tau)c(\tau + s) \quad \forall s \geq 0 \]
If equilibrium exists

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\[ \Rightarrow \quad w(\tau, \tau + s) = a(\tau)c(\tau + s) \quad \forall s \geq 0 \quad \Rightarrow 2 \text{ state variables only!} \]
If equilibrium exists

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- Contracts:

\[ e^{-\rho s \left( \frac{w(\tau, \tau + s)}{w(\tau, \tau)} \right)} - \gamma = \frac{p(\tau + s)}{p(\tau)} = e^{-\rho s \left( \frac{c(\tau + s)}{c(\tau)} \right)} - \gamma \]

\[ \Rightarrow w(\tau, \tau + s) = a(\tau)c(\tau + s) \quad \forall s \geq 0 \quad \Rightarrow \text{2 state variables only!} \]

- Unique equilibrium

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EQUILIBRIUM PROPERTIES

If equilibrium exists

- Can aggregate entrepreneurs
- Unique contract
- Contracts:
  
  \[ e^{-\rho s \left( \frac{w(\tau, \tau + s)}{w(\tau, \tau)} \right) - \gamma} = \frac{p(\tau + s)}{p(\tau)} = e^{-\rho s \left( \frac{c(\tau + s)}{c(\tau)} \right) - \gamma} \]

  \[ \Rightarrow w(\tau, \tau + s) = a(\tau)c(\tau + s) \forall s \geq 0 \Rightarrow 2 \text{ state variables only!} \]

- Unique equilibrium

- Wage commitments state variable \( \sim \) employment dynamics
\( \gamma = 0, 1, 2, 3, 4 \). Model produced data aggregated to quarterly, logged, HP-filtered.
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**Incomplete Markets and Volatility**

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**Incomplete Markets** vs **Complete Markets**

- **$sd(\theta)/sd(z)$**
- **$sd(w)/sd(z)$**

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**Incomplete Markets and Volatility**

- **complete vs incomplete markets**
- **worker vs entrepreneur risk aversion**

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INCOMPLETE MARKETS AND VOLATILITY

- complete vs incomplete markets
- worker vs entrepreneur risk aversion
- impatient workers

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CONCLUSIONS

Tractable model of market incompleteness, wages with aggregate shocks
CONCLUSIONS

**Tractable** model of market incompleteness, wages with aggregate shocks

**Quantitative results**: Incomplete markets moves volatilities in right direction, by moderate amount
Single Contract Problem

Given prices $p(t)$ and beliefs $\Theta(\sigma(t), t)$ for all $t$

$$\max_{\sigma(t)} -p(t)\kappa + q(\Theta(\sigma(t), t)) E_t \int_0^\infty p(t + s)e^{-\delta s}[z(t + s) - w(t, t + s)]ds$$
**Single Contract Problem**

Given prices $p(t)$, value of unemployment $V^u(t)$ for all $t$

$$\max_{\sigma(t)} -p(t)\kappa + q(\theta(t))E_t \int_0^\infty p(t + s)e^{-\delta s}[z(t + s) - w(t, t + s)]ds$$

$$\rho V^u(t) = u(b) + \mu(\theta(t))[E_t \int_0^\infty e^{-(\rho+\delta)s}[u(w(t, t + s)) + \delta V^u(t + s)]ds - V^u(t)]$$

$$+ \eta[E_+ ,t V^u(t) - V^u(t)] + \frac{d}{dt} V^u(t)$$
Firm problem developed

Worker utility from $\sigma(t)$: $E_t \int_0^\infty e^{-(\rho+\delta)s} u(w(t, t + s))ds$

Subproblem of firm problem:

$$\text{Profit}(V) = \max_{\sigma(t)} E_t \int_0^\infty p(t + s)e^{-\delta s}[z(t + s) - w(t, t + s)]ds$$

$$E_t \int_0^\infty e^{-(\rho+\delta)s} u(w(t, t + s))ds \geq V$$

2nd subproblem of firm problem:

$$\max_{\sigma(t)} q(\theta) \text{Profit}(V)$$

$$\mu(\theta)(V - V^u) = \text{given}$$