The Long and The Short of Asset Prices: Using Long-Run Consumption-Return Correlations to Test Asset Pricing Models

Jianfeng Yu

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Motivation

- **External Habit-Formation Models:**
  - Key mechanism: Agents care about consumption above a certain "habit" level
  - The relative difference between consumption and habit level (surplus ratio) becomes a determinant of risk aversion
  - This produces time variation in the Sharpe ratio, expected returns, making returns volatile, and raising risk premium

- **Trend-Cycle Models:**
  - Common Theme: Small predictable consumption component
  - Production Based Model of Panageas and Yu (2005): Time varying presence of growth options over the medium run produces time varying risk premia
  - Common Implication: Expected returns depend on whether consumption is above or below its stochastic trend
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Key idea:

- Both models derive return implications from slow moving processes that are related to consumption (backward-looking surplus ratio /forward-looking expected consumption growth)
- The two models have different implications for long run and short run correlations between consumption and returns.
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Summary of Main Results

Stylized Facts in the data

- Short run correlations are lower than long run correlations between consumption and returns.
- Stock market returns "lead" rather than "lag" consumption growth.

Performance of the models

- External habit formation models have a difficult time matching the correlation over different horizons and the lead-lag relation between consumption growth and asset returns.
- By contrast, trend cycle models can match both the short run and the long run feature of the data.
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Roadmap

- Stylized facts in the data, both time-domain analysis and the frequency domain analysis
  - Long-horizon implications for habit models
  - Long-horizon implications for trend-cycle models
  - Conclusions
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Conclusions
Correlation and Covariance Over Long Horizons: Data

Both cumulative correlation \( \text{corr} \left( \sum_{j=1}^{K} r_{t+j}, \sum_{j=1}^{K} g_{c,t+j} \right) \) and cumulative covariance \( \text{cov} \left( \sum_{j=1}^{K} r_{t+j}, \sum_{j=1}^{K} g_{c,t+j} \right) \) are increasing with horizon.
Correlation between consumption growth and asset returns:

- **High Frequency Correlation (with cycle b/t 0.5 and 3 years):** 0.114
- **Low Frequency Correlation (with cycle longer than 3 years):** 0.342

Granger’s Causality Test:

- **Consumption does NOT Granger-cause returns**  
  \( p\text{-value} = 0.4482 \)
- **Asset returns DO Granger-cause consumption**  
  \( p\text{-value} = 4.3770 \times 10^{-4} \)
Band-Pass Filter and Granger Causality Test: Data

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Coherency of Consumption and Returns: Quarterly Data

The Long and The Short of Asset Prices
Cospectrum of Consumption and Returns: Quarterly Data

Jianfeng Yu  The Long and The Short of Asset Prices
Phase Spectrum of Consumption and Returns: Quarterly Data

Phase Spectrum

Phase (degrees)

frequency

Jianfeng Yu  The Long and The Short of Asset Prices
External Habit-Formation: Model Setup

- Consumption growth

\[ g_{c,t} = \mu_c + \epsilon_{c,t} \]

- The cointegrating constraint

\[ d_t = \mu_{dc} + c_t + \delta_t \]
\[ \delta_t = \rho \delta_{t-1} + \epsilon_{\delta,t}, \]
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Model Setup

- The agent’s problem

\[ E_t \sum_{k=0}^{\infty} \delta^k \frac{(C_{t+k} - X_{t+k})^{1-\gamma} - 1}{1-\gamma} \]

- The log surplus ratio \( s_t = \log \left( \frac{C_t - X_t}{C_t} \right) \)

\[ s_{t+1} = (1 - \phi) \bar{s} + \phi s_t + \lambda (s_t) \epsilon_{c, t+1} \]

- The sensitivity function \( \lambda (s) \)

\[ \lambda (s_t) = \begin{cases} 
\frac{1}{\bar{s}} \sqrt{1 - 2(s_t - \bar{s})} - 1, & s_t \leq s_{\text{max}} \\
0, & s_t \geq s_{\text{max}} 
\end{cases} \]

with

\[ s_{\text{max}} = \bar{s} + \frac{1}{2} \left( 1 - \bar{s}^2 \right), \quad \bar{s} = \sigma_c \sqrt{\frac{\gamma}{1 - \phi}} \]
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Log-linear Approximation

- Assume linear approximation on pd ratio

\[ z_t \equiv \log(P_t/D_t) = a_0 + a_1 s_t + a_2 \delta_t \]

- The asset returns can be approximated by

\[ r_{t+1} \approx \alpha - \beta_S \tilde{S}_{t+1} + u_{t+1} \]
\[ \tilde{S}_t \approx \sum_{j=1}^{\infty} \phi^{j-1} g_{c,t-j} \]
\[ u_{t+1} \equiv \tau \epsilon_{c,t+1} + [1 + a_2 \rho] \epsilon_{\delta,t+1} \]
\[ \beta_S = \frac{a_1 (\rho \phi - 1)}{\tilde{S}} > 0 \quad \text{iff} \quad a_1 > 0 \]
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Increasing Cospectrum, Negative Low-Frequency Correlation

Proposition

- As long as the model produces countercyclical equity premium, more covariation between consumption and returns comes from high frequency components (i.e. the cospectrum is an increasing function of the frequency.)
- As long as the model produces a reasonable equity premium, the correlations between consumption growth and asset returns at low frequencies (or long horizons) are negative.

Both implications are the opposite of the data.
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## Parameter Choices

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<thead>
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<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean consumption growth (%)</td>
<td>$g_c$</td>
<td>1.89</td>
</tr>
<tr>
<td>Std. of consumption growth (%)</td>
<td>$\sigma_c$</td>
<td>1.22</td>
</tr>
<tr>
<td>Log risk-free rate (%)</td>
<td>$r^f$</td>
<td>0.94</td>
</tr>
<tr>
<td>Persistence coefficient in habit</td>
<td>$\phi$</td>
<td>0.87</td>
</tr>
<tr>
<td>Persistence coefficient in $\delta_t$</td>
<td>$\rho_\delta$</td>
<td>0.89</td>
</tr>
<tr>
<td>Std. of the innovation in $\delta_t$</td>
<td>$\sigma_\delta$</td>
<td>0.112</td>
</tr>
<tr>
<td>Risk aversion coefficient</td>
<td>$\gamma$</td>
<td>2</td>
</tr>
<tr>
<td>Correlation b/t innovations in $c_t$ and $\delta_t$</td>
<td>$\rho_{c,\delta}$</td>
<td>-0.1</td>
</tr>
<tr>
<td>Subjective discount factor</td>
<td>$\delta$</td>
<td>0.89</td>
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All the quantities are annualized.
## Summary Statistics

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<th>Postwar Sample</th>
<th>Long Sample</th>
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<tr>
<td>$E(g_c)$</td>
<td>1.90</td>
<td>1.89</td>
<td>1.72</td>
</tr>
<tr>
<td>$\sigma(g_c)$</td>
<td>1.22</td>
<td>1.22</td>
<td>3.32</td>
</tr>
<tr>
<td>$E(r^f)$</td>
<td>0.94</td>
<td>0.94</td>
<td>2.92</td>
</tr>
<tr>
<td>$E(r - r^f)$</td>
<td>6.71</td>
<td>6.69</td>
<td>3.90</td>
</tr>
<tr>
<td>$\sigma(r - r^f)$</td>
<td>15.34</td>
<td>15.7</td>
<td>18.0</td>
</tr>
<tr>
<td>$\exp[E(p - d)]$</td>
<td>18.30</td>
<td>24.7</td>
<td>21.1</td>
</tr>
<tr>
<td>$\sigma(p - d)$</td>
<td>0.31</td>
<td>0.26</td>
<td>0.27</td>
</tr>
<tr>
<td>$AC_1(p - d)$</td>
<td>0.84</td>
<td>0.87</td>
<td>0.78</td>
</tr>
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### Annualized Quantities
Long-Horizon Correlations

Habit Model 1
Habit Model 2
Habit Model 3
Data

Quarters
Correlations

0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
1 2 3 4 5 6 7 8
Spectra For the Model and Data


The Histogram of Differences Between Low- and High-Frequency Correlations

Data

Jianfeng Yu  The Long and The Short of Asset Prices
Consumption Dynamics

- Consumption growth follows ARMA(2,2):

\[
g_{c,t} - \mu_c = \rho_{c,1} (g_{c,t-1} - \mu_c) + \rho_{c,2} (g_{c,t-2} - \mu_c) + \epsilon_{c,t} + \theta_{c,1}\epsilon_{c,t-1} + \theta_{c,2}\epsilon_{c,t-2}
\]

where \( \epsilon_{c,t} \sim WN(0, \sigma^2_c) \).

- Equivalent to trend-cycle representation for log consumption:

\[
c_t = T_t + x_t
\]
\[
T_t = T_{t-1} + \mu + \xi_t
\]
\[
x_t = \rho_{x,1}x_{t-1} + \rho_{x,2}x_{t-2} + \epsilon_{x,t}
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\[ + \theta_{c,1} \epsilon_{c,t-1} + \theta_{c,2} \epsilon_{c,t-2} \]

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## Consumption Dynamics: Estimation From Quarterly Data

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<th>$\rho_{c,1}$</th>
<th>$\rho_{c,2}$</th>
<th>$\theta_{c,1}$</th>
<th>$\theta_{c,2}$</th>
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<tr>
<td><strong>Estimate</strong></td>
<td>0.006</td>
<td>1.304</td>
<td>-0.554</td>
<td>-1.029</td>
<td>0.436</td>
<td>0.004</td>
</tr>
<tr>
<td><strong>Standard Error</strong></td>
<td>0.001</td>
<td>0.376</td>
<td>0.239</td>
<td>0.366</td>
<td>0.149</td>
<td>2 \cdot 10^{-4}</td>
</tr>
<tr>
<td><strong>Trend + AR(2)</strong></td>
<td>$\mu_c$</td>
<td>$\rho_{x,1}$</td>
<td>$\rho_{x,2}$</td>
<td>$\sigma_{\epsilon_x}$</td>
<td>$\sigma_\xi$</td>
<td>$\rho_{\epsilon_x,\xi}$</td>
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<td><strong>Implied Value</strong></td>
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<td>0.005</td>
<td>0.007</td>
<td>-0.957</td>
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Reduced Form of Trend-Cycle Model

- Consumption and dividend dynamics

\[ c_t = T_t + x_t \]
\[ T_t = T_{t-1} + \mu + \xi_t \]
\[ x_t = \rho_{x,1}x_{t-1} + \rho_{x,2}x_{t-2} + \epsilon_{x,t} \]
\[ \delta_t = d_t - c_t - \mu_{dc} = \rho\delta \delta_{t-1} + \epsilon_{\delta,t} \]

- Expected return

\[ E_{t-1}(r_t) = -\beta_C x_{t-1}, \quad \text{where} \quad \beta_C > 0 \]

- Realized returns:

\[ r_t \approx -\beta_C x_{t-1} + u_t \]
\[ u_t = \epsilon_{\delta,t}\bar{\psi} + \epsilon_{x,t} \cdot (\rho^* - \beta_C \rho \bar{\rho}) + \xi_t \]
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\[ c_t = T_t + x_t \]
\[ T_t = T_{t-1} + \mu + \xi_t \]
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\[ \delta_t \equiv d_t - c_t - \mu dc = \rho \delta \delta_{t-1} + \epsilon\delta_t \]

- Expected return

\[ E_{t-1} (r_t) = -\beta_C x_{t-1}, \quad \text{where} \quad \beta_C > 0 \]

- Realized returns:

\[ r_t \approx -\beta_C x_{t-1} + u_t \]
\[ u_t = \epsilon_{\delta,t} \bar{\psi} + \epsilon_x,t \cdot (\rho^* - \beta_C \rho \bar{\rho}) + \xi_t \]
Reduced Form of Trend-Cycle Model

- Consumption and dividend dynamics

\[ c_t = T_t + x_t \]
\[ T_t = T_{t-1} + \mu + \xi_t \]
\[ x_t = \rho x_{t-1} + \rho x_{t-2} + \epsilon_x, t \]
\[ \delta_t \equiv d_t - c_t - \mu dc = \rho \delta \delta_{t-1} + \epsilon_{\delta, t} \]

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The parameter values $\beta_C = 2$, $\rho_{u,\xi} = 0$ and $\rho_{u,\xi} = 0$. 

Jianfeng Yu  The Long and The Short of Asset Prices
The parameter values $\beta_C = 2$, $\rho_{u,\xi} = 0.2$ and $\rho_{u,\epsilon_x} = -0.2$. 
The parameter values $\beta_C = 2$, $\rho_{u,\xi} = 0.5$ and $\rho_{u,\epsilon_x} = -0.5$. 
The parameter values $\beta_C = 5$, $\rho_{u,\xi} = 0$ and $\rho_{u,\epsilon_x} = 0$. 

Coherency

Cospectrum

Phase Spectrum
Habit-Formation Models: ARMA(2,2) Consumption Growth
Band-Pass Filter Analysis of Different Models

Correlations:

<table>
<thead>
<tr>
<th></th>
<th>C-C</th>
<th>IID</th>
<th>ARMA</th>
<th>B-Y</th>
<th>P-Y</th>
<th>data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-Freq. Corr.</td>
<td>0.65</td>
<td>0.78</td>
<td>0.72</td>
<td>0.15</td>
<td>0.61</td>
<td>0.34</td>
</tr>
<tr>
<td>High-Freq. Corr.</td>
<td>0.75</td>
<td>0.89</td>
<td>0.81</td>
<td>-0.04</td>
<td>0.25</td>
<td>0.11</td>
</tr>
<tr>
<td>95% Diff. from M.C.</td>
<td>0.02</td>
<td>0.02</td>
<td>0.04</td>
<td>0.34</td>
<td>0.50</td>
<td>0.23</td>
</tr>
</tbody>
</table>
Conclusion

- Develop a new test to evaluate two types of leading asset pricing models.
  - External Habit-Formation Models:
    - does not match the correlation over different horizons and the lead-lag relation b/t consumption growth and asset returns
  - Long-Run Risk and Trend-Cycle Models:
    - can match the short-run and long-run features of the data
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