An Equilibrium Model of Credit Risk and Asset Pricing

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June 2008
Construct equilibrium model quantitatively consistent with salient features of asset and credit markets
Benchmarks

Equity Premium Puzzle
Equity premia implied by standard models fall short of historical equity premia ($\approx 6\%$)

Credit Spread Puzzle
Average credit spreads implied by structural models of default ($\approx 20$ bp) fall short of historical average credit spreads ($\approx 100$ bp)
Credit spreads are high in spite of low historical default rates

- Default losses are countercyclical and investors require default risk premium
This paper

Construct tractable dynamic equilibrium model with heterogeneous firms featuring
- Optimal capital structure
- Default
- Investment
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Investigate quantitatively and qualitatively implications for
- Equity risk premia and credit spreads
- Aggregate fluctuations
- Capital structure dynamics
Related Literature

- Equilibrium Asset Pricing with Risk of Default

- Capital Structure and Credit Spreads

- Financing Frictions and Macroeconomics

- Equilibrium Asset Pricing with heterogeneous firms
Environment

- One (representative) household and a continuum of firms
- Entry and exit
- Investment in the form of entry costs
- Firms issue debt because of tax advantage
- Aggregation and market clearing
Firm $i$ draws stochastic entry cost $e_{it}$ and decides whether to enter.

- purchases capital stock normalized to one
- generates output

$$y_{it} = x_t z_{it}$$

where

- aggregate shock $x_t$
- idiosyncratic shock $z_{it}$
Decisions:
- How much entry cost to pay and when to enter?
- How much debt to take on (coupon $b$)?
- When to default?

Aggregate State: $s \equiv (x, \mu)$
- $x$: aggregate shock
- $\mu$: distribution of firms over $z$ and $b$
Strategy: Solve for optimal decisions backwards

Equity value for incumbent firm with coupon $b$

$$V(s, z, b) = \max\{0, (1 - \tau)(xz - b) + E[\beta M(s, s') V(s', z', b)]\}$$

- $M(s, s')$ is the stochastic discount factor, to be determined in equilibrium
- because of corporate taxes $\tau$ firm has an incentive to take on debt
- max operator reflects default option and yields default cutoff shock $\bar{z}(s, b)$
Bond Valuation

Given default decision determine corporate bond value for every coupon $b$

$$B(s, z, b) = (b + E[\beta M(s, s')B(s', z', b)])\chi\{V > 0\} + (1 - \theta)(xz)(1 - \chi\{V = 0\})$$

- $\chi$ is a default indicator function
- $\theta$ are bankruptcy costs

Value of riskless consol bond

$$\tilde{B}(s, z, b) = b + E[\beta M(s, s')\tilde{B}(s', z', b)]$$

Credit spread

$$\text{spread} = \frac{b}{B} - \frac{b}{\tilde{B}}$$
Firms draw initial idiosyncratic shock $z$ from stationary distribution $G(z)$
the value is only observed after entry

Before observing $z$ firms decide to issue how much debt

Ex ante firm value conditional on $s$ and $b$

$$A(s, b) = \int V(s, z, b) + B(s, z, b) dG(z)$$

**Optimal Capital Structure**
Firms issue debt in order to maximize firm value

$$A_0(s) = \max_{b \geq 0} \left\{ \int V(s, z, b) + B(s, z, b) dG(z) \right\}$$
Firm entry is subject to stochastic entry cost $e$

- $e$ is uniformly distributed on $[0, E]$.

Firms enter if and only if $e$ is less or equal the ex ante firm value $A_0(s)$.

**Entry condition**

$$e \leq \bar{e}(s) = A_0(s)$$
The solution of the firm problem is characterized by

- An entry condition
  \[ e \leq \bar{e}(s) \]

- An optimal coupon
  \[ b = \bar{b}(s) \]

- An optimal default cutoff
  \[ z = \bar{z}(s, b) \]

All these decisions depend on the aggregate state of the economy!
Aggregate Quantities

- Aggregate Investment is the sum over entry costs
  \[ I(s) = \int_0^{\bar{e}(s)} e \, de \]

- Bankruptcy costs are
  \[ \Theta(s) = \int xz\theta(1 - \chi\{z > \bar{z}(s,b)\}) \, d\mu \]

- Adding the book values of defaulting firms to output, total output in the economy is then simply
  \[ Y(s) = \int xz \, d\mu \]

- Aggregate consumption is
  \[ C(s) = Y(s) - \Theta(s) - I(s) \]
Households
The representative household maximizes lifetime utility given as
\[ \sum_{t=0}^{\infty} \beta^t u(C_t) \]
and
\[ u(C) = \frac{C^{1-\gamma}}{1-\gamma} \]

Equilibrium
In equilibrium we require that
\[ M(s, s') = \frac{u'(C(s'))}{u'(C(s))} \]
and that \( \mu \) follows the law of motion
\[ \mu' = \Gamma(x, \mu) \]
Computational complexity arises from endogeneity of the pricing kernel, captures market clearing conditions and depends on $\mu$

- use Krusell-Smith, Khan-Thomas tricks
- rewrite problems in units of utils
- forecast next period consumption
- simulate and iterate on forecasting rule

$$\tilde{V}(x, z, b, C) = \max\{0, (1-\tau)(xz-b)u'(C)+E[\beta \tilde{V}(x', z', b, C')]\}$$

and

$$C' = \alpha_0 + \alpha_1 C + \alpha_2 x$$
The model is calibrated at annual frequency

<table>
<thead>
<tr>
<th>Parameter Values</th>
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<tbody>
<tr>
<td>( \beta )</td>
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<tr>
<td>( \gamma )</td>
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<tr>
<td>( \tau )</td>
</tr>
<tr>
<td>( \theta )</td>
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<tr>
<td>( \rho_x )</td>
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<tr>
<td>( \sigma_x )</td>
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<td>( \rho_z )</td>
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<td>( \sigma_z )</td>
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## Aggregate Moments

### Return Moments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>Model</th>
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</thead>
<tbody>
<tr>
<td>$E[r_f]$</td>
<td>1.8</td>
<td>1.92</td>
</tr>
<tr>
<td>$\sigma[r_f]$</td>
<td>3.0</td>
<td>4.21</td>
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<tr>
<td>$E[r_e - r_f]$</td>
<td>6.18</td>
<td>3.09</td>
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<tr>
<td>$\sigma[r_e]$</td>
<td>16.54</td>
<td>12.78</td>
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### Macro Moments

<table>
<thead>
<tr>
<th>Variable</th>
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<tbody>
<tr>
<td>$\sigma[\Delta C]$</td>
<td>2.54</td>
<td>2.36</td>
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<tr>
<td>$\sigma[\Delta Y]$</td>
<td>0.51</td>
<td>0.42</td>
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<tr>
<td>$\sigma[\Delta I]$</td>
<td>2.56</td>
<td>2.91</td>
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<tr>
<td>$\lambda$</td>
<td>0.19</td>
<td>0.22</td>
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Traditional capital structure model counterfactually imply

- Low credit spreads
- High market leverage ratios ($\approx 60 - 70\%$)

Accounting for risk premia improves the quantitative fit!

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
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<tbody>
<tr>
<td>Default rate</td>
<td>1.48</td>
<td>1.39</td>
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<tr>
<td>Credit Spread</td>
<td>104</td>
<td>81</td>
</tr>
<tr>
<td>Market Leverage</td>
<td>0.35</td>
<td>0.39</td>
</tr>
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</table>
Do financing frictions amplify and propagate shocks through the economy? Bernanke, Gertler, Gilchrist (1998), Cooley, Marimon, Quadrini (2004): Financing frictions are propagated through small entrepreneurial firms and add to output volatility.

This model: Capital structure choice increases output volatility by 17%.

Effect is propagated through credit risk premia on investment in large public firms.
Traditional models of capital structure have problems replicating stylized facts concerning corporations financing policies

- Countercyclical market leverage ratios
- Procyclical equity issuances

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<th>Model</th>
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<tbody>
<tr>
<td>Investment</td>
<td>+</td>
<td>0.72</td>
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<tr>
<td>Book leverage</td>
<td>+/-</td>
<td>0.87</td>
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<tr>
<td>Market leverage</td>
<td>-</td>
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<tr>
<td>Equity Issuance</td>
<td>+</td>
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<tr>
<td>Default rate</td>
<td>-</td>
<td>-0.91</td>
</tr>
<tr>
<td>Credit Spread</td>
<td>-</td>
<td>-0.77</td>
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Conclusion

The paper integrates trade-off model of capital structure into dynamic general equilibrium production economy:
- generates sizeable risk premia in equity and credit markets
- identifies financial accelerator mechanism propagated through large firms
- replicates cyclical patterns of firm financing

A step towards integrating asset pricing, corporate finance and macro!