The Value of Useless Information

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Outline

Motivation

Basic Model

Applications

Discussion
Example: Dying Agent

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Example: Dying Agent

- The agent is on his deathbed, dying of unknown causes.
- It could be a genetic disease that might affect his son with whom he has completely lost touch.
- He prefers it if it his son is not affected.
- If his doctor knows what the causes are, should the agent ask to be informed?
Realistic Applications

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- ‘Fear of failure’: choosing not to perform a task or to deliberately impede the chance of success.
- Political ignorance: deliberately ignoring a politician’s or a government’s actions.
- Not wanting to know what is in your food, or the extent of animal testing used by your shampoo brand.
- Wanting to know whether your soccer team won.
Issues

- It might seem that the agent should be indifferent between knowing that he has a disease and remaining in doubt, from a standard vNM Expected Utility viewpoint.
- Information only seems to have instrumental value. If it is not useful for a decision, then it has no value.
For the case of the doctor, one might say that if he could resolve his uncertainty, he has expected utility (with objective probabilities):

\[ U_R = p_g u(g) + (1 - p_g) u(b) \]
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which is the same as if he remains in doubt:

\[ U_{NR} = p_g u(g) + (1 - p_g) u(b) \]
Issues

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- The expected utility function $U_R$ can be interpreted as the expectation of the utility that he obtains once he observes the true outcome.
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- But $U_{NR}$ is not necessarily the right function to use in this setting.
- The expected utility function $U_R$ can be interpreted as the expectation of the utility that he obtains once he observes the true outcome.
- $U_{NR}$ does not have a similar interpretation, as he does not observe the 'true' outcome if he remains uninformed.
- The only outcome that matters is the one that the agent perceives.
- His ex-post utility is not necessarily $u_g$ if he does not know that the outcome is $g$. 
The relevant framework is the agent’s. It is his frame of reference that matters, as it is his decision that is being evaluated.

In the examples above, there is no clear notion of how the agent perceives the world if he does not observe the true consequence. There is no aggregation tool.

It is hence unclear what $U_{NR}$ should be. The standard vNM model is silent on the issue.
Expanding the outcome space

- Attempting to solve the problem by adding the observation as part of the outcome:

  \[ Z = \{ \text{Good and Informed (} g_\text{I}\text{), Bad and Informed (} b_\text{I}\text{), Good and Not informed (} g_\text{N}\text{), Bad and Not informed (} b_\text{N}\text{)} \} \].

- He would have the following utilities, when choosing between remaining informed and uninformed:

  \[ U_R = p_g u(g_\text{I}) + (1 - p_g) u(b_\text{I}) \]

  and

  \[ U_{NR} = p_g u(g_\text{N}) + (1 - p_g) u(b_\text{N}) \]

- and he would compare the two.
Expanding the outcome space

- But we encounter the following problem:
- Let $p_g = 1$. Then there is no intrinsic difference between observing and not observing the outcome. Hence:

\[ U_R = u(g_I) = u(g_N) = U_{NR} \]

- Similarly, for $p_t = 0$, we have $u(b_I) = u(b_N)$
- Hence:

\[ U_R = p_g u(g_I) + (1 - p_g) u(b_I) = p_g u(g_N) + (1 - p_g) u(b_N) = U_{NR} \]

for all $p_g$. 
Expanding the outcome space

- The value that the agent places on these two extra outcomes is completely pinned down, he is not allowed to have any other preferences.
- As a result, $U_R = U_{NR}$ for all values of $p_e$.
- The extra outcomes $\{g_N, b_N\}$ have to have the same values as $\{g_I, b_I\}$, respectively, and the fix has essentially added notation.
- It is not implicit in the standard Expected Utility.
Objective

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- Further assumptions are then made over the preferences associated with unresolved lotteries, to obtain a more specific representation.
- A few applications are then considered.
Basic Model

- $\mathcal{Z} = [z, \bar{z}] \subset \mathcal{R}$ is the outcome space.
- $\mathcal{L}_o$ is the set of simple probability measures on $\mathcal{Z}$. For $f = (z_1, p_1; z_2, p_2; \ldots; z_m, p_m) \in \mathcal{L}_o$, $z_i$ occurs with probability $p_i$.
- $\mathcal{L}_1$ is the set of simple lotteries over $\mathcal{Z} \cup \mathcal{L}_o$. For $X \in \mathcal{L}_1$, the notation $X = (z_1, q^l_1; z_2, q^l_2; \ldots; z_n, q^l_n; f_1, q^N_1; f_2, q^N_2; \ldots; f_m, q^N_m)$ is used.
- $\succeq$ denotes the agents preferences over $\mathcal{L}_1$. 
Basic Model

Figure: Lottery $X = (z_1, \frac{1}{2}; z_2, \frac{1}{4}; f_1, \frac{1}{4})$, where $f_1 = (z_3, \frac{1}{3}; z_4, \frac{2}{3})$
Basic Model: Assumptions

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- **AXIOM A.1 (Weak Order):** \( \succeq \) is complete and transitive.

- **AXIOM A.2 (Continuity):** \( \succeq \) is continuous in the weak convergence topology. That is, for each \( X \in \mathcal{L}_1 \), the sets \( \{ X' \in \mathcal{L}_1 : X' \succeq X \} \) and \( \{ X' \in \mathcal{L}_1 : X \succeq X' \} \) are both closed in the weak convergence topology.

- **AXIOM A.3 (Certainty):** Take any \( z_i \in Z \), and let \( X = \delta z_i = (z_i, 1) \) and \( X' = (\delta z_i, 1) \). Then \( X \sim X' \).

- **AXIOM A.4 (Independence):** For all \( X, Y, Z \in \mathcal{L}_1 \) and \( \alpha \in (0, 1] \), if \( X \succ Y \) implies \( \alpha X + (1 - \alpha) Z \succ \alpha Y + (1 - \alpha) Z \).
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▶ **AXIOM A.4 (Independence):** For all $X, Y, Z \in \mathcal{L}_1$ and $\alpha \in (0, 1]$, $X \succ Y$ implies $\alpha X + (1-\alpha)Z \succ \alpha Y + (1-\alpha)Z$. 
Lemma (Informed Certainty Equivalent)

Suppose axioms A.1 through A.3 hold. There exists an \( H: \mathcal{L}_\circ \to \mathbb{Z} \) such that for all \( f \in \mathcal{L}_\circ \),

\[
\delta_H(f) \sim \delta_f
\]
Basic Model: Representation Theorem Template

Representation Theorem

Suppose axioms A.1 through A.4 hold. Then there exist a continuous and bounded function \( u: \mathcal{Z} \to \mathbb{R} \), and an \( H: \mathcal{L}_0 \to \mathcal{Z} \) such that for all \( X, Y \in \mathcal{L}_1 \),

\[
X \succ Y \text{ if and only if } W(X) > W(Y)
\]

where \( W \) is defined to be: for all

\[
X = (z_1, q_1^l; \ldots; z_n, q_n^l; f_1, q_1^N; \ldots; f_m, q_m^N),
\]

\[
W(X) = \sum_{i=1}^{n} q_i^l u(z_i) + \sum_{i=1}^{m} q_i^N u(H(f_z))
\]

Moreover \( u \) is unique up to positive affine transformation. If \( H(f) \) has more than one element, then any element can be chosen arbitrarily.
Representations of H: Recursive Expected Utility

$\succeq_N$ is defined to be the preference relation over purely unresolved lotteries, that is: $\delta_f \succeq \delta_{f'}$ implies $f \succeq_N f'$. 
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Representations of H: Recursive Expected Utility

- $\succeq_N$ is defined to be the preference relation over purely unresolved lotteries, that is: $\delta_f \succeq \delta_{f'}$ implies $f \succeq_N f'$.

- Suppose the independence axiom holds over $\succeq_N$:

**AXIOM H.1 (Independence for $\succeq_N$):** For all $f, f', f'' \in \mathcal{L}_o$ and $\alpha \in (0, 1]$, $f \succ_N f'$ implies $\alpha f + (1 - \alpha) f'' \succ_N \alpha f' + (1 - \alpha) f''$. 
“$\succeq_N$ is defined to be the preference relation over purely unresolved lotteries, that is: $\delta_f \succeq \delta_{f'}$ implies $f \succeq_N f'$.

Suppose the independence axiom holds over $\succeq_N$:

**AXIOM H.1 (Independence for $\succeq_N$):** For all $f, f', f'' \in \mathcal{L}_0$ and $\alpha \in (0, 1]$, $f \succ_N f'$ implies $\alpha f + (1 - \alpha) f'' \succ_N \alpha f' + (1 - \alpha) f''$.

Then all the axioms hold for $\succeq_N$ to have the standard EU form (Kreps-Porteus (1978), Grant-Kajii-Polak (1998)).
Let \( \nu \) be the utility function associated with \( \succeq_N \). Then:

\[
H(f) = \nu - 1(E \nu) = \nu - 1(\sum_{z \in Z} \nu(z) f(z))
\]
Let \( v \) be the utility function associated with \( \succeq_N \). Then:

\[
H(f) = v^{-1}(Ev) = v^{-1}\left(\sum_{z \in \mathcal{Z}} v(z)f(z)\right)
\]
Problem with this approach:

\[ f = (t_b, 1/3; t_m, 1/3; t_g, 1/3), \quad f' = (t_b, 1/2; t_g, 1/2) \]

<table>
<thead>
<tr>
<th>( \delta_m )</th>
<th>( f )</th>
<th>( f' )</th>
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<tbody>
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<td>( \prec )</td>
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But then \( \succeq \) violates independence (and the weaker Dekel betweenness axiom).

Essentially, \( v \) contains too many notions that cannot be disentangled.
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Consider the genetic disease example: say he has a bad genetic trait \( (t_b) \), a mediocre one \( (t_m) \), or a good one \( (t_g) \).
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There are three lotteries over outcomes:

\[
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The preferences \( \delta_m \succeq f \succeq \delta_{f'} \) and \( f \succ_N f' \succ_N \delta_m \) can both appear plausible.

But then \( \succeq_N \) violates independence (and the weaker Dekel betweenness axiom).

Essentially, \( \nu \) contains too many notions that cannot be disentangled.
Representations of H: RDU

As an alternative, consider rank-dependent utility:

**Definition (RDU) (Abdellaoui)** Rank-dependent utility holds if there exists a strictly increasing continuous probability weighting function \( w : [0, 1] \rightarrow [0, 1] \) with \( w(0) = 0 \) and \( w(1) = 1 \) and a strictly increasing utility function \( v : \mathbb{Z} \rightarrow \mathbb{R} \) such that for all \( f, f' \in \mathcal{L}_o \),

\[
f \succ^N f' \quad \text{if and only if} \quad V_{RDU}(f) > V_{RDU}(f')
\]

where \( V_{RDU} \) is defined to be:

for all \( f = (z_1, p_1; z_2, p_2; \ldots; z_m, p_m) \),

\[
V_{RDU}(f) = v(z_1) + \sum_{i=2}^{m} [v(z_i) - v(z_{i-1})]w(p_i^*)
\]

Moreover, \( v \) is unique up to positive affine transformation.
In this case, the $H$ function takes the following form:

$$H(f) = v^{-1}(V_{RDU}(f)) =$$

$$v^{-1}\left( v(z_1) + \sum_{i=2}^{m} [v(z_i) - v(z_{i-1})] w(p_i^*) \right)$$
Theorem
Suppose that axioms A.1 through A.4 and the RDU axioms hold, and let $u$ and $v$ be the utility functions associated with the informed and uninformed lotteries, respectively, and $w$ be the decision weight associated with the uninformed lotteries. In addition, suppose that $u$, $v$ are both differentiable. Then:

If $\succeq$ displays doubt-proneness and $\succeq^N$ mean-preserving risk-aversion, then $V_{\text{RDU}}$ must be of the EU form. That is, $w(p) = p$ for all $p \in \mathcal{L}_\circ$. It also follows that both $u$ and $v$ are concave, and that $u = \lambda \circ v$ for some continuous, concave, and increasing $\lambda$. 
Exending the model

- The general independence axiom can also be relaxed. Assuming the following:

\[ \text{AXIOM E.1 (Unresolved lottery equivalent)} \]

For all \( f \in L_0 \) such that \( \delta f \sim \delta H(f) \), and for all \( X, \tilde{X} \in L_1 \) such that:

\( X = (z_1, q_{I_1}; \ldots; z_n, q_{I_n}; \ f_1, q_{f_1}; \ldots; f_m, q_{N_m}) \)

and

\( \tilde{X} = (z_1, q_{I_1}; \ldots; z_n, q_{I_n}; \ H(f_1), q_{f_1}; \ldots; f_m, q_{N_m}) \)

the following holds:

\( X \sim \tilde{X} \).

Other models can be extended to include unresolved lotteries (this is discussed in the main paper).
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  and
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Discussion
Example 1: Self-handicapping

- Suppose that an agent can conduct a task which requires effort \( e \in \{e_l, e_m, e_h\} \subset \mathbb{R} \), with \( e_l < e_m < e_h \). The more effort he puts in, the more likely he is to succeed at his task.
- He can get either $0 or $100.

<table>
<thead>
<tr>
<th>Effort ( e )</th>
<th>( p(0) )</th>
<th>( p(100) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_l )</td>
<td>( \frac{2}{3} )</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>( e_m )</td>
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<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( e_h )</td>
<td>( \frac{1}{3} )</td>
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</tr>
</tbody>
</table>

Table: Self-handicapping

- If effort does not cost him anything, he chooses effort \( e_h \).
Self-handicapping

- Suppose now that he also has a talent $T \in [t_b, t_g] \subset \mathbb{R}$, which he cannot observe directly.
- However, his success depends on a combination of talent, effort, and luck.
- He cares about his talent for its own sake, not just for its instrumental value.
- His prior is $t_b = \frac{1}{2}, t_g = \frac{1}{2}$.
- Assume his utility of money is linearly separable from his utility over talent.
## Self-handicapping

| Effort $e$ | $p(0)$ | $p(100)$ | $p(100|t_g)$ | $p(0|t_g)$ |
|------------|--------|----------|---------------|------------|
| $e_l$      | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ |
| $e_m$      | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $e_h$      | $\frac{1}{3}$ | $\frac{2}{3}$ | $1$           | $0$        |

**Table:** Self-handicapping
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| Effort $e$ | $p(0)$ | $p(100)$ | $p(100|t_g)$ | $p(0|t_g)$ | $p(t_g|100)$ | $p(t_g|0)$ |
|-----------|--------|----------|--------------|------------|--------------|------------|
| $e_l$     | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | 1            | $\frac{1}{4}$ |
| $e_m$     | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $e_h$     | $\frac{1}{3}$ | $\frac{2}{3}$ | 1            | 0          | $\frac{3}{4}$ | 0          |
Let his utility for $100 be 100, and his utility for $0 be 0. Then, letting $W$ be his value function:

$$W(e_l) = \frac{1}{3} [100 + u(t_g)] + \frac{2}{3} \left[u \left( v^{-1} \left( \frac{3}{4} v(t_b) + \frac{1}{4} v(t_g) \right) \right)\right]$$

$$W(e_m) = 50 + u \left( v^{-1} \left( \frac{1}{2} v(t_b) + \frac{1}{2} v(t_g) \right) \right)$$

$$W(e_h) = \frac{1}{3} u(t_b) + \frac{2}{3} \left[100 + u \left( v^{-1} \left( \frac{1}{4} v(t_b) + \frac{3}{4} v(t_g) \right) \right)\right]$$

Depending on $u$ and $v$, you could now have:

- $e_l$ preferred to $e_m$ preferred to $e_h$, if finding out he is untalented is too disappointing.
- $e_l$ preferred to $e_h$ preferred to $e_m$, if he places high importance on finding out that he is talented.
- $e_h$ preferred to $e_m$ preferred to $e_l$, if it is important to find out that he is talented, but not extremely so.
Self-handicapping

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$$W(e_m) = 50 + u \left( v^{-1} \left( \frac{1}{2} v(t_b) + \frac{1}{2} v(t_g) \right) \right)$$

$$W(e_h) = \frac{1}{3} u(t_b) + \frac{2}{3} \left[ 100 + u \left( v^{-1} \left( \frac{1}{4} v(t_b) + \frac{3}{4} v(t_g) \right) \right) \right]$$

Depending on $u$ and $v$, you could now have:

- $e_m$ preferred to $e_h$ preferred to $e_l$, if finding out he is untalented is too disappointing.
- $e_l$ preferred to $e_h$ preferred to $e_m$, if he places high importance on finding out that he is talented.
- $e_h$ preferred to $e_l$ preferred to $e_m$, if it is important to find out that he is talented, but not extremely so.
Self-handicapping

Figure: Lotteries for efforts $e_l$, $e_m$ and $e_h$
Example 2: Political Ignorance

- $N$ citizens care about issue $\gamma \in [0, 1]$, determined by a politician that they vote for.
- Two candidates, $C_1$ and $C_2$, one votes for $\gamma = 1$, one for $\gamma = 0$.
- Voters don’t know which is which ($\frac{1}{2}$ probability $C_1$ implements $\gamma = 1$, etc.).
- Can choose to observe what $\gamma$ the politician would implement, at no cost.
Political Ignorance

1) Each voter decides whether or not to observe where candidates C1 and C2 stand.

2) Each voter votes sincerely, i.e. he votes for the candidate on whom he places a higher probability of implementing policy $\gamma$ that he prefers. If he is indifferent or if he places equal probability on either candidate implementing his preferred policy, then he tosses a coin and votes for C1 if heads, C2 if tails.

3) The candidate who obtains the majority wins the vote. In case of a tie, a coin toss determines the winner. The winner then implements the policy he prefers. There is no possibility of reelection.
Political Ignorance

- Suppose every voter prefers $\gamma = 1$.
- Suppose also that they are doubt-prone.
- There exist equilibria where a strict majority gets informed and vote for the right candidate.
- There also exists an equilibrium where no one gets informed, and the wrong candidate wins with probability $\frac{1}{2}$. 
Outline

Motivation

Basic Model

Applications

Discussion
Discussion

- An agent’s choice to ignore free information or to acquire useless information does require manipulable beliefs or self-deception of any kind.
- This structure has a number of applications (status quo bias, self-handicapping, political ignorance).
- Extending other models to allow for unresolved lotteries can also be done in a straightforward way.