Money and Credit with Limited Commitment and Theft

Daniel Sanches and Stephen Williamson
Washington University in St. Louis

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Ideas

- Understand the roles of money and credit in transactions, and the interaction between the two.
- Construct a model where money and credit are both robust.
- Show that, for robustness, memory must be imperfect, and there must be costs to using money - consider theft.
- Given this, what are the implications for monetary policy?
Private information and spatial separation are insufficient to generate a welfare-improving role for money.


Typical result: Efficient monetary policy drives out any transactions role for credit - no costs associated with monetary exchange.
Model

- Basic structure: Rocheteau and Wright (2005).
- Time is discrete and continues forever.
- Two subperiods: day and night.
- Two types of agents: buyers and sellers (continuum of each type with measure one).
- Buyers want to consume during the day but can produce only at night.
- Sellers can produce during the day but want to consume at night.
\begin{itemize}
  \item Technology: one unit of labor produces one unit of the unique perishable consumption good.
  \item Random bilateral meetings during the day (each buyer is matched with a seller).
  \item Centralized markets at night.
\end{itemize}
• Buyer’s preferences:

\[ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ u(q_t) - n_t \right] \right\}, \]

where \( u(\cdot) \) is strictly increasing, strictly concave, and twice continuously differentiable, with \( u(0) = 0 \) and \( u'(0) = \infty \).

• Seller’s preferences:

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t (-l_t + x_t) \right]. \]
Planner’s Problem with Full Commitment

- Restrict attention to stationary allocations.
- A stationary allocation is a pair \((q, x)\), with \(q\) being the seller’s production during the day and \(x\) the buyer’s production at night.
- Agents can commit to the planner’s proposed allocation.
- Participation constraints:

\[
  u(q) - x \geq 0
\]

and

\[
  -q + x \geq 0.
\]

- Efficient allocations satisfy these constraints and \(q = q^*\).
Planner’s Problem with Limited Commitment

- Add incentive constraints:

\[-x + v \geq 0\]

and

\[-q + x + w \geq 0,\]

where \(v\) and \(w\) are the buyer’s and seller’s continuation values, respectively.

- Simplifying, we obtain

\[q \leq x \leq \beta u(q).\]

- \(q^*\) is the solution to \(u'(q^*) = 1\) and \(q^{**}\) is the solution to \(q^{**} = \beta u(q^{**}).\)

- If \(\beta\) is close to one, \(q^* \leq q^{**}\). If \(\beta\) is small, \(q^{**} < q^*\).

- Limited commitment matters if and only if \(q^{**} < q^*\).
Figure 1: Efficiency When $q^{**} > q^*$
Figure 2: Efficiency When $q^{**} < q^{*}$
Equilibrium Allocations with Perfect Memory

- Bargaining protocol during the day:
  - Sellers announce willingness to trade.
  - Buyer makes a take-it-or-leave-it offer.
- No bargaining inefficiencies.
- Each buyer is endowed with one unit of fiat money at the beginning of the first day.
- Money stock grows at gross rate $\mu > 0$.
- New money is injected through lump-sum transfers at night.
- Limited commitment with respect to tax liabilities as well as private liabilities (Andolfatto, 2007).
- Default by any buyer triggers global autarky.
- Bellman equation for buyers:

$$v = \max_{m, l} \left\{ -m + \beta \left[ u \left( \frac{m}{\mu} + l \right) - l + \gamma + v \right] \right\},$$

subject to

$$-l + \gamma + v \geq 0$$

and

$$l \geq 0.$$
Equilibria if \( q^* \leq q^{**} \):

- If \( \mu > \beta \), no monetary equilibrium.
- If \( \mu = \beta \), continuum of monetary equilibria indexed by \( m \).

Equilibria if \( q^* > q^{**} \):

- If \( \mu > \beta u'(q^{**}) \), no monetary equilibrium.
- If \( \mu = \beta u'(q^{**}) \), continuum of monetary equilibria indexed by \( m \).

Money is held only under Friedman rule or Andolfatto rule.

Results consistent with Kocherlakota (1998):

- No social role for money with perfect memory.
- Limited commitment is insufficient to make money useful.
Imperfect Memory and Autarkic Punishments

- Fraction $\rho$ of sellers cannot be monitored.
- Fraction $1 - \rho$ of sellers have monitoring potential.

Bargaining protocol

- If there is a monitoring opportunity, the buyer chooses whether or not the interaction with the seller will be monitored.
- If monitored, the seller observes the buyer's history (record of monitored transactions).
- Seller announces willingness to trade.
- Buyer makes a take-it-or-leave-it offer.

- Interactions of buyers with the government are public information.
• A stationary monetary equilibrium is a pair \((x, y)\) satisfying optimality

\[
\rho u'(x) + (1 - \rho) u'(y) = \frac{\mu}{\beta},
\]

nonnegativity of consumption and loan quantity

\[
0 \leq x \leq y \leq q^*,
\]

and incentive compatibility

\[
\beta \rho [u(x) - x] + (1 - \rho) \beta u(y) - (1 - \beta \rho) y \geq (1 - \beta) \hat{v},
\]

with \(y = q^*\) if IC does not bind.

• \(\hat{v} = 0\): default on tax or private liabilities triggers global autarky.
Results:

- Money and credit coexist for $\mu > \beta u' \min (q^*, q^{**})$.
- $\mu = \beta u' \min (q^*, q^{**})$ is optimal and eliminates credit.

Credit is not robust, and an efficient economy runs without it.

At the optimum, money is memory - it recovers the solutions to the planner’s problem with perfect memory.
Imperfect Memory and Non-Autarkic Punishments

- Punishment equilibrium has valued money.
- Default triggers equilibrium where sellers will not trade if the buyer wants monitoring.
- Punishment equilibrium must be sustainable - no incentive to default on tax liabilities.
- Continuation value with punishment:

\[
\hat{v} = -m(\mu) + \beta \left[ u \left[ \frac{m(\mu)}{\mu} \right] + \left( 1 - \frac{1}{\mu} \right) m(\mu) + \hat{v} \right],
\]

where

\[
u' \left[ \frac{m(\mu)}{\mu} \right] = \frac{\mu}{\beta}.
\]

- Sustainability

\[
\left( 1 - \frac{1}{\mu} \right) m(\mu) + \hat{v} \geq \hat{v}.
\]
Results:

- Only equilibrium is the monetary equilibrium with $\mu = 1$ (incentive constraint is satisfied if and only if $\mu = 1$).
- No credit activity.
Theft

- So far, there is no cost to monetary exchange other than the implicit foregone interest.
- A sufficiently high rate of return on money eliminates the implicit opportunity cost, and money drives out credit.
- In reality, there are costs to monetary exchange, such as counterfeiting, risk of loss, costs of maintaining the currency stock, and theft.
- Focus here on theft.
- $\tau =$ cost of theft for a seller during the day (can only steal in a non-monitored transaction).
- $\alpha =$ fraction of sellers who steal the buyer’s money during the day.
A stationary monetary equilibrium is a list \((x, y, \alpha)\) satisfying optimality
\[
\rho(1 - \alpha)u'(x) + (1 - \rho)u'(y) = \frac{\mu}{\beta},
\]
nonnegativity of consumption and loan quantity
\[
0 \leq x \leq y \leq q^*,
\]
incentive compatibility,
\[
\beta[\rho(1 - \alpha)u(x) + (1 - \rho)u(y)] - \rho\beta x - (1 - \rho\beta)y \geq \hat{v}(1 - \beta),
\]
with \(y = q^*\) if IC does not bind, and optimal theft

- if \(\alpha = 0\), then \(x \leq \tau\),
- if \(0 < \alpha < 1\), then \(x = \tau\),
- if \(\alpha = 1\), then \(x \geq \tau\).
The government chooses \((x, y, \alpha, \mu)\), where \((x, y, \alpha)\) is an equilibrium, given \(\mu\), to maximize

\[
W = \rho (1 - \alpha) [u(x) - x] + (1 - \rho) [u(y) - y] - \rho \alpha \tau.
\]

Solution has \(\alpha = 0\).
Theft with Autarkic Punishment

- Theft matters if and only if $\tau < \min (q^*, q^{**})$.
- Suppose theft matters.
- Case $q^* \leq q^{**}$
  - $x = \tau$
  - $y \leq q^*$, with strict inequality if $\rho$ is close to one or $\tau$ is sufficiently small.
  - $\mu = \beta \left[ \rho u' (\tau) + (1 - \rho) u' (y) \right] > \beta u' \left[ \min (q^*, q^{**}) \right]$.
- Case $q^* > q^{**}$
  - $x = \tau$
  - $y < q^*$ always.
  - $\mu = \beta \left[ \rho u' (\tau) + (1 - \rho) u' (y) \right] > \beta u' \left[ \min (q^*, q^{**}) \right]$. 
Welfare is increasing in $\tau$.

Lower $\tau$ implies lower $x$ and $y$, but $l = y - x$ increases.

Less costly theft promotes the use of credit.
Theft with Non-Autarkic Punishment

- Punishment equilibrium
  - If $\tau \geq \hat{x}$, then $x = \hat{x}$, $\mu = 1$, and $(1 - \beta) \hat{v} = \beta u(\hat{x}) - \hat{x}$.
  - If $\tau < \hat{x}$, then $x = \tau$, $\mu = \beta u'(\tau)$, and
    $$(1 - \beta) \hat{v} = \beta \{ -\tau [(1 - \beta) u'(\tau) + 1] + u(\tau) \}.$$  
- $u'(\hat{x}) = 1/\beta$.
- Equilibrium if $\tau \geq \hat{x}$: $x = y = \hat{x}$ and $\mu = 1$.
- Equilibrium if $\tau < \hat{x}$: $x = \tau$ and $y$ satisfies
  $$\beta (1 - \rho) u(y) - (1 - \rho \beta) y \geq \beta (1 - \rho) [u(\tau) - \tau] - \beta (1 - \beta) \tau u'(\tau).$$
• $y$ could increase with a decrease in $\tau$ - theft is a greater problem in the punishment equilibrium, which implies a higher money growth rate with punishment (tends to promote credit).
• $y > \tau$ for $\tau < \hat{x}$, so that when theft matters, credit is supported at the optimum.
Results:

- When theft matters, the Friedman rule is suboptimal, and may not be feasible.
- The optimal money growth rate tends to rise as the cost of theft falls.
- An increase in the cost of theft increases consumption in non-monitored trades, and typically decreases the quantity of lending.
- With non-autarkic punishments, the money growth rate is higher in the punishment equilibrium, as the theft problem is more severe - theft acts to discipline the credit market.
Conclusion

Without costs of monetary exchange, Friedman rule or Andolfatto rule is optimal, and this drives credit out.

If theft is sufficiently low-cost:

- Friedman rule is never optimal.
- Theft can discipline credit market behavior.
- Money and credit always coexist at the optimum.
- The optimal money growth rate tends to rise as the cost of theft falls.