Persuasion and Limited Communication

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Persuasion is important in a wide array of political and economic environments.

What makes an argument persuasive?

Endogenize persuasiveness so it is optimal for the listener.
A speaker makes a request of a listener. Whether the listener would like to accept the request depends on facts known only to the speaker. The speaker presents evidence. A persuasion rule specifies evidence accepted by listener. An argument is persuasive if it is accepted by the optimal rule. Can the listener credibly commit to such a rule? Glazer and Rubinstein (2001,2004,2006) say “yes.”
Results

1. Present an Algorithm for Finding Optimal Rule and Speaker Strategy in the Credible Implementation, under a “No time constraints” assumption.

2. Extend Credibility to a Model with Multiple Actions Under a Concavity Assumption.

3. Qualitative Properties of Persuasion:
   1. Symmetry:
      - No time constraints:
      - ∃ optimal rule treating equivalent evidence equivalently.
      - Time constraints: this fails.
   2. Monotone Comparative Statics
All results depend on relation between the Persuasion Problem and the Maximum Flow Problem.
The Persuasion Problem

- Speaker and listener; speaker makes a request.
- There is a finite set $X$ of states.
- $A \subseteq X = \text{states where listener would like to accept.}$
- $R = X \setminus A = \text{states where the listener would like to reject.}$
- $p = (p_x)_{x \in X} = \text{probability distribution on states.}$
- $\sigma(x) = \text{set of messages available at state } x.$
- $M = \text{all possible messages (finite).}$
The Persuasion Problem

- $f : M \rightarrow [0, 1]$ is a **persuasion rule**.
- $f(m)$ = probability that listener accepts given message $m$.
- $f$ is **deterministic** if $\forall m, f(m) \in \{0, 1\}$.
The Persuasion Problem

- $\alpha(f, x) := \max_{m \in \sigma(x)} f(m) =$ probability of acceptance at $x$ induced by $f$.

- The error at state $x$ induced by $f$ is:

  $$\mu_x(f) = \begin{cases} 
  1 - \alpha(f, x), & \text{if } x \in A; \\
  \alpha(f, x), & \text{if } x \in R.
  \end{cases}$$

- The listener’s problem is:

  $$\min_{f \in F} \sum_{x \in X} p_x \mu_x(f)$$

  where $F =$ set of all persuasion rules.
GR (2006) establish that

1. There always exists an optimal deterministic rule.
2. There always exists a **credible implementation** of the optimal rule: i.e., a equilibrium of the game without commitment which induces the same outcome as the optimal rule.
Definition

A message structure \((X, M, \sigma)\) is normal if:

\[
\forall x \in X, \exists m_x \in \sigma(x), \forall y \in X, m_x \in \sigma(y) \Rightarrow \sigma(x) \subseteq \sigma(y).
\]

\(m_x\) is \(x\)’s maximal message.
Examples

- Normal: Speaker observes a sequence \((x_1, \ldots, x_n)\) of random length \(n\), and can report any subsequence.
- Not Normal: Speaker observes sequence \((x_1, \ldots, x_n)\) of fixed length, and may report \(k < n\) components.
Maximum Flow
A **flow** is a function $\varphi$ from the edges of the graph to $\mathbb{R}_+$

- Push as much flow from the source $s$ to the sink $t$, subject to:
- **capacity constraints**: the flow along any edge is no more than its capacity.
- **flow conservation constraints**: for any vertex $v$ other than $s, t$, the flow into $v$ equals the flow out of $v$. 
Ford-Fulkerson Algorithm

1. Start with zero flow.

2. Find a path from $s$ to $t$, and push as much flow through it as possible.

3. Find a path from $s$ to $t$ in the residual graph, where:
   
   1. Existing flow is subtracted from capacity.
   2. For any flow along an edge there is capacity going in the other direction so that it can be undone.

   and push as much flow through it as possible.

4. Repeat until there is no such path in the residual graph.
### Persuasion Problem: An Example

<table>
<thead>
<tr>
<th>State</th>
<th>A/R</th>
<th>Prob</th>
<th>Messages</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>3/16</td>
<td>$m_1, m_2, m_3$</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>2/16</td>
<td>$m_2, m_3$</td>
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<tr>
<td>3</td>
<td>R</td>
<td>1/16</td>
<td>$m_3$</td>
</tr>
<tr>
<td>4</td>
<td>A</td>
<td>2/16</td>
<td>$m_1, m_2, m_3, m_4, m_5$</td>
</tr>
<tr>
<td>5</td>
<td>A</td>
<td>1/16</td>
<td>$m_3, m_5$</td>
</tr>
<tr>
<td>6</td>
<td>R</td>
<td>4/16</td>
<td>$m_2, m_3, m_6$</td>
</tr>
<tr>
<td>7</td>
<td>R</td>
<td>2/16</td>
<td>$m_1, m_2, m_3, m_7$</td>
</tr>
<tr>
<td>8</td>
<td>R</td>
<td>1/16</td>
<td>$m_1, m_2, m_3, m_5, m_8$</td>
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Persuasion and Limited Communication
Applying any algorithm for the max flow problem to the network corresponding to the persuasion problem, one can find:

1. An optimal persuasion rule.
2. A speaker strategy in the credible implementation of the optimal rule.
3. The error probability at the optimal rule.
Maximum Flow

1
3/16
2/16
2/16
1/16

S

2

1

3

1/16
1/16
4/16

6

4/16
2/16

7

2/16

8

T

Persuasion and Limited Communication
In the optimal rule, in states $x \in A$, the speaker tells the truth, and in $x \in R$ the speaker randomizes over lies.
Fix a normal persuasion problem satisfying $m_x = m_y \Rightarrow x = y$. Let $\varphi$ be a maximum flow corresponding flow problem. Then:

$$f(m) := \begin{cases} 1, & \text{if } m = m_x \text{ for some } x \in A \cap V_\varphi; \\ 0, & \text{otherwise.} \end{cases}$$

is an optimal persuasion rule. A speaker strategy in the credible implementation is given by:

$$\zeta(x, m) := \begin{cases} 1, & \text{if } x \in A, m = m_x. \\ \frac{\varphi(y, x)}{\varphi(x, t)}, & \text{if } x \in R, m = m_y \in \sigma(x) \text{ with } y \in A; \\ 0, & \text{otherwise.} \end{cases}$$

unless $x \in R$ and $\varphi(x, t) = 0$, in which case the strategy at $x$ is arbitrary. The error probability is given by $\text{value}(\varphi)$. 

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Multiple Actions

- Listener has a finite set of actions $J = \{1, \ldots, n\}$.
- Speaker has utility function $u : J \rightarrow \mathbb{R}$ and prefers higher to lower actions.
- Listener has utility function $v : J \times X \rightarrow \mathbb{R}$.
- A persuasion rule is a function $f : M \rightarrow \Delta(J)$.
Credibility

**Assumption**

A. There exists a strictly increasing function \( r : J \rightarrow \mathbb{R} \) and for all \( x \in X \), there exists a concave function \( c_x : \mathbb{R} \rightarrow \mathbb{R} \) such that for all \( a \in J \), \( v(a, x) = c_x(r(a)) \).

**Assumption**

B. For all \( x \in X \), there exists a concave function \( c_x : \mathbb{R} \rightarrow \mathbb{R} \) such that for all \( a \in J \), \( v(a, x) = c_x(u(a)) \).

**Theorem**

Assume A. Every persuasion rule which is **optimal among deterministic rules** is credible.

**Theorem**

Assume B. There is an optimal rule which is deterministic. Every optimal deterministic rule is credible.
Conclusion

1. Present an Algorithm for Finding Optimal Rule and Speaker Strategy in the Credible Implementation, under a “No time constraints” assumption.

2. Extend Credibility to a Model with Multiple Actions Under a Concavity Assumption.

3. Qualitative Properties of Persuasion:
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