Procurement Auctions

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Introduction

Procurement has at least one key difference from most standard auctions

May care about who you buy from

Example: change over costs to use a new supplier, or preference for a more reliable supplier

How to procure in such settings?
Our setting: *Know* your preferences

(i.e., this is not a setting of hidden quality as in

Manelli and Vincent (Ecta ’95))
We will

- Look at the optimal mechanism
  - These are conceptually simple, but may be hard to explain to bidders and implement

- Look at some simple mechanisms
  - Request for Proposals
  - First Price with Bonus (of which the RFP is a special case)
  - Second Price with Bonus (Clock)

- Relate the two, and arrive at a surprisingly strong conclusion
Main Results

- In these auctions, FP and SP mechanisms are far from equivalent

- In every setting we have analytic results for, SP is better than FP or RFP (even though the latter are much more commonly used)

- In other settings, exploration of (lots) of examples says that not only is SP better than first, it is pretty close to optimal
Model Setup

- 1 buyer

- 2 suppliers, 1 and 2

- Supplier $i$ has cost $c_i \in [0, 1]$, private information
  - i.i.d. distribution $F$, density $f$
  - note symmetry

- Buyer must purchase input from one of the two suppliers
  - Puts value $v >> 0$ on seller 2
  - Puts value $v + \Delta$ on seller 1
Basic Analysis

- Standard assumption:
  \[ F/f \] is increasing, or equivalently, \( F \) is log-concave

- Virtual Cost (Myerson) is \( c + \frac{F(c)}{f(c)} \)

- \( c + \frac{F(c)}{f(c)} \) is increasing
The optimal mechanism is to buy from 1 iff

\[ c_1 + \frac{F(c_1)}{f(c_1)} \leq c_2 + \frac{F(c_2)}{f(c_2)} + \Delta. \]

Thus, if \( \Delta = 0 \), optimal auction is “buy from the low \( c \)”.

Proof of this is pretty standard.
The “Myerson” line $\Phi_M(\cdot)$ is implicitly defined by

$$c_1 + \frac{F(c_1)}{f(c_1)} = \Phi_M(c_1) + \frac{F(\Phi_M(c_1))}{f(\Phi_M(c_1))} + \Delta.$$
A typical Myerson line

\[ c_1 + \frac{F(c_1)}{f(c_1)} = \Phi_M(c_1) + \frac{F(\Phi_M(c_1))}{f(\Phi_M(c_1))} + \Delta \]
Properties of the Myerson Line

Under log-concavity, \( 0 < c_1 - \Phi_M(c_1) < \Delta \)
An Important Property of the Myerson Line

\[ \Phi'_M(c_1) \geq 1 \iff \frac{F}{f} \text{ is convex} \]

\[ \Phi'_M(c_1) \leq 1 \iff \frac{F}{f} \text{ is concave} \]

Why is this true?
Properties of the Myerson Line

Consider the case where $\frac{F}{f}$ is convex,

Similarly, if $\frac{F}{f}$ is concave, then $\Phi'_M(c_1) \leq 1$. 
Properties of the Myerson Line

For \( \frac{F}{T} \) to be convex, a sufficient (but far from necessary) condition is \( f \) is decreasing.
Linear Distribution Functions

Example: \( f = 1, \ F = x, \) so \( \frac{F}{f} = x \)

\( \Phi_M(c_1) \) is given by

\[
c_1 + \frac{F(c_1)}{f(c_1)} = c_2 + \frac{F(c_2)}{f(c_2)} + \Delta
\]

\[
2c_1 = 2c_2 + \Delta
\]

Thus,

\[
c_1 - c_2 = \frac{\Delta}{2}
\]
The Picture
What are some “simple” mechanisms?
Request for Proposals:

- used all the time

- suppliers submit bid $b_1, b_2$

- buyer chooses his favorite and pays $b_i$

- so, if $b_1 - b_2 \leq \Delta$, choose 1
  
  if $b_1 - b_2 > \Delta$, choose 2

This has the flavor of a first price auction.
Standard First Price Auction

Standard FPA:

- bidders submit $b_i$’s

- buyer *committed* to choose the lowest $b_i$ and pay $b_i$

This matters!

After bids are submitted, buyer wants to change mind if $b_2 + \Delta > b_1 > b_2$
Efficiency vs. competition:

- **RFP**
  - Good news: $\Pr(\text{buy from 1}) > \frac{1}{2}$
  - Bad news: less competition, buyer pays more on average

- **Standard FPA**
  - Good at competition (like Bertrand with undifferentiated products)
  - But, $\Pr(\text{buy from 1}) = \frac{1}{2}$
First Price Bonus Auctions

The trade-off between RFP and FPA suggests FPBA (first price bonus auctions)

Fix $A \geq 0$,

- if $b_1 < b_2 + A$, buy from 1
- if $b_1 \geq b_2 + A$, buy from 2, ($b_1 \leq 1$, reserve of 1)

Note that $A = 0$ is FPA, $A = \Delta$ is RFP.

“Scoring rules” effectively do this.
Bonus Clock Auctions

How about simple “second price” mechanisms for procurement?

A standard clock auction is:

- clock starts at a price
- clock ticks
- at some point someone says “yours”
- winner (remaining player) makes sale at currently displayed price $p$

or, sealed bid version ...
Our twist:

if 2 wins, 2 gets $p$

if 1 wins, 1 gets $p + A$, where $A$ is the bonus

Note that if $A = 0.3$, can start clock at $1 - A = 0.7$ (no need to pay more than 1).
Simple dominant strategies:

- player 2 hangs in until $c_2$
- player 1 hangs in until $c_1 - A$
In general, SPBA is not fully optimal. If $\frac{F}{f}$ is concave,
SPBA for General Distribution Functions

If \( \frac{F}{f} \) is convex,
The bidding functions for bidder 1 and 2 are $\beta_1(\cdot)$, $\beta_2(\cdot)$ respectively.
Characterization of the Equilibrium

Define function $\phi_{FP}(\cdot) = \phi(\cdot)$ by

\[
\beta_1(\phi(c)) = \beta_2(c) + A \quad \phi'(c) = \frac{\beta'_2(c)}{\beta'_1(\phi(c))}
\]
What do we know about $\phi$?

- $c \leq \phi(c) \leq c + A$, strict except at 0 and $1 - A$
- $\phi' > 0$
- $\phi(0) = 0$
- $\phi(1 - A) = 1$

Note that $\phi' > 1$ on average.
So FPBA is lousy when realized costs are low, no efficiency gain, but paying too much on average.

In general, lots of distortion at high costs, very little at low costs.

The optimal mechanism, in contrast, has more even distortion.
Comparing the FPBA and SPBA

We know $\phi' > 1$ on average.

Assume $\phi' > 1$ pointwise.
Comparing the FPBA and SPBA

Claim: If $\frac{F}{f}$ is convex and $\phi' > 1$, then any FPBA is dominated by a SPBA.

Proof:
So, for carefully chosen $A_S^*$, SPBA

*never* does worse and often does better.

(This is overkill, of course)
So, how about $\phi'$?

Fundamental problem:

- FPBA is profoundly unwieldy to analyze
**Proof of $\phi' > 1$ everywhere**

A sketch of one way to show that $\phi' > 1$ everywhere for one case

- **Claim 1:** $\phi'(0) > 1$
- **Claim 2:** as $c_2 \to 1 - A$, $\phi'(c_2) \to \infty$
Proof of $\phi' > 1$ everywhere

So imagine that $\phi' < 1$ somewhere.
Let $r \in (0, 1 - A)$ be a global minimizer of $\phi'(\cdot)$. 

![Graph showing $\phi'(c_2)$, $\phi''(r) = 0$, and $r \in (0, 1 - A)$]
Lot’s of grinding from FOC says

\[
\phi'(r) = \frac{\int_{\phi(r)}^{1} \bar{F}(\psi(s)) ds \frac{f(r)}{\bar{F}^2(r)}}{\int_{r}^{1-A} \bar{F}(\phi(s)) ds \frac{f(\phi(r))}{\bar{F}^2(\phi(r))}} = \frac{T}{B}
\]

where

\[\psi = \phi^{-1}\]

and

\[\bar{F} = \text{reverse cumulative}\]
\[
\phi'(r) = \frac{1}{B} \left( \int_{\phi(r)}^{1} \bar{F}(\psi(s))ds \frac{f(r)}{\bar{F}^2(r)} - \int_{r}^{1-A} \bar{F}(\phi(s))ds \frac{f(\phi(r))}{\bar{F}^2(\phi(r))} \right)
\]

1’s surplus at \( c_1 = \phi(r) \)

2’s surplus at \( c_2 = r \)
Some grinding gets that ...
Proof of $\phi' > 1$ everywhere

\[ \frac{\phi''}{\phi'}(r) = \frac{T_x}{T}(r) - \frac{B_x}{B}(r) \]

\[ > \left( \frac{\bar{F}(r)}{\bar{F}(r + \delta)} \right) \frac{1}{\phi(r)B}^{-2} \frac{f(r)}{f(r + \delta)} \]

where $\delta \equiv \phi(r) - r$.

Strategy: show that this expression is positive.

But then, $\phi'' > 0$, a contradiction.
Yet, more grinding ...

a sufficient condition for this is that

$$\bar{F}(x) \frac{1}{\phi'(r)B^2} f(x)$$

is log-concave.

To get much further, need a bound on $B$. 
A lemma:

\[
\phi'(r) \int_r^{1-A} \bar{F}(\phi(s)) ds \frac{f(r + \delta)}{\bar{F}^2(r + \delta)} \leq \int_{r+\delta}^1 \bar{F}(s) ds \frac{f(r + \delta)}{\bar{F}^2(r + \delta)}
\]

Define

\[
W(r) = \int_{r+\delta}^1 \bar{F}(s) ds \frac{f(r + \delta)}{\bar{F}^2(r + \delta)}
\]
Proof of the lemma

**Proof:** By a change of variables

\[
\int_{r}^{1-A} \bar{F}(\phi(s))ds = \int_{\phi(r)}^{1} \frac{\bar{F}(s)}{\phi'(s)}ds < \frac{1}{\phi'(r)} \int_{\phi(r)}^{1} \bar{F}(s)ds
\]

(Note that this is a pretty loose bound.)
So, for example, assume $f_x \geq 0$. Then ...
\[ \int_{\phi(r)}^{1} \bar{F}(s) ds \]
$\bar{F}(\phi(r)) / f(\phi(r))$
Thus, \( W(r) \leq \frac{1}{2} \).

So, since \( \phi'(r)B < W(r) \), we have \( \frac{1}{\phi'(r)B} - 2 > 0 \), and thus

\[
\bar{F}(x)\frac{1}{\phi'(r)B}^{-2}f(x)
\]

is log-concave.
We have proofs for some other cases, and have checked numerical solutions for a lot of examples, and always have $\phi' > 1$. 
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Conclusion

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- SPBA is simple to play and performs better
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Maybe $\Delta$ can be “designed”
• Qualification process
• The Boeing 787’s interchangeable jet engine