MONETARY POLICY, SHORT-RUN DYNAMICS, AND WELFARE

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Motivation

• Many economists agree that monetary policy has persistent nominal and real effects

• Existing models have difficulty generating enough persistence

• Most existing explanations lack micro-foundations
  ◦ NOT explicitly model the role of money
  ◦ NOT explicitly model the process of exchange
  ◦ BUT impose ad-hoc nominal rigidities
Search Theory of Money

- Role of money explicitly modeled

- Due to technical difficulties, short-run implications have not been fully explored

- Existing work has
  - focused on long-run (steady-state) analysis
    - Lagos and Wright (2005), Molico (2006)
  - assumed indivisibility
    - Wallace (1997)
  - assumed perfect insurance
    - Shi (1999), Aruoba and Wright (2003)

- Therefore, distributional and propagational effects of monetary shocks not well understood
What we do

- Develop a micro-founded search model in which agents are subject to idiosyncratic liquidity shocks, as well as aggregate productivity and monetary shocks
- Propose a numerical algorithm to solve the model
- Study the dynamics of the model
- Impulse response to unanticipated inflation shocks
- Welfare costs of anticipated inflation
- Welfare evaluation of monetary policy rules
How the model works

- Agents face idiosyncratic uncertainty regarding their trading opportunities as well as aggregate monetary and technology uncertainty.
- Agents are unable to insure against idiosyncratic risk due to market incompleteness implied by the frictions that generate the role of money.
- Heterogenous idiosyncratic histories imply a non-degenerate distribution of money holdings which become a state variable summarizing the past histories.
- Redistributive monetary policy has persistent (real) effects on output and prices: aggregate shocks will propagate and diffuse gradually as the money distribution adjusts over time.
Roadmap

- Model
- Numerical Method
- Numerical Findings
- Conclusion and Future Works
Environment

- Continuum of infinitely lived agents
- Agents specialize in the consumption and production of goods
- Preferences: $u(x_t) - c(h_t)$
- Production function: $y_t = z_t h_t$
- Anonymous bilateral matching
- Lack of double coincidence of wants
- Buyer with probability $\sigma$
- Seller with probability $\sigma$
- Take-it-or-leave-it offer from buyers (no “hold-up problem”)
- Lump-sum money injection before trade $M' = \mu M$
Aggregate Shocks

• Aggregate productivity shock $z \in \{z_L, z_H\}$ follow a Markov chain with transition probabilities given by

$$
\Pi_{zz'} = \begin{bmatrix}
\pi_{LL} & \pi_{LH} \\
\pi_{HL} & \pi_{HH}
\end{bmatrix}
$$

• Monetary policy rule: $\mu = G(z, \lambda) + \epsilon_{\mu}$ where $G(z, \lambda)$ is a monetary policy rule and $\epsilon_{\mu}$ is an i.i.d. money growth shock. Also, $\epsilon_{\mu} \in \{-\Delta, 0, \Delta\}$ with $Pr(\epsilon = -\Delta) = Pr(\epsilon = \Delta) = \tau < \frac{1}{2}$. 
Model

Timing

- Money shock $\varepsilon_{\mu}$
- Productivity shock $z$
- Agents meet and trade goods
- Money transfer $\mu = G(z, \lambda) + \varepsilon_{\mu}$
- $\lambda' = H(\varepsilon_{\mu}, z, \lambda)$

$t$

$\lambda$

$V(m, \varepsilon_{\mu}, z, \lambda)$

$t+1$
Terms of Trade

Terms of trade determined by take-it-or-leave-it offers from buyers:

\[ d(m_b, m_s; \epsilon, z, \lambda) \text{ and } q(m_b, m_s; \epsilon, z, \lambda) \text{ solve} \]

\[
\max_{0 \leq d \leq m_b + \mu - 1, q \geq 0} u(q) + E_{\epsilon|z} E_{z'} [V\left(\frac{m_b + \mu - 1 - d}{\mu}; \epsilon', z', \lambda'\right)] ,
\]

subject to

\[
\begin{align*}
c\left(\frac{q}{z}\right) &= E_{\epsilon|z} E_{z'} \left\{ V\left[\frac{m_s + \mu - 1 + d}{\mu}; \epsilon', z', \lambda'\right] - V\left[\frac{m_s + \mu - 1}{\mu}; \epsilon', z', \lambda'\right] \right\} \\
\lambda' &= H(\epsilon, z, \lambda) \\
\mu &= G(z, \lambda) + \epsilon
\end{align*}
\]
Value Function

\[ V(m; \epsilon_\mu, z, \lambda) = \sigma \beta \int_0^\infty \{ u[q(m, m_s; \epsilon_\mu, z, \lambda) \}
\]
\[ + E_{\epsilon'_{\mu}} E_{z'|z} V \left( \frac{m + \mu - 1 - d(m_b, m_s; \epsilon_\mu, z, \lambda)}{\mu} ; \epsilon'_{\mu}, z', \lambda' \right) \]
\[ - V \left[ \frac{m + \mu - 1}{\mu} ; \epsilon'_{\mu}, z', \lambda' \right] \lambda(d m_s) \]
\[ + \beta E_{\epsilon'_{\mu}} E_{z'|z} V \left[ \frac{m + \mu - 1}{\mu} ; \epsilon'_{\mu}, z', \lambda' \right], \]

where

\[ \lambda' = H(\epsilon_\mu, z, \lambda), \mu = G(z, \lambda) + \epsilon_\mu \]
Computing Equilibria

• Solving this stochastic heterogeneous agents model with a continuum of agents constitutes an impossible technical problem given that the state vector includes the whole distribution of money, which is an infinite-dimensional object.

• Recently, algorithms have been proposed which summarize the cross-section distribution of agents’ characteristics by a small number of moments, therefore making the dimension of the state space tractable, and calculate the law of motion for these state variables using simulation procedures. The best known examples of this approach are those of den Hann (1997), Rios-Rull (1997), and Krusell and Smith (1998).
• An added difficulty of computing equilibria in our model is that it requires us to compute the expected gain from trade given the random matching.

• To exactly compute that expectation might require us to keep track of a large number of moments.

• To overcome this difficulty we follow Algan, Allais and Den Haan (2007) by parameterizing the cross-sectional distribution and using reference moments as in Reiter (2002).

• Parameterizing the cross-sectional distribution allows us to obtain a numerical solution using standard quadrature and projection techniques. Using reference moments allows us to get a better characterization of the cross-sectional distribution without increasing the number of state variables.
Parameterization

- We use the second moment as a state variable and the 3rd to 5th moments as reference moments

- \( G(z, \lambda) = \bar{\mu} \) such that \( \mu = \bar{\mu} + \epsilon_\mu \)

- \( \bar{\mu} = 1.005 \) (quarterly rate) and \( \epsilon_\mu \in \{0.0025, 0, -0.0025\} \)

- Only monetary shocks, for illustration (but we have solved the model with both shocks)

Preferences:

- \( u(q) = A \ln(1 + q) \) and \( c(q) = \frac{1}{\bar{q} - q} - \frac{1}{q} \) (as in Molico) with \( A = 100, \bar{q} = 1 \)

- \( \sigma = 0.5, \beta = 0.99 \) (quarterly model)
Equilibrium

- We find that the law of motion of the endogenous aggregate state is well approximated by a linear function, e.g.

\[ s'_2 = 0.001706 - 0.266 \cdot \epsilon_\mu + 0.986 \cdot s_2 \]

- \( R^2 \) of 0.999984

- Note that a positive monetary shock decrease the variance of the distribution of money and has (very) persistent effects.
Impulse Responses: One-time money growth shock

- Distribution Variance
- Aggregate Output
- Average Period Utility
- Velocity
- P/M
Numerical Example

Terms of Trade

Quantities Traded

$q(mb,ms)$

Money buyer

Money seller

$q(mb,ms)$
Concluding Remarks

• We developed a micro-founded search model of money with idiosyncratic and aggregate uncertainty, and found that monetary policy can have persistent effects on output, prices and welfare.

• To do:
  – Proper calibration of the model
  – Analyze the sensitivity of the results to changes in the number of reference moments used
  – Welfare costs of anticipated inflation taking into account transitions
  – Welfare evaluation of different policy rules