Long Term vs Short Term comovements in stock markets: the use of markov switching multifractal models

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Aim of the paper

- To give some new tools for assessing market risk, volatility and extreme comovements
- new tools:
  - volatility cycles
  - crisis probability
  - crisis transmission probability
  - probabilities of long term high (low) volatility cycles.
Illustration 1
Introduction
Fractal series & models
Comovement analysis & results
Conclusions

Simulations
A possible view of the market

Price

![Price Chart](chart.png)
Illustration

Long term vs Short term comovements...
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$k=2$
k=3

Long term vs Short term comovements...
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$k=4$

Long term vs Short term comovements...
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$k=5$

![Graph showing long term vs short term comovements...](image-url)
k=6
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$k=7$

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$k=8$

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$k=9$

Long term vs Short term comovements...
k=10

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$k=12$
Returns

Long term vs Short term comovements...
Price

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Price

Simulated returns
Normal distribution

![Graph showing the distribution of simulated returns compared to a normal distribution.](image-url)
Price

- Long run shifts on the Low frequencies
  - climatic; demographic; fundamentals factors; accounting announcements; macroeconomic statistics; long term position of investors; market organization, industry consolidation.

- Transient shifts on the High frequencies
  - XR movements; arbitrages

- not dependent on the data frequency that is used!
Short vs Long (ex ante)

- Infinite # types of investors
- Infinite # types of information
- Infinite # types of investment incentives
- Continuous scale of horizons from the short term to the long term

... with everything superposed as Strata
Returns formulation

\[ R_t = \left( \prod_{k=1}^{\bar{k}} M_{k,t} \right)^{1/2} \sigma \varepsilon_t \]

with \( \varepsilon \) following a standard Gaussian.

- The components \( M_{k,t} \) allow for jumps at different frequencies to different values.
- Implication: there is a Markov switching process behind each component.
Transition frequencies

Each component $M_k$ with $k = 1...\bar{k}$ is characterized by its jump frequency modelled as

$$\gamma_k = 1 - (1 - \gamma_1)^{b(k-1)}$$

with $\gamma_1 \in [0;1]$, $b \in ]0;1]$ so that $\gamma_k \in [0;1]$

$M_1$ is short lasting while $M_{\bar{k}}$ is long lasting: continuous discrimination between long and short terms.
An empirical illustration (CAC), MLE estimation

Long term vs Short term comovements...
An empirical illustration (Components correlations in CAC DAX FTSE)

<table>
<thead>
<tr>
<th></th>
<th>$M_{1}^{cac}$</th>
<th>$M_{2}^{cac}$</th>
<th>$M_{3}^{cac}$</th>
<th>$M_{1}^{dax}$</th>
<th>$M_{2}^{dax}$</th>
<th>$M_{3}^{dax}$</th>
<th>$M_{1}^{ftse}$</th>
<th>$M_{2}^{ftse}$</th>
<th>$M_{3}^{ftse}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{1}^{cac}$</td>
<td>1</td>
<td>0.69</td>
<td>0.36</td>
<td>0.86</td>
<td>0.60</td>
<td>0.34</td>
<td>0.79</td>
<td>0.63</td>
<td>0.46</td>
</tr>
<tr>
<td>$M_{2}^{cac}$</td>
<td>0.69</td>
<td>1</td>
<td>0.71</td>
<td>0.62</td>
<td>0.92</td>
<td>0.67</td>
<td>0.79</td>
<td>0.89</td>
<td>0.79</td>
</tr>
<tr>
<td>$M_{3}^{cac}$</td>
<td>0.36</td>
<td>0.71</td>
<td>1</td>
<td>0.33</td>
<td>0.76</td>
<td>0.98</td>
<td>0.57</td>
<td>0.78</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Stronger correlation between components of the same series, and between components of different series for the same frequency. Interesting: correlations are high for long term component between the series: short run differences are not persistent.
Calvet 2006 JoF
Returns formulation

\[ x_t = \left( \prod_{k=1}^{\tilde{k}} M_{k,t}^\alpha \prod_{k=1}^{\tilde{k}} M_{k,t}^\beta \right)^{1/2} \ast \varepsilon_t \]

with \( \varepsilon \) following a bivariate Gaussian \((0, \Sigma)\).

\[
\Sigma = \begin{pmatrix}
\sigma_\alpha^2 & \rho_\varepsilon \sigma_\alpha \sigma_\beta \\
\rho_\varepsilon \sigma_\alpha \sigma_\beta & \sigma_\beta^2
\end{pmatrix}
\]

- Need to extend the former Markov switching process to its bivariate form (paper).
Two main sources of correlation

1. Unconditional cross correlation between residuals $\rho_\varepsilon$
Two main sources of correlation

1. Unconditional cross correlation between residuals $\rho_\varepsilon$

2. Correlation between information arrivals (jumps in components) $\lambda$ such that

$$\Pr(D_k^\beta = 1 \mid D_k^\alpha = 1) = (1 - \lambda)\gamma_k + \lambda.$$
We note $\text{MSM}(\bar{k})$ the model with $\bar{k}$ frequencies (model selection procedure).

Component $M_k$ are stage invariant in a sense that their distribution is invariant.

This distribution is set to a binomial distribution taking value $m_0$ or $2 - m_0$ with equal probability.

$m_0 \in [1; 2]$ is the high value of components so that $E(M_k) = 1$ i.e. the volatility is not explosive.

the number of states is finally $4^{\bar{k}}$. 
Data and estimations

- Estimated by maximum likelihood with Bayesian updating on MatLab 7.1
- Four stock indexes: CAC40, DAX, FTSE, NYSE 15:00 GMT.
- Daily frequencies between 1994 and 2008
- Models estimated for various \( \bar{k} \), model selection tests (Vuong HAC), in bivariate forms.
  - Univariate case: 32 models estimated from \( k = 1 \) to 8 for 4 indexes.
  - Bivariate case: 30 models estimated from \( k = 1 \) to 5 for 6 pairs of indexes.
- \( \bar{k} = 3 \) is optimal for all models so that we have 64 states in volatilities.
## Estimated parameters MSM(3)

<table>
<thead>
<tr>
<th></th>
<th>CAC-DAX</th>
<th>CAC-FTSE</th>
<th>CAC-NYSE</th>
<th>DAX-FTSE</th>
<th>DAX-NYSE</th>
<th>FTSE-NYSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_0^\alpha$</td>
<td>1.433 (0.0003)</td>
<td>1.432 (0.0003)</td>
<td>1.433 (0.0003)</td>
<td>1.468 (0.0003)</td>
<td>1.467 (0.0003)</td>
<td>1.452 (0.0003)</td>
</tr>
<tr>
<td>$m_0^\beta$</td>
<td>1.468 (0.0002)</td>
<td>1.459 (0.0004)</td>
<td>1.441 (0.0005)</td>
<td>1.460 (0.0004)</td>
<td>1.441 (0.0004)</td>
<td>1.442 (0.0003)</td>
</tr>
<tr>
<td>$\sigma_\alpha$</td>
<td>1.533 (0.0039)</td>
<td>1.538 (0.004)</td>
<td>1.506 (0.0024)</td>
<td>1.669 (0.0045)</td>
<td>1.637 (0.005)</td>
<td>1.187 (0.0020)</td>
</tr>
<tr>
<td>$\sigma_\beta$</td>
<td>1.667 (0.0047)</td>
<td>1.336 (0.0081)</td>
<td>1.220 (0.0028)</td>
<td>1.330 (0.0082)</td>
<td>1.225 (0.0026)</td>
<td>1.193 (0.0039)</td>
</tr>
</tbody>
</table>

*Table 3: Models estimations for MSM(3)*
### Estimated Dependence Structure

<table>
<thead>
<tr>
<th></th>
<th>CAC-DAX</th>
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<th>DAX-NYSE</th>
<th>FTSE-NYSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>0.253 (0.0718)</td>
<td>0.314 (0.0962)</td>
<td>0.278 (0.0961)</td>
<td>0.313 (0.102)</td>
<td>0.306 (0.082)</td>
<td>0.330 (0.086)</td>
</tr>
<tr>
<td>(\gamma_1)</td>
<td>0.041 (0.0093)</td>
<td>0.032 (0.0095)</td>
<td>0.051 (0.0139)</td>
<td>0.033 (0.0083)</td>
<td>0.045 (0.0086)</td>
<td>0.044 (0.0125)</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>0.956 (0.482)</td>
<td>0.984 (0.487)</td>
<td>0.899 (0.461)</td>
<td>0.903 (0.437)</td>
<td>0.788 (0.414)</td>
<td>0.903 (0.453)</td>
</tr>
<tr>
<td>(\rho_\varepsilon)</td>
<td>0.909 (0.0031)</td>
<td>0.837 (0.0055)</td>
<td>0.784 (0.0077)</td>
<td>0.814 (0.0065)</td>
<td>0.787 (0.0077)</td>
<td>0.779 (0.0079)</td>
</tr>
<tr>
<td>lnL</td>
<td>-7132.4</td>
<td>-6982.8</td>
<td>-7201.9</td>
<td>-7357.5</td>
<td>-7405.0</td>
<td>-6544.5</td>
</tr>
</tbody>
</table>
Volatilities

**Figure:** volatility MSM(3)
Volatilities

Figure: volatility MSM(3)
State correlations

\[
\text{Cov}_t \left( x_t^\alpha, x_t^\beta \right) = \rho \sigma_\alpha \sigma_\beta \prod_{k=1}^{\bar{k}} E \left[ \left( M_{k,t}^\alpha M_{k,t}^\beta \right)^{1/2} \right]
\]

\[
\text{Var}_t(x_t^c) = \sigma_c^2 E(M_t^c)
\]

so that

\[
\text{Corr}_t \left( x_t^\alpha, x_t^\beta \right) = \rho \frac{\prod_{k=1}^{\bar{k}} E \left[ \left( M_{k,t}^\alpha M_{k,t}^\beta \right)^{1/2} \right]}{\left( E(M_t^\alpha) E(M_t^\beta) \right)^{1/2}}
\]
State correlations

Figure: correlation MSM(3)
State correlations

**Figure:** correlations MSM(3)
### Shared volatility cycles (days)

<table>
<thead>
<tr>
<th></th>
<th>CAC-DAX</th>
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<th>DAX-NYSE</th>
<th>FTSE-NYSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>24.3</td>
<td>30.6</td>
<td>19.2</td>
<td>30.2</td>
<td>21.9</td>
<td>22.7</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>94.4</td>
<td>96.4</td>
<td>67.9</td>
<td>95.6</td>
<td>70.5</td>
<td>67.8</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>371.1</td>
<td>305.5</td>
<td>242.7</td>
<td>304.5</td>
<td>229.2</td>
<td>204.5</td>
</tr>
</tbody>
</table>

Volatility shared cycles length between indexes (days)
Low (High) volatility long run cycles

Definition

Returns are on a low (high) long run volatility cycle when the component of the lowest frequency is at its lowest (highest) value.

\[ \Pr(\text{LLRC})_t = \Pr(M^\alpha_{k,t} = M^\beta_{k,t} = 2 - m^\alpha_0) \]

\[ \Pr(\text{HLRC})_t = \Pr(M^\alpha_{k,t} = M^\beta_{k,t} = m^\alpha_0) \]
long run cycles

Figure: High long run cycles MSM(3)
**Figure:** High long run cycles MSM(3)
long run cycles

Figure: High long run cycles MSM(3)
Crisis probability

**Definition**

A crisis is defined when all components are at all frequencies and simultaneously for both market at their highest value.

\[
\Pr(\text{crisis})_t = \Pr(M^\alpha_{1,t} = \ldots M^\alpha_{k,t} = m_0^\alpha \text{ and } M^\beta_{1,t} = \ldots M^\beta_{k,t} = m_0^\beta)
\]
The Spielberg example:

- climatic catastrophe,
The Spielberg example:

- climatic catastrophe,
- chain bankruptcy,
The Spielberg example:

- climatic catastrophe,
- chain bankruptcy,
- hedge fund disengagement,
The Spielberg example:

- climatic catastrophe,
- chain bankruptcy,
- hedge fund disengagement,
- currency arbitrages
The Spielberg example:

- climatic catastrophe,
- chain bankruptcy,
- hedge fund disengagement,
- currency arbitrages
- ... and the little cat is dead.
Joint crisis probability

Figure: crisis probability MSM(3)
Joint crisis probability

**Figure**: crisis probability MSM(3)
Probability of extreme comovements

Definition

The probability of conditionnal extreme comovements is defined as the probability to switch to a crisis period in market $\beta$ when market $\alpha$ is in a crisis situation.

$$
\Pr(\text{ext.com.})_t = \Pr(M^\alpha_{1,t} = ... M^\alpha_{k,t} = m_0^\alpha \mid M^\beta_{1,t} = ... M^\beta_{k,t} = m_0^\beta)
$$

$$
= \frac{\Pr(M^\alpha_{1,t} = ... M^\alpha_{k,t} = m_0^\alpha \text{ and } M^\beta_{1,t} = ... M^\beta_{k,t} = m_0^\beta)}{\Pr(M^\beta_{1,t} = ... M^\beta_{k,t} = m_0^\beta)}
$$
Probability of extreme comovements

Figure: extreme comovements MSM(3)
Probability of extreme comovements

**Figure:** extreme comovements MSM(3)
Probability of extreme comovements

Figure: extreme comovements MSM(3)
Probability of extreme comovements

Figure: extreme comovements MSM(3)
Probability of extreme comovements

Figure: extreme comovements MSM(3)
Conclusions

- Derived new indicators from a new class of multifractal multivariate models
- Improve the vision of long and short term comovements and long run common cycles
- Applied on stock indexes, with capital cycles estimated longer than those for XR data
- Give insights about crisis, and shared cycles and extreme comovements in a unified econometric framework
- Further research on this class of models is needed and under work
- On the current crisis: what we can say is that 2007 is a transition period and that 2008 switch to a long term high volatility cycle with higher extreme comovements probability.