Fixed Costs and Long-Lived Investments

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Background

Neoclassical investment models predict that firm-level investment should be frequent and small.

Firm-level data show that many firms make infrequent, large adjustments to their capital stock.
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Fixed costs seem to play a role in plant-level investment.

Solving equilibrium models with fixed costs is complicated.
Background
Background

adjust

$k^*$  \(\delta\)  \(\bar{k}\)
Background

Steady State $f_t(k)$
Background

Out-of-steady-state $f_t(k)$
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**Question**: Are fixed costs (and the distribution of capital holdings) important for understanding aggregate investment and policy?
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Caballero and Engel [1999] – Yes.
Bachmann, Caballero and Engel [2006] – Yes.
Question: Are fixed costs (and the distribution of capital holdings) important for understanding aggregate investment and policy?

Thomas [2002] – calibrated DSGE model

In general equilibrium, [consumption smoothing] offsets changes in aggregate investment demand implied by [lumpy investment]. Adjustments in wages and interest rates yield quantity dynamics that are virtually indistinguishable from the standard model, and lumpy investment appears largely irrelevant for equilibrium business cycle analysis.
This paper

Analysis of investment model with fixed costs.

Focus on long-lived capital (low depreciation).
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1. Distribution of capital holdings has no bearing on the equilibrium.

2. Aggregate behavior is virtually identical to the neoclassical model.
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Analysis of investment model with fixed costs.

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1. Distribution of capital holdings has no bearing on the equilibrium.

2. Aggregate behavior is virtually identical to the neoclassical model.

3. However, results flow from near infinite elasticity of investment timing. Not consumption smoothing per se.
Model: Overview

• Investment Supply and Demand. (partial equilibrium).
Model: Overview

Demand Side:

- Fixed number of firms.
- Flow profits $A(t)k(t)^\alpha$
- Capital depreciation $\dot{k}(t) = -\delta k(t)$
- $T$: time of adjustment, $\bar{k}$: reset level of capital.
- Cost of adjusting capital: $F + p(T)[\bar{k} - k(T)]$
Model: Overview

Supply Side:

\[ p(t) = z(t) \cdot S(I(t)) \]

- \( p(t) \): Market price of new capital.
- \( I(t) \): Aggregate investment.
- \( z(t) \): Supply shock.

\[ S(0) = 0, \quad S' > 0 \]
Model: Demand Side

Incentive to delay or accelerate adjustment depends on difference between $MP^k$ at $T$ and the user cost of capital “Jorgenson Gap”

$$G(\delta, T) = \alpha k(T)^{\alpha-1} - (r + \delta)$$
Model: Demand Side
Model: Demand Side

\[ \alpha k^{\alpha - 1} \]

\[ r + \delta \]
Model: Demand Side

\[ \alpha k^{\alpha - 1} \]
Model: Demand Side
Model: Demand Side

\[ G(\delta, T) \]

\[ r + \delta \]

\[ k(T), k^J, \bar{k} \]

\[ \alpha k^{\alpha-1} \]
Model: Demand Side

The Jorgenson Gap is small (per $T$)

$$G(\delta, T) = \alpha k(T)^{\alpha^{-1}} - (r + \delta)$$

$$\approx \delta (r + \delta)(1 - \alpha)T$$
Intertemporal Elasticity of Substitution
Intertemporal Elasticity of Substitution

Relative to avg annual profit flow, loss from adjusting at $T + dT$ is approximately,

$$\frac{1}{2} \delta \alpha \left[ \frac{G(\delta, T)}{G(\delta, T) + (1-\alpha)(r + \delta)} \right] (dT)^2$$
Intertemporal Elasticity of Substitution

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$$\frac{1}{2} \delta \alpha \left[ \frac{G(\delta, T)}{G(\delta, T) + (1 - \alpha)(r + \delta)} \right] (dT)^2 < \frac{1}{2} \delta \alpha (dT)^2$$
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Relative to avg annual profit flow, loss from adjusting at $T + dT$ is approximately,

$$\frac{1}{2} \delta \alpha \left[ \frac{G(\delta, T)}{G(\delta, T) + (1 - \alpha)(r + \delta)} \right] (dT)^2 < \frac{1}{2} \delta \alpha (dT)^2$$

E.g. $dT = 1$ (one year delay), $\delta = .04$, and $\alpha = .5$, the loss is less than 1% of annual profits.
Intertemporal Elasticity of Substitution

$\varepsilon$-equilibrium with $\varepsilon = .01$ : $dT \leq \left[ \sqrt{2\varepsilon/\alpha\delta} \right] = 1$
Intertemporal Elasticity of Substitution

$\varepsilon$-equilibrium with $\varepsilon = 0.01$:

$$dT \leq \sqrt{2\varepsilon / \alpha \delta} = 1$$
Intertemporal Elasticity of Substitution

Price change \((dp)\) which makes the firm indifferent between adjusting now or delaying adjustment \((dT)\)

\[
dp \approx - \frac{G(\delta, T)}{2T} (dT)^2
\]
Intertemporal Elasticity of Substitution

Price change \((dp)\) which makes the firm indifferent between adjusting now or delaying adjustment \((dT)\)

\[
dp \approx -\frac{1}{2} \delta (r + \delta)(1 - \alpha)(dT)^2
\]

E.g. \(\delta = .04, \ \alpha = .5, \ r = .02\). Then, for a one year delay, the firm would require \(dp = -0.0006\) (6 basis points).
Comparison to neoclassical investment model:

Shadow value of capital:

\[ q_t = \int_0^\infty e^{-(r+\delta)s} \frac{\partial F(t+s)}{\partial k} \, ds \]

Marginal cost of capital: \( p_t \)

\[ p_t = q_t \]

For sufficiently long-lived (low \( \delta \)) capital, \( q_t \) is roughly constant.
Predictions in the low-depreciation limit:

1. **Supply shocks** $\rightarrow p$ unchanged, $I$ falls by the amount of the shock.

2. **Demand shocks** $\rightarrow p$ unchanged, $I$ unchanged.

3. **Variations in the initial distribution** have no effect on $p$ or $I$. 
Numerical Model

Follows King and Thomas [2006].

Discount rate ($r$) = 0.02

Elasticity of supply ($\xi$) = 1.00.

Curvature of profit function ($\alpha$) = 0.35.

Depreciation rate (baseline) ($\delta$) = 0.05.
Out-of-Steady-State Distribution

Initial Distribution

Aggregate Investment: Alternate Depreciation Rates

Aggregate Investment: Alternate Supply Elasticities
Comparison with the Neoclassical Model

- **Aggregate Investment**
  - Neoclassical Model
  - Fixed-Cost Model

- **Prices**
  - Neoclassical Model
  - Fixed-Cost Model
Comparison to DSGE Models:
Comparison to DSGE Models:

DSGE framework:

\[ E_t \left[ \sum_{j=0}^{\infty} \beta^j \left\{ C_{t+j}^{1-\frac{1}{\sigma}} - \phi N_{t+j}^{1+\frac{1}{\eta}} \right\} \right] \]

Production

\[ Y_t = AK_t^{\alpha} N_t^{1-\alpha} \]

Resource constraint

\[ Y_t = C_t + I_t \]
Comparison to DSGE Models:

Marginal cost of investment (in utility) \( u'(C_t) \)
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Marginal cost of investment (in utility): $u'(C_t)$

Elasticity of supply

$$\xi = \frac{\eta}{1 + \alpha \eta} \left(1 - \alpha \right) \frac{Y}{I} + \sigma \frac{C}{I}$$
Comparison to DSGE Models:

Marginal cost of investment (in utility) : \( u'(C_t) \)

Parameterization:

\[
\eta = \frac{1}{2}, \quad \sigma = .2, \quad \alpha = \frac{1}{3}
\]

\[
\frac{Y}{I} < 7, \quad \frac{C}{I} < 5
\]
Comparison to DSGE Models:

Marginal cost of investment (in utility): $u'(C_t)$

Parameterization:

$$\eta = \frac{1}{2}, \quad \sigma = .2, \quad \alpha = \frac{1}{3}$$

$$\frac{Y}{I} < 7, \quad \frac{C}{I} < 5$$

$$\xi \approx 3$$
Comparison to DSGE Models:

Marginal cost of investment (in utility) : \( u'(C_t) \)

Alt. parameterization:

\[ \eta = 1, \quad \sigma = 1, \quad \alpha = \frac{1}{3} \]
Comparison to DSGE Models:

Marginal cost of investment (in utility) : $u'(C_t)$

Alt. parameterization:

$\eta = 1, \ \sigma = 1, \ \alpha = \frac{1}{3}$

$\xi \approx 8.5$
Comparison to DSGE Models:

Marginal cost of investment (in utility) : $u'(C_t)$

Bachmann, Caballero and Engel (2006)
Comparison to DSGE Models:

Marginal cost of investment (in utility) : $u'(C_t)$

Bachmann, Caballero and Engel (2006)

$\eta = \infty, \ \sigma = 10, \ \alpha = \frac{1}{3}$
Comparison to DSGE Models:

Marginal cost of investment (in utility): $u'(C_t)$

Bachmann, Caballero and Engel (2006)

$\eta = \infty, \sigma = 10, \alpha = \frac{1}{3}$

$\xi \approx 64$
Conclusion:

- For long-lived capital goods, the intertemporal elasticity of substitution for investment timing is virtually infinite.

- Since firms are willing to re-time investment, the distribution of capital is not an important state variable.

- Neoclassical investment models also have a near infinite elasticity of investment timing. As a result, the two models exhibit nearly identical reactions to supply/demand shocks.

- Aggregate investment best analyzed with neoclassical models.