Labor market search and interest rate policy

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June 20, 2008
Growing interest in the role of labor market search and matching frictions in DSGE models with sticky prices

- Employment and output fluctuations
- Inflation dynamics
Motivation

✔ Growing interest in the role of labor market search and matching frictions in DSGE models with sticky prices

✘ Employment and output fluctuations

✘ Inflation dynamics

✔ But many monetary policy studies consider competitive labor markets

✘ Some exceptions focus on optimal policy with labor market frictions
With the labor market frictions, temporary demand shocks have opposing effect on present and future real marginal cost.
Motivation (cont.)

✔ With the labor market frictions, temporary demand shocks have opposing effect on present and future real marginal cost

✔ A temporary consumption decline

✘ reduces hiring cost and thus marginal cost of production temporarily

✘ reduces employment and output persistently
Motivation (cont.)

✔ With the labor market frictions, temporary demand shocks have opposing effect on present and future real marginal cost

✔ A temporary consumption decline
  ❌ reduces hiring cost and thus marginal cost of production temporarily
  ❌ reduces employment and output persistently

✔ Subsequent consumption recovery
  ❌ requires recovery of future employment and output from diminished level
  ❌ strongly raises future hiring cost and future MC
A real rate rise may increase inflation (expectations), suggesting that monetary policy may become a source of instability.
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What are monetary policy implications of the labor market frictions in terms of stability of REE?
Overview

✔ Sticky price model with labor market search and matching frictions
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✔ Determinacy analysis of three specifications of interest rate policy

✘ forward-looking policy
✘ current-looking policy
✘ backward-looking policy
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  ❌ forward-looking policy
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✔ Determinacy analysis under flexible forward-looking policy
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- Sticky price model with labor market search and matching frictions
- Determinacy analysis of three specifications of interest rate policy
  - forward-looking policy
  - current-looking policy
  - backward-looking policy
- Determinacy analysis under flexible forward-looking policy
- E-stability analysis of the indeterminacy problem
The model

Introduction

Model and main determinacy analysis

The model

The model: labor market
The model: labor market (cont.)
The model: monetary policy, market clearing
Log-linearized equilibrium conditions
Demand channel of monetary policy
Vacancy channel of monetary policy
Baseline calibration for quarterly model
Indeterminacy under forward-looking policy
Indeterminacy under forward-looking policy (cont.)
Determinacy under non-forward-looking policies

Monetary Authority

Households

Large family: unemployment insurance
Consume and save
Work or search for jobs

Retail firms

Wholesale firms
The model

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Households

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Labor only input, frictional labor market
Unmatched firms post vac. at cost $\gamma$
$n$ worker–firm matches produce
Competitive output market
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Monetary Authority

Households

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Work or search for jobs

Retail firms

Relative price z

Wholesale firms

Transform wholesale into diff. good
Calvo price stickiness
Monopolistically comp. output market

Labor only input, frictional labor market
Unmatched firms post vac. at cost $\gamma$
$n$ worker--firm matches produce
Competitive output market
Population size is normalized to 1. Unemployment rate is \( U_t = 1 - n_t \)
The model: labor market

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At the start of period $t$, a proportion $\rho$ of $n_{t-1}$ matches is destroyed.
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- Given \( u_t = 1 - (1 - \rho)n_{t-1} \) searching workers and \( v_t \) vacancies, 
  \( m_t = \psi u_t^{\xi} v_t^{1-\xi} \) new matches are created.
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✔ New matches become productive instantaneously, \( n_t = (1 - \rho) n_{t-1} + m_t \)

✔ Labor market tightness is measured by \( \theta_t = v_t / u_t \)
Firm's matching (vacancy filling) rate is

\[ q_t \equiv \frac{m_t}{v_t} = \psi \theta_t^{-\xi} \]
The model: labor market (cont.)

Firm's matching (vacancy filling) rate is
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Job creation condition
\[ \frac{\gamma}{q_t} = z_t - w_t + E_t \beta_{t,t+1} (1 - \rho) \frac{\gamma}{q_{t+1}} \]
The model: labor market (cont.)

- Firm’s matching (vacancy filling) rate is
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- Job creation condition
  \[ \frac{\gamma}{q_t} = z_t - w_t + E_t \beta_{t,t+1}(1 - \rho) \frac{\gamma}{q_{t+1}} \]

- Wage determination via Nash bargaining
  \[ w_t = \eta \left[ z_t + E_t \beta_{t,t+1}(1 - \rho) p_{t+1} \frac{\gamma}{q_{t+1}} \right] + (1 - \eta) b \]
The model: monetary policy, market clearing

✔ Monetary authority: Three specifications of interest rate policy ($\phi_{\pi} > 0$)

\[
\text{forward-looking: } \quad R_t = R\left(\frac{E_t \pi_{t+1}}{\pi}\right)^{\phi_{\pi}} \\
\text{current-looking: } \quad R_t = R\left(\frac{\pi_t}{\pi}\right)^{\phi_{\pi}} \\
\text{backward-looking: } \quad R_t = R\left(\frac{\pi_{t-1}}{\pi}\right)^{\phi_{\pi}}
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- forward-looking: $R_t = R(E_t\pi_{t+1}/\pi)^{\phi_{\pi}}$
- current-looking: $R_t = R(\pi_t/\pi)^{\phi_{\pi}}$
- backward-looking: $R_t = R(\pi_{t-1}/\pi)^{\phi_{\pi}}$

✔ Resource constraint: $y_t = Y_t + \gamma v_t$

✔ Retail goods market clearing: $Y_t(i) = C_t(i) \ \forall i \in [0, 1]$
Log-linearized equilibrium conditions

✔ Labor market conditions

\[
\hat{\theta}_t = \hat{v}_t + \frac{1 - u}{u} \hat{n}_{t-1}
\] : labor market tightness

\[
\hat{n}_t = (1 - \rho)\hat{n}_{t-1} + \rho(\hat{v}_t - \xi \hat{\theta}_t)
\] : employment law of motion

\[
\xi \hat{\theta}_t = \chi \hat{z}_t - \beta (1 - \rho) (1 - \eta \rho) (\hat{R}_t - E_t \hat{\pi}_{t+1}) + \beta (1 - \rho) (\xi - \eta \rho) E_t \hat{\theta}_{t+1}, \quad \chi > 0
\] : job creation

\[
\hat{n}_t = s_c \hat{C}_t + s_v \hat{v}_t
\] : resource constraint
Log-linearized equilibrium conditions

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+ \beta (1-\rho)(\xi - \eta p) E_t \hat{\theta}_{t+1}, \quad \chi > 0 \quad : \text{ job creation }
\]

\[
\hat{n}_t = s_c \hat{C}_t + s_v \hat{v}_t \quad : \text{ resource constraint }
\]

✔ The rest of conditions is the same as with frictionless labor market

\[
\hat{C}_t = E_t \hat{C}_{t+1} - \sigma^{-1}(\hat{R}_t - E_t \hat{\pi}_{t+1}) + (g_t - E_t g_{t+1}), \quad g_t = \rho_g g_{t-1} + \varepsilon_t
\]

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{z}_t
\]

\[
\hat{R}_t = \phi_\pi E_t \hat{\pi}_{t+j}, \quad j = -1, 0, 1
\]
Demand channel: \((\hat{R}_t - E_t \hat{\pi}_{t+1}) \uparrow \Rightarrow \hat{\pi}_t \downarrow\)
Demand channel of monetary policy

✔ Demand channel: \((\hat{R}_t - E_t\hat{\pi}_{t+1}) \uparrow \Rightarrow \hat{\pi}_t \downarrow\)

✔ Why? \((\hat{R}_t - E_t\hat{\pi}_{t+1}) \uparrow\)

\(\Rightarrow\) aggregate demand and production \(\downarrow\)

\(\hat{C}_t \downarrow: \hat{C}_t = E_t\hat{C}_{t+1} - \sigma^{-1}(\hat{R}_t - E_t\hat{\pi}_{t+1}) + \ldots\)

\(\hat{n}_t \downarrow: \hat{n}_t = s_c\hat{C}_t + s_v\hat{\nu}_t\)

\(\Rightarrow\) vacancies, labor market tightness and real MC \(\downarrow\)

\(\hat{\nu}_t \downarrow: \hat{n}_t = (1 - \rho)\hat{n}_{t-1} + \rho(\hat{\nu}_t - \xi\hat{\theta}_t)\)

\(\hat{\theta}_t \downarrow: \hat{\theta}_t = \hat{\nu}_t + \frac{1 - u}{u}\hat{n}_{t-1}\)

\(\hat{\zeta}_t \downarrow: \xi\hat{\theta}_t = \chi\hat{\zeta}_t - \beta(1 - \rho)(1 - \eta\eta)(\hat{R}_t - E_t\hat{\pi}_{t+1}) + \ldots\)

\(\Rightarrow\) inflation \(\downarrow\)

\(\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{\zeta}_t\)
Vacancy channel: 

\[ (\hat{R}_t - E_t \hat{\pi}_{t+1}) \uparrow \Rightarrow E_t \hat{\pi}_{t+1} \uparrow \& \hat{\pi}_t \uparrow \]
Vacancy channel of monetary policy

✔ Vacancy channel: \( (\hat{R}_t - E_t\hat{\pi}_{t+1}) \uparrow \Rightarrow E_t\hat{\pi}_{t+1} \uparrow \& \hat{\pi}_t \uparrow \)

✔ Why? \( (\hat{R}_t - E_t\hat{\pi}_{t+1}) \uparrow \)

⇒ expected recovery of demand

\[ E_t\hat{C}_{t+1} - \hat{C}_t \uparrow: E_t\hat{C}_{t+1} - \hat{C}_t = \sigma^{-1}(\hat{R}_t - E_t\hat{\pi}_{t+1}) + \ldots \]

⇒ expected vacancies, labor market tightness and real MC \( \uparrow \)
due to sluggish labor market adjustment

\[ E_t\hat{v}_{t+1} \uparrow: E_t\hat{n}_{t+1} - \hat{n}_t = -\rho\hat{n}_t + \rho (E_t\hat{v}_{t+1} - \xi E_t\hat{\theta}_{t+1}) \]

\[ E_t\hat{\theta}_{t+1} \uparrow: E_t\hat{\theta}_{t+1} = E_t\hat{v}_{t+1} + \frac{1 - u}{u} \hat{n}_t \]

\[ E_t\hat{z}_{t+1} \uparrow: \xi E_t\hat{\theta}_{t+1} = \chi E_t\hat{z}_{t+1} + \ldots \]

⇒ inflation \( \uparrow \)

\[ \hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{z}_t \]
### Baseline calibration for quarterly model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>relative risk aversion</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>consumption demand elasticity</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>retail firms’ probability of not reoptimizing prices</td>
</tr>
<tr>
<td>$\eta$</td>
<td>worker bargaining power</td>
</tr>
<tr>
<td>$\xi$</td>
<td>elasticity of matching</td>
</tr>
<tr>
<td>$q$</td>
<td>firm matching rate</td>
</tr>
<tr>
<td>$\rho$</td>
<td>job destruction rate</td>
</tr>
<tr>
<td>$U$</td>
<td>steady state unemployment rate</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>flow cost of a vacancy</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>AR(1) parameter for preferences shocks</td>
</tr>
</tbody>
</table>
Indeterminacy under forward-looking policy

Under baseline calibration, determinacy is ensured iff

\[ 1 < \phi_\pi < 1.16 \]
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In stark contrast to the case of frictionless labor market in which determinacy is ensured iff (Bullard & Mitra, 02; Woodford, 03)

\[ 1 < \phi_\pi < 1 + 2(1 + \beta)/\kappa = 25 \]
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✔ In the presence of the vacancy channel, forward-looking policy renders REE indeterminate
Indeterminacy under forward-looking policy (cont.)

✔ Expected growth of consumption but sluggish output adjustment ⇒ vacancy channel

✗ Output adjusts more sluggishly with small $\rho$, $\xi$ or large $U$

✗ Expected consumption grows faster with small $\sigma$
Indeterminacy under forward-looking policy (cont.)

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✔ Robustness: determinacy interval of $\phi_\pi$ for alternative parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Determinacy Interval of $\phi_\pi$</th>
</tr>
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<tbody>
<tr>
<td>$\rho = 0.07$</td>
<td>$1 &lt; \phi_\pi &lt; 1.13$</td>
</tr>
<tr>
<td>$\rho = 0.15$</td>
<td>$1 &lt; \phi_\pi &lt; 1.20$</td>
</tr>
<tr>
<td>$\xi = 0.235$</td>
<td>$1 &lt; \phi_\pi &lt; 1.11$</td>
</tr>
<tr>
<td>$\xi = 0.5$</td>
<td>$1 &lt; \phi_\pi &lt; 1.26$</td>
</tr>
<tr>
<td>$U = 0.12$</td>
<td>$1 &lt; \phi_\pi &lt; 1.10$</td>
</tr>
<tr>
<td>$U = 0.03$</td>
<td>$1 &lt; \phi_\pi &lt; 1.29$</td>
</tr>
<tr>
<td>$\sigma = 0.16$</td>
<td>$1 &lt; \phi_\pi &lt; 1.03$</td>
</tr>
<tr>
<td>$\sigma = 5$</td>
<td>$1 &lt; \phi_\pi &lt; 1.35$</td>
</tr>
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</table>
Determinacy under non-forward-looking policies

- ✔ Under baseline calibration, current-looking policy ensures determinacy as long as Taylor principle \( \phi_\pi > 1 \) is satisfied

- ✘ Same conditions as with frictionless labor market, because feedback from real MC on policy subdues real interest rate change
Determinacy under non-forward-looking policies

✔ Under baseline calibration, current-looking policy ensures determinacy as long as Taylor principle \( \phi_\pi > 1 \) is satisfied

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✔ Backward-looking policy ensures determinacy under active but not too strong policy responses

✘ Same conditions as with frictionless labor market, because feedback from real MC on policy subdues real rate change in next period
Prescriptions for indeterminacy problem

✔ Three prescriptions for indeterminacy problem induced by forward-looking policy

  a-1. Policy response to expected future output

  \[ \hat{R}_t = \phi_\pi E_t \hat{\pi}_{t+1} + \phi_Y E_t \hat{Y}_{t+1} \]

  a-2. Policy response to current output

  \[ \hat{R}_t = \phi_\pi E_t \hat{\pi}_{t+1} + \phi_Y \hat{Y}_t \]

  b. Policy response to unemployment

  \[ \hat{R}_t = \phi_\pi E_t \hat{\pi}_{t+1} - \phi_U \hat{U}_t \]

  c. Interest rate smoothing

  \[ \hat{R}_t = \phi_R \hat{R}_{t-1} + \phi_\pi E_t \hat{\pi}_{t+1} \]
Mild policy response to expected future output ameliorates indeterminacy problem
Policy response to current output overcomes indeterminacy problem
Policy response to current unemployment overcomes indeterminacy problem
Strong interest rate smoothing ameliorates indeterminacy problem
E-stability analysis of REE

Does forward-looking policy generate a unique E-stable fundamental REE even in cases of indeterminacy?

- E-stability as an REE selection criterion
- Focus on ‘fundamental’ (ie. Evans & Honkapohja’s (2001) ‘MSV’) RE solutions
- If fundamental REE is E-stable, it is least-squares learnable
Active but not too strong policy response solely to expected future inflation ensures a unique E-stable fundamental REE
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\[ 1 < \phi_\pi < 7.25 \]
Active but not too strong policy response solely to expected future inflation ensures a unique E-stable fundamental REE

$1 < \phi_\pi < 7.25$

In the case of frictionless labor market, Taylor principle is a necessary and sufficient condition for the unique E-stable REE (Bullard & Mitra, 2002). Here, it is no longer a sufficient condition.
**E-stability analysis of REE (cont.)**

- Active but not too strong policy response solely to expected future inflation ensures a unique E-stable fundamental REE

- $1 < \phi_{\pi} < 7.25$

- In the case of frictionless labor market, Taylor principle is a necessary and sufficient condition for the unique E-stable REE (Bullard & Mitra, 2002). Here, it is **no longer a sufficient condition**

- The indeterminacy problem is not critical from the perspective of E-stability
In the presence of search frictions, current- and backward-looking policies ensure determinacy of REE under same conditions as without the frictions, while forward-looking policy renders REE indeterminate.
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Indeterminacy is due to a vacancy channel of monetary policy that stems from the labor market frictions and renders inflation expectations self-fulfilling.
Concluding remarks

✔ In the presence of search frictions, current- and backward-looking policies ensure determinacy of REE under same conditions as without the frictions, while forward-looking policy renders REE indeterminate.

✔ Indeterminacy is due to a vacancy channel of monetary policy that stems from the labor market frictions and renders inflation expectations self-fulfilling.

✔ The indeterminacy can be overcome if forward-looking policy responds also to output or unemployment or contains interest rate smoothing.

✘ Policy-induced fluctuations in aggregate demand should be moderate to ensure determinacy.
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The indeterminacy is not critical from the perspective of learnability.