Ambiguity, Learning and Life-Cycle Portfolio Choice

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Introduction

- Abundant evidence documents systematic patterns in household portfolio allocation
- Some of this empirical evidence is hard to explain
- I construct a life-cycle portfolio choice model with learning over ambiguous stock returns
- I find that ambiguity + learning help explaining some otherwise puzzling features of the data
Works by Heaton and Lucas (2000), Ameriks and Zeldes (2002), Guiso et al. (2000) document a number of facts about portfolio choice

- Participation in the stock market is moderate ($\leq 50\%$), hump-shaped in age and increasing in wealth
- The portfolio share of stocks is moderate (50-60\%)
- The portfolio share of stocks is mildly increasing in age
- The portfolio share of stocks is mildly increasing in wealth
Theoretical literature: Portfolio choice

- Early theoretical work: Samuelson (1969), Merton (1971)
- More recent work trying to address some unsolved issues: Benzoni et al. (2004), Lynch and Tan (2004), Wachter and Yogo (2007)
Theoretical literature: Model Uncertainty and Learning


- Ambiguity + Learning: Epstein and Schneider (2005), Leippold et al. (2005), Miao and Ju (2007)

- Maenhout (2004) and Cagetti et al. (2003) use robust control approach in endowment and production economy respectively
The present research

- **Use standard framework**
  - Realistic life-cycle profile of income
  - Uninsurable idiosyncratic earnings risk
  - Consumption-saving decision and asset allocation decision between risk-free and risky financial asset
  - Market frictions

- **Add key new features**
  - Stock returns are perceived as ambiguous and agents are averse to ambiguity
  - Learning allows agents to reduce extent of ambiguity
  - Information about return process flows only to participating households
The model

• **Time structure**
  - Time is discrete
  - Finite horizon with uncertain lifespan

• **Preferences and Learning**
  - Max-min utility
  - Learning model follows Epstein and Schneider (2005, 2007)
  - Under the learning model used here the set of posteriors depends only on the past fraction of high returns
Income and assets

Labor income and pensions

- Earnings given by $Y_t = G(t)z_t$ where
  \[ \ln(z_t) = \rho \ln(z_{t-1}) + \varepsilon_t \]
- After retirement fixed pension benefit $Y_t = Y_{ss}$

Financial assets

- One period risk free bond with price $q$
- Risky stock with return $R_{t+1}^s$
- Minimum size of stock investment for participants and no borrowing / short sale constraints
The household’s optimization problem

Participation indirect utility

\[ V_{t}^{\text{part}}(X_t, z_t, \phi_t, n_t) = \max_{c_t, B_{t+1}, S_{t+1}} \min_{\rho_t \in \mathcal{P}_t} \left\{ u(c_t) + \right. \]
\[ + \beta \pi_{t+1} \mathbb{E} V_{t+1}(X_{t+1}, z_{t+1}, \phi_{t+1}, n_{t+1}) \left. \right\} \]

Subject to the following constraints and laws of motion:

\[ c_t + qB_{t+1} + S_{t+1} \leq X_t + Y_t \]
\[ X_{t+1} = B_{t+1} + R(s_{t+1})S_{t+1} \]
\[ \phi_{t+1}(s_{t+1}) = \frac{n_t \phi_t + s_{t+1}}{n_{t+1}} \]
\[ n_{t+1} = n_t + 1 \]
Non participation indirect utility

\[ V_{t}^{nopart}(X_t, z_t, \phi_t, n_t) = \max_{c_t, B_{t+1}} \left\{ u(c_t) + \beta \pi_{t+1} \mathbb{E} V_{t+1}(X_{t+1}, z_{t+1}, \phi_t, n_t) \right\} \]

Subject to the following constraints and laws of motion:

\[ c_t + q B_{t+1} \leq X_t + Y_t \]

\[ X_{t+1} = B_{t+1} \]

Value function

\[ V_{t}(X_t, z_t, \phi_t, n_t) = \max \left\{ V_{t}^{nopart}(X_t, z_t, \phi_t, n_t), V_{t}^{part}(X_t, z_t, \phi_t, n_t) \right\} \]
Parameter choice

**General parameters.** Mostly standard with two exceptions

- Equity premium set at 5.5 %
- Risk aversion coefficient = 3

**Learning parameters**

- $\lambda = 0.01$ implying long run “uncertainty” about equity premium of 64 basis points
- $\alpha$ (controls the speed of learning) is 0.2
- Number of observations used to form initial prior = 20
Results: No learning models

Figure: Conditional stock shares: Model without ambiguity and with ambiguity but no learning

- Participation rate: ≈ 42 % with cost of 12.5 and 1 % of average income respectively
- Worst-case equity premium 1%
Results: Baseline case with learning

Figure: Conditional stock shares

- Average participation ≈ 43 %
- Conditional share ≈ 58 %
Understanding the mechanism

**Figure:** Worst case equity premium

**Figure:** Optimal stock share decisions
Sensitivity analysis I: Initial ambiguity

- Participation rate: baseline ≈ 42%, low initial ambiguity ≈ 50.6%
- Conditional share: baseline ≈ 58.2%, low initial ambiguity ≈ 65.4%

Figure: Conditional stock shares
Sensitivity analysis II: Speed of learning

Figure: Conditional stock shares

- Participation rate: baseline $\approx 42\%$, faster learning $\approx 45.3\%$
- Conditional share: baseline $\approx 58.2\%$, faster learning $\approx 69.9\%$
Sensitivity analysis III: Risk aversion

Figure: Conditional stock shares

- Participation rate: baseline ≈ 42%, high RA ≈ 48%
- Conditional share: baseline ≈ 58.2%, high RA ≈ 36.3%
## Average conditional shares

<table>
<thead>
<tr>
<th></th>
<th>Wealth groups</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0-25</td>
</tr>
<tr>
<td>No ambiguity</td>
<td>n.a</td>
</tr>
<tr>
<td>No learning</td>
<td>n.a</td>
</tr>
<tr>
<td>Learning, baseline</td>
<td></td>
</tr>
<tr>
<td>Learning, LIA</td>
<td>67.9</td>
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<tr>
<td>Learning, faster</td>
<td>64.8</td>
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<tr>
<td>Learning, HRA</td>
<td>34.2</td>
</tr>
</tbody>
</table>
I have solved a life-cycle portfolio choice model with ambiguous stock returns and learning by ambiguity averse agents.

- Ambiguity per se reduces the average stock share but it makes it counter-factually decreasing over the life-cycle.
- Adding learning creates an increasing life-cycle profile of conditional stock shares.
- The conditional share is less declining over wealth but still not increasing as in the data.