Individual and Aggregate Money Demands: Theory and an Application to the Welfare Cost of Inflation

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Objective

How to model the demand for money?

Implications

How much does inflation cost society?
What is the optimum quantity of money?
What are the effects of monetary policy shocks?
Findings

Tractable model in which agents have different money demands: heterogeneity

Market segmentation

Use the model to compare predictions of endogenous and fixed market segmentation

Use the model to calculate the welfare cost of inflation

Silva (2008): calculates the effects of changes in the interest rate
Data: Real Balances

$M_1/PY$, United States, 1900-1997

Interest rate: short commercial paper rate
Data: Real Balances

$M1/PY$, United States, 1900-1997

Interest rate: short commercial paper rate
Model

General equilibrium Baumol-Tobin model
Intertemporal discount, infinite-lived agents, consumption smoothing
Assumptions common in other macroeconomic models
   But not made so far in a Baumol-Tobin model for money demand
   Jovanovic (1982), Romer (1986)
Transfer cost in goods
   Explicit about calibration
   Comparison of predictions and data
   Convergence after interest rate shocks
Model

Baumol-Tobin model: periodic exchanges of bonds for money

Implies endogenous transfer periods
  Endogenous segmentation

With fixed transfer periods
  Fixed segmentation
  Real balances increasing with the interest rate: counterfactual
  Robust to high intertemporal substitution? To intertemporal discounting?

Welfare cost of inflation
Agents need money to buy goods
  Cash-in-advance constraint: but agents decide when to exchange bonds for money
Bonds receive interest
Money does not receive interest
Asset Market
  Exchange bonds and money: transfer cost
General equilibrium
  Some agents exchange bonds for money
  Others do the opposite: money for bonds
  Deposit earnings from work in a brokerage account

Baumol-Tobin Model
Details

Preferences

\[ U(c) = \int_{0}^{\infty} e^{-\rho t} u(c(t))ds \]

where

\[ u(c) = \begin{cases} 
\frac{c^{1-1/\eta}}{1-1/\eta} & \text{for } \eta \neq 1 \\
\log c & \text{for } \eta \neq 1 
\end{cases} \]

\( \eta \) : Elasticity of Intertemporal Substitution
Details

At time zero,

\[ M_0 : \text{money} \]
\[ W_0 : \text{assets} \]

\[ F : \text{distribution of } (M_0, W_0) \]

CIA constraint

\[ \dot{M}(t, M_0, W_0) = -P(t)c(t, M_0, W_0), \quad t \neq T_1, T_2, \ldots \]

Agents produce \( Y \) units of the good in each period
Individual Maximization

Agents choose transfer times and consumption

\[
\max_{c,\{T_j\}} \sum_{j=0}^{\infty} \int_{T_j}^{T_{j+1}} e^{-\rho t} u(c(t)) dt
\]

subject to

\[
\int_{0}^{T_1} P(t)c(t) dt \leq M_0 \quad \sum_{j=1}^{\infty} Q(T_j) \left[ \int_{T_j}^{T_{j+1}} P(t)c(t) dt + P(T_j)\gamma Y \right] \leq W_0
\]
Closing the Model

Market Clearing for goods

\[ \int c(t, M_0, W_0; P) dF(M_0, W_0) + \gamma Y \times \#\text{Transfers}(t; P) = Y \]

\( F \) : distribution of \((M_0, W_0)\)
Market Clearing for money and bonds

Money: \[ M(t, M_0, W_0; P)dF(M_0, W_0) = M^S(t) \]

Bonds: \[ B_0(M_0, W_0; P)dF(M_0, W_0) = B_0^S \]

Government budget constraint

\[ B_0^S = \int_0^\infty Q(t)P(t)\frac{\dot{M}(t)}{P(t)} dt \]
Results

Proposition 1. Expression for the optimal interval between transfers $N$.

Proposition 2. $N$ exists and is unique for all $r$ (nominal interest rate), $\eta$ (EIS), $\rho$ (discounting) and $\gamma$ (transfer cost).

Proposition 3. $N$ is such that

$$\frac{\partial N}{\partial r} < 0 \quad \frac{\partial N}{\partial \gamma} > 0 \quad \lim_{\gamma \to 0} N = 0 \quad \lim_{r \to 0} rN > 0$$
Consumption within Holding Periods

\[ r = 4\% \text{ p.a.}, \, \rho = 3\% \text{ p.a.}, \text{ and } \gamma = 1.79 \]
Real Balances

Distribute agents along $[0,N)$

Find optimal consumption for each agent

Find optimal transfer times for each agent

Calculate $M(t, n) = \int_t^{T_{j+1}(n)} P(t)c(t, n), n \in [0, N)$

$\implies$ Expression for Individual Real Balances
Distribution of Real Balances

Steady state: agents are uniformly distributed along \([0,N)\)

They have real balances from 0 to \(m_H > 0\)

Obtain the distribution of real balances with the distribution of agents and the expression of individual real balances

The distribution of real balances along 0 to \(m_H\) is not uniform

It is close to a uniform when \(\eta\) decreases: \(c(t)\) constant
Distribution of Money Holdings

$r = 4\% \text{ p.a.}, \rho = 3\% \text{ p.a.}, \text{ and } \gamma \text{ calibrated for each } \eta$
Aggregate Real Balances

Sum individual real balances over $n \in [0,N)$

Obtain

$$m(r,Y,\rho,\eta,\gamma) = \frac{c_0(r,Y,\rho,\eta,\gamma)}{\rho - r(1 - \eta)} \left( \frac{1 - e^{-\eta r N(.)}}{\eta r N(.)} + e^{-\eta r N(.)} \frac{1 - e^{(r - \rho)N(.)}}{(r - \rho) N(.)} \right)$$
Data: Real Balances

$M1/PY$, United States, 1900-1997
Data and Predictions: Real Balances $M1/PY$, United States, 1900-1997

$\circ$: Geometric Mean of the Data
Calibration

\( \rho \): an interest rate of 3% p.a. implies zero inflation

\( \gamma \): money-income ratio passes through the geometric mean of the data

Implies \( \rho = 3\% \) p.a. and \( \gamma = 1.79 \)
Aggregate Real Balances

Endogenous $N$

$\eta = 0.1, 1, 10$
Aggregate Real Balances

Fixed $N$

- $\eta = 0.1, 1$
- $\eta = 0.1, 1, 10$


$\frac{M}{PY}$ (years)

Nominal interest rate (% p.a.)

$N$ (days)

Nominal interest rate (% p.a.)
Aggregate Real Balances

Fixed $N$

$\eta = 0.1, 1$

$\eta = 10$

$\eta = 50$

$\eta = 100$

$\eta = 0.1, 1, 10$

Aggregate Real Balances

Fixed $N$

$\eta = 0.1, 1$

$\eta = 10$

$\eta = 50$

$\eta = 100$

$\eta = 0.1, 1, 10$
Effects of Endogenous Segmentation: In Another Way

![Graph showing the relationship between Interest Elasticity of Money Demand and Elasticity of Intertemporal Substitution, $\eta$. The graph includes data points for different interest rates: r = 0.1% p.a. and r = 4% p.a. The y-axis represents the Interest Elasticity of Money Demand, ranging from -0.9 to 0.1, with grid lines at intervals of 0.1. The x-axis represents the Elasticity of Intertemporal Substitution, $\eta$, ranging from 0 to 100, with grid lines at intervals of 10. The graph includes markers for different scenarios, indicating changes in interest elasticity with varying elasticity of substitution.]
Effects of Endogenous Segmentation: In Another Way

Interest Elasticity of Money Demand

Elasticity of Intertemporal Substitution, $\eta$

-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1

N Fixed

N Endogenous

$N_{\text{Fixed}}$

$r = 0.1\% \text{ p.a.}$

$r = 4\% \text{ p.a.}$

$r = 0.1\% \text{ p.a.}$

$r = 4\% \text{ p.a.}$
Large $\gamma$?

$\gamma = 1.79 = 1.8$ working days per transfer

Implies $N = 180$ days with inflation of 1% p.a.:

Large?

No!
1. Transfers are not ATM withdrawals. They are exchanges from high-yielding assets to money. Evidence that agents trade these assets less than once a year.

2. Agents represent consumers and firms.

3. With 5 working days per week and 52 weeks per year, implies 1.38% of the total working time for financial transfers. About 30 minutes per week. Calibrated shopping time models find similar numbers. Ex Lucas (2000) finds 1% for transfers. Same number if we use 360 working days.

4. Note that we can interpret $\gamma$ in this model.
Optimum Quantity of Money

Optimal monetary policy

Set money growth at $-\rho$

Implies inflation $= -\rho$, and $r = 0$

Friedman rule

Optimum quantity of money

$r \rightarrow 0$ implies Real Balances equal to $\frac{1}{\rho} \frac{Y}{\rho} Y$

Real balances equal to the present value of future production
Welfare Cost of Inflation

Percentage income compensation to leave consumers indifferent between \( r > 0 \) and \( r = 0 \)

Result

\[
1 + w(r) = \left[ \left( 1 - \frac{\gamma}{N} \right) \frac{\eta r N}{1 - e^{\eta r N}} \right]^{-1} \left[ \left( 1 - \frac{\gamma}{N} \right) \frac{\eta r N}{1 - e^{\eta r N}} \right] \left[ \frac{r N}{r N} \frac{1 - e^{\tilde{r}N(1-\eta)}}{1 - e^{r N(1-\eta)}} \right]^{1 - \frac{1}{1 - 1/\eta}}
\]
Welfare Cost Relative to $r = 0\%$ p.a.
Welfare Cost Relative to $r = 3\% \text{ p.a.}$
Conclusions

Model with heterogeneity in money demand
Money demand changes explain the real effects of money
Endogenous segmentation has powerful effects
Predictions on real balances conform to data with $N$ endogenous
Real balances unresponsive to interest rates with $N$ fixed.
Even when $N$ is large
Inflation: effects on consumption variability and on financial transfers. The effects on financial transfers is larger.
END