Incomplete Information
In a Long Run Risks Model of Asset Pricing

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Introduction

Long run risks model of asset pricing pioneered by Bansal and Yaron (2004) shows great deal of promise in resolving several empirical puzzles in asset pricing.

Bansal (2007) provides a brief review of this literature.

Long run risks model well suited for exploration of effects of incomplete information.
Classical **incomplete information** setting studied in Dothan and Feldman (1986), Detemple (1986), Gennotte (1986), and more recently, in Brennan and Xia (2001).

More recent literature includes David (1997), Veronesi (1999), and Bidarkota et al. (2007).


Brandt et al. (2004) study effects of incomplete information in asset pricing model with recursive utility which does not capture long run risks.
Linear Gaussian conditionally homoskedastic long run risks model

with incomplete information

Variance of filter density becomes constant once Kalman filter reaches steady state.

Approximate analytical solution method of B-Y (2004) can then be used to solve model.

We characterize solution to such a model, calibrate to data, and study equilibrium implications.
**Incomplete information in the long run risks model**


Bansal and Shaliastovich (2008) endogenously generate jumps in stock prices in a long-run risks model where agents occasionally choose to incur a cost to learn the true value of relevant fundamentals in an incomplete information framework.

Croce et al. (2006) study implications of incomplete information in a long-run risks model on cross-sectional properties of stock returns and cash flow duration.
Organization

- Description of economic environment and asset pricing model
- Model solution
- Model calibration
- Analysis of model implied rates of return
2. Asset Pricing Model

Similar to one studied in Bansal and Yaron (2004).

First order condition for representative agent with Epstein and Zin (1989) and Weil (1989) recursive preferences:

\[
E_t \left[ \delta^\theta G_{t+1} \psi^{-\gamma(1-\theta)} R_{a,t+1} R_{i,t+1} \right] = 1 \tag{1}
\]

where: $0 < \delta < 1$ is time discount factor, $\psi \geq 0$ is IES, and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$ with $\gamma \geq 0$ being risk aversion parameter.
Analytical Approximations

\[ z_t \equiv \ln\left( \frac{P_{a,t}}{C_t} \right) \] - log price-consumption ratio

\[ r_{a,t+1} \equiv \ln\left( R_{a,t+1} \right) \] - continuous return

First-order Taylor’s series approximation:

\[ r_{a,t+1} \cong k_0 + k_1 z_{t+1} - z_t + g_{t+1} \]  \hspace{1cm} (2)

\( k_0 \) and \( k_1 \) are approximating constants - depend on average level of \( z \).
Define analogous quantities $z_{m,t} \equiv \ln \left( \frac{P_{m,t}}{D_t} \right)$ and $r_{m,t+1} \equiv \ln \left( R_{m,t+1} \right)$ on market portfolio.

$$r_{m,t+1} \approx k_{0,m} + k_{1,m}z_{m,t+1} - z_{m,t} + g_{d,t+1}$$  \hspace{1cm} (3)

Approximating constants $k_{0,m}$ and $k_{1,m}$ - depend on average level of $z_m$.

Logarithm of IMRS:

$$m_{t+1} = \theta \ln (\delta) - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1}$$  \hspace{1cm} (4)

$g_{t+1} \equiv \ln \left( G_{t+1} \right)$ - log of gross consumption growth rate.
2.2 Complete Information

\[ g_{t+1} = x_t + \sigma \eta_{t+1} \]  
\[ x_{t+1} - \mu = \rho (x_t - \mu) + \varphi_e \sigma e_{t+1} \]  
\[ g_{d,t+1} = \phi x_t + \varphi_d \sigma u_{t+1} \]  
\[ \eta_{t+1}, e_{t+1}, u_{t+1} \sim \text{iid } N(0,1) \]

\( x_t \) is interpreted as conditional mean growth rate of consumption, assumed known at time \( t \) and \( \sigma^2 \) is its conditional variance.
Model Solution

Under complete information $x_t$ is the state variable

Endogenous solution to price dividend ratios on claims to aggregate consumption and market returns can be fully characterized in terms of $x_t$.

\[ z_t = A_0 + A_1 x_t \quad \text{(6a)} \]

\[ z_{t,m} = A_{0,m} + A_{1,m} x_t. \quad \text{(6b)} \]

\[ A_1 = \frac{1 - \frac{1}{\psi}}{1 - k_1 \rho}, \quad A_{1,m} = \frac{\phi - \frac{1}{\psi}}{1 - k_{1,m} \rho}. \quad \text{(7)} \]
2.3 Incomplete Information

\[ g_t = x_t + \sigma \eta_t \quad (8a) \]
\[ x_t - \mu = \rho(x_{t-1} - \mu) + \phi_e \sigma e_t \quad (8b) \]
\[ g_{d,t} = \phi x_t + \phi_d \sigma u_t \quad (8c) \]
\[ \eta_t, e_t, u_t \sim \text{iid } N(0,1) \]

View \( x_t \) as unobservable state variable driven by dynamics in Equation (8b).

\( x_t \) can only be inferred through Bayesian filtering process.
Simplification

Assume investors ignore measurement Equation (8c) while trying to learn about $x_t$.

In deriving rates of return and risk premia on market portfolio, investors are assumed to fully take into account the process for dividend growth rates specified in Equation (8c).
Filtering

Let $Y_t \equiv \{g_1, g_2, \ldots, g_t\}$ denote history of consumption growth rates.

Filter density of $x_t$, $p(x_t \mid Y_t) \sim N(a_t, P_t)$

Once filter reaches steady state, filter variance $\bar{P}$
Kalman filter updating formula for filter mean:

\[ a_t = (1 - \rho)\mu + \rho a_{t-1} + \left( \frac{\rho^2 P + \phi_e^2 \sigma^2}{\rho^2 P + \phi_e^2 \sigma^2 + \sigma^2} \right) \{\rho (x_{t-1} - a_{t-1}) + \varphi_e \sigma e_t + \sigma \eta_t\}. \]  

(9a)

Steady state filter variance obtained by solving following Riccati equation for \( P_t = P_{t-1} = \bar{P} \):

\[ P_t = \left( \frac{\rho^2 P_{t-1} + \phi_e^2 \sigma^2}{\rho^2 P_{t-1} + \phi_e^2 \sigma^2 + \sigma^2} \right) \sigma^2. \]

(9b)
Model Solution

Under incomplete information, filter mean of \( x_t \) is relevant state variable

Endogenous solution to price dividend ratios on claims to aggregate consumption and market returns can be fully characterized in terms of filter mean.

\[
z_t = A_0 + A_1 a_t \quad \text{(10a)}
\]

\[
z_{t,m} = A_{0,m} + A_{1,m} a_t. \quad \text{(10b)}
\]

\[
A_1 = \frac{\rho (1 - 1/\psi)}{1 - k_1 \rho}, \quad A_{1,m} = \frac{\rho (1 - 1/\psi)}{1 - k_{1,m} \rho}. \quad \text{(11)}
\]
Model-Implied Rates of Return

In incomplete information case, relevant state variable is filter mean $a_t$, not unobservable $x_t$.

Using Bansal and Yaron (2004) methods, one can derive expressions for relevant moments of endogenous variables of interest.
Table 1. Model Parameterization

<table>
<thead>
<tr>
<th>Preference Parameters</th>
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<tbody>
<tr>
<td>$\delta$</td>
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<td>$\gamma$</td>
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<td>$\psi$</td>
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<tr>
<th>Parameters of Stochastic Process for Consumption</th>
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<tr>
<td>$\mu$</td>
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<td>$\rho$</td>
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<td>$\sigma$</td>
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<td>$\varphi_e$</td>
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<table>
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<tr>
<th>Parameters of Stochastic Process for Dividends</th>
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<tr>
<td>$\phi$</td>
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<td>$\varphi_d$</td>
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Table 2. Asset Pricing Implications

<table>
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<tr>
<th></th>
<th>( E(R_m - R_f) )</th>
<th>( E(R_f) )</th>
<th>( \sigma(R_m) )</th>
<th>( \sigma(R_f) )</th>
<th>( \sigma(p-d) )</th>
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<td><strong>Complete</strong></td>
<td>( \gamma ) ( \psi )</td>
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<td></td>
<td></td>
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<tr>
<td>Information</td>
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<td>13.43</td>
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<td>17.82</td>
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