Equilibrium Indeterminacy and Asset Price Fluctuation in Japan:
A Bayesian Investigation*

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Abstract

This paper investigates sources of asset price fluctuation and a long-lasting recession in Japan using an estimated financial accelerator model. For explicit treatment of expectational beliefs, the model is analyzed over the parameter space where the equilibrium can be indeterminate. We show that indeterminacy arises if the financial accelerator effect is sufficiently large. According to our Bayesian estimation results, Japan’s economy was affected by sunspots. But the contribution of the sunspots to the asset price volatility was low. Rather, net worth and cost shocks drove the asset price fluctuation. We also find that the sunspots substantially affected the capital investment. (100 words)

Keywords: Asset Price, Financial Accelerator, Indeterminacy, Sunspot Shocks, Bayesian Analysis

JEL Classification: C11, C62, E30, E44

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1 Introduction

In the late 1980s, Japan’s economy experienced the rapid expansion and the sharp downfall in asset prices, which have been often considered as non-fundamental phenomena. The U.S. economy also suffered the remarkable decline of stock prices in 2000. But, unlike the recent experience in the U.S., the adjustment of the Japan’s asset prices led to a long-term economic slowdown (see Figure 1). In this paper, we investigate causes of boom and bust in asset prices followed by the prolonged sever recession in Japan.

About Japan’s asset price fluctuation in the late half of the 1980s, Okina and Shiratsuka (2003) have noted that it was not a rational bubble as modeled in Blanchard and Watson (1982) but that it was characterized by excessively optimistic expectations with respect to future economic fundamentals. Therefore, it should be essential to consider a role of such beliefs for an analysis of asset price movements. Almost all the literature that studies non-fundamental asset price fluctuation, however, deals with its source just as an exogenous shock, which does not represent agents’ beliefs in a formal and explicit way. The primary contribution of this paper is to investigate sources of the asset price fluctuation in the rational expectations framework by allowing sunspots, which are non-fundamental disturbances including expectational beliefs, to affect economic dynamics.

The recent macroeconomics has developed theoretically and empirically appealing ways of modeling interaction between asset prices and economic fundamentals. Bernanke and Gertler (1989) have emphasized that the existence of asymmetric information between lenders and borrowers gives rise to an agency problem. In their model, agency costs make external funding sources for firms more expensive than internal sources. As a consequence, a firm’s balance sheet affects the cost of finance. Carlstrom and Fuerst (1997) have incorporated the agency problem into a real business cycle framework and demonstrated the quantitative importance of credit frictions. Bernanke, Gertler, and Gilchrist (1999) (henceforth BGG) have combined nominal rigidities with the agency cost model and developed a tractable dynamic general equilibrium model. In contrast to Carlstrom and Fuerst (1997) model, the BGG model exhibits a countercyclical external finance premium\(^1\), and hence the empirical literature has favored the latter model\(^2\). Based on their mechanism, a rise in asset prices improves balance sheets of firms and declines the external finance premium, which stimulates investment. Using the BGG model,

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\(^1\)For comparison and implication of a Carlstrom and Fuerst type model and a BGG type model, see Faia and Monacelli (2005).

\(^2\)In recent studies, Christensen and Dib (2006) have estimated a BGG type model using a maximum likelihood procedure in order to assess the importance of credit market frictions. Based on Japan’s economic data, Fukunaga (2002) has calibrated the BGG model to explain the large volatility of Japan’s corporate investment. Fuchi, Muto, and Ugai (2005) have estimated a modified version of the BGG model by means of GMM and historically evaluated the financial accelerator effects in Japan.
Bernanke and Gertler (1999) have analyzed effects of an asset price bubble. They have, however, assumed a stochastic process of the bubble exogenously, as is the case with the other literature. Under the rational expectations assumption, agents can fully coordinate with such an exogenous process if the equilibrium is determinate. In their approach, therefore, agents' beliefs are not specifically taken into consideration.

A distinctive feature of our analysis is that the BGG model is investigated in the parameter space where sunspot shocks can affect the equilibrium dynamics, i.e., the model has properties of equilibrium indeterminacy. This assumption is motivated by the following three reasons. First, a property of indeterminacy allows for explicit treatment of expectational beliefs. According to a solution strategy developed by Blanchard and Kahn (1980), Sims (2002), and Lubik and Schorfheide (2003), indeterminacy of rational expectations equilibria arises when associated rational expectations errors are not fully pinned down by fundamental disturbances. Hence, the expectations errors under indeterminacy can be specified by belief shocks. Second, the model under indeterminacy may be well fitted to the data. Generally, models under indeterminacy exhibit more persistent dynamics than those under determinacy. Hence, the model under indeterminacy may be a good candidate for explaining the long-lasting severe recession after the adjustment of asset prices in Japan. Last but not least, if we estimated the model only under determinacy, the financial accelerator effect might be underestimated. This motivation comes from our finding; that is, equilibrium indeterminacy arises if the financial accelerator effect is sufficiently large, regardless of an aggressive monetary policy. This paper is, to the best of our knowledge, the first attempt to consider indeterminacy generated by the financial accelerator mechanism.

Another feature of this paper is that the financial accelerator model is estimated using a Bayesian method in order to empirically examine the origin of the asset price fluctuation and the prolonged recession in Japan\(^3\). Bayesian estimation strategies help to estimate dynamic stochastic general equilibrium (DSGE) models with cross-equation restrictions, coping well with misspecification and identification problems. Furthermore, following Lubik and Schorfheide (2004), we estimate distributions of the structural parameters over the parameter space in which the equilibrium can be both determinate and indeterminate. Since sunspots come into effect only under indeterminacy, this estimation strategy allows us to test whether sunspots can affect the equilibrium dynamics and to what extent sunspots have caused economic fluctuations if they have existed.

The main findings are as follows. According to our estimation results, Japan’s economy had been affected by sunspots but the contribution of the sunspot shock to the asset price volatility

\(^3\)Neri (2004) has estimated a model with Carlstrom and Fuerst (1997) style credit frictions by Bayesian techniques. Queijo (2005) has estimated a modified version of the BGG model using Bayesian methods. Both papers aim to evaluate the importance of frictions in credit markets for business cycles in the U.S. or the Euro area.
had been negligible. Rather, the asset price fluctuation had been mainly driven by shocks on firm’s net worth and cost shocks. This result is new since it contrasts with the standard discussion that the rapid expansion of Japan’s asset prices in the late 1980s is generated by optimistic beliefs of the agents. On the other hand, the sunspots had driven capital investment in substantial degree. This finding provides a novel view of investment fluctuations since the previous studies attempt to explain the volatility of investment only by fundamental factors.

This paper is structured as follows. The next Section presents a financial accelerator model that is used for our analysis. In Section 3, we review a full set of solutions for rational expectations models and how agents’ expectational beliefs are related with sunspots. Also, we investigate the sources of indeterminacy in the financial accelerator model. Section 4 explains the Bayesian estimation methodology. In Section 5, our benchmark estimation results are presented and we explore the driving forces of asset price fluctuations in Japan. Section 6 is to check robustness of our empirical analysis. Section 7 is the conclusion.

2 The Model

The model is based on the BGG model, which has been considered as one of the standard DSGE models that analyze macroeconomic fluctuations associated with credit market frictions and asset prices. The model is a standard dynamic New Keynesian model modified to allow for financial accelerator effects.

Infinitely lived households work, consume, and save. Firms are owned by entrepreneurs who have finite expected life, which prevents them from reaching an entirely self-financing steady state. Firms purchase the stock of physical capital, financing through internally generated funds and by borrowing from the public. Their accumulated capital is used in combination with hired labor to produce the output. A government conducts fiscal and monetary policy. There is no foreign sector. Following Calvo (1983), BGG assume the staggered nominal price settings, which allow monetary policy to have real effects.

Credit market frictions are characterized by asymmetric information and agency problem between borrowers and lenders. The existence of credit frictions leads to a financial accelerator that magnifies and propagates shocks to economic dynamics. In particular, credit market frictions cause external finance premium, i.e. external financing becomes more costly than internal finance. This premium affects the overall cost of capital and, hence, firms’ investment schedules. A positive shock to the asset price, for example, improves balance sheets of the firms and declines the external finance premium, which stimulates investment. Also, endogenous firms’ net worth gives rise to additional feedback effects of the shock.

\[4\text{This effect of asset price is similar to the credit-cycle phenomenon stressed by Kiyotaki and Moore (1997).}\]
2.1 Households

The representative households infinitely live and derive utility from consumption $C_t$, real money balances $M_t/P_t$, and leisure $1 - H_t$. They maximize the following expected utility function:

$$E_t \sum_{k=0}^{\infty} \beta^k \left[ \log (C_{t+k}) + \mu \log \left( \frac{M_{t+k}}{P_{t+k}} \right) + \eta \log (1 - H_{t+k}) \right],$$

where $\beta \in (0, 1)$ is the discount factor, and $\mu > 0$ and $\eta > 0$ are associated with elasticities of substitution against consumption. The budget constraint is:

$$C_t + D_t + \frac{M_t}{P_t} \leq W_t H_t + R_{t-1} D_{t-1} + \frac{M_{t-1} - T_t + V_t}{P_t},$$

where $D_t$ is real deposits in the financial intermediary, $W_t$ is the real wage, $R_t$ denotes the real interest rate, $T_t$ is a lump-sum tax, and $V_t$ is dividends received from retailers.

The first order conditions for the households' optimization problem are:

$$\frac{1}{C_t} = \beta \frac{1}{E_t C_{t+1}} R_t,$$

$$\frac{M_t}{P_t} = \mu C_t \frac{R^n_t}{R^n_t - 1},$$

$$W_t \frac{1}{C_t} = \eta \frac{1}{1 - H_t},$$

where $R^n_t$ is the gross nominal interest rate.

2.2 Entrepreneurs

Entrepreneurs manage firms that produce wholesale goods. At the end of each period, in order to produce the goods, the entrepreneurs purchase capital $K_{t+1}$ at the price $Q_t$ from capital producers. The entrepreneurs finance the capital from financial intermediaries in addition to their own net worth $N_{t+1}$. Thus, the amount of their borrowing is $Q_t K_{t+1} - N_{t+1}$.

The entrepreneurs’ demand for capital depends on marginal return and the marginal financial cost. Assuming that they are risk-neutral, the interest rate on external funds $F_t$ and the marginal productivity of capital $R^k_t$ satisfy:

$$F_t = \frac{R^k_t + (1 - \delta) Q_t}{Q_{t-1}},$$

where $\delta \in (0, 1)$ is the depreciation rate of capital.
It is assumed that, due to the existence of credit-market imperfections, external finance is more expensive than internal funds. BGG have shown that the existence of imperfect information between lenders and borrowers gives rise to an agency problem and have characterized optimal contract such that external finance premium depends on the borrower’s leverage ratio $N_{t+1}/Q_tK_{t+1}$. Consequently, the external finance premium, i.e. the ratio of the marginal external financing cost to the riskless interest rate, satisfies the following optimality condition:

$$\frac{E_tF_{t+1}}{R_t} = S\left(\frac{N_{t+1}}{Q_tK_{t+1}}\right),$$

with $N_{t+1} \leq Q_tK_{t+1}$, $S' (\cdot) < 0$, $S (1) = 1$.

We assume that each entrepreneur survives to the next period with a constant probability $\gamma$\textsuperscript{5}. Then, the aggregate entrepreneurial net worth evolves according to:

$$N_{t+1} = \gamma [F_tQ_{t-1}K_t - E_{t-1}F_t (Q_{t-1}K_t - N_t)] + (1 - \gamma) X_t,$$

where $F_t$ is the ex post real return of capital at $t$ and $E_{t-1}F_t$ is the ex ante cost of borrowing at $t - 1$ toward $t$. $X_t$ is the transfer to the new generation of entrepreneurs to begin operations. For simplicity, we ignore $X_t$ hereafter, while BGG deal with $X_t$ explicitly by introducing entrepreneurs consumption\textsuperscript{6}. Instead, a net worth shock $NX_t$ is introduced, which follows the first order autoregressive process:

$$\log (NX_t) = (1 - \rho_n) \log (NX) + \rho_n \log (NX_{t-1}) + \varepsilon_{nt},$$

$$\varepsilon_{nt} \sim N(0, \sigma^2_n),$$

where $\rho_n \in (0, 1)$ and $NX > 0$ is the steady state level of $NX_t$. $\varepsilon_{nt}$ is a serially uncorrelated random disturbance.

Each entrepreneur maximizes the following expected discounted profits:

$$E_t \sum_{k=0}^{\infty} \Lambda_{t,k} \left(Y_{t+k} - R_{t+k}^k K_{t+k} - W_{t+k} H_{t+k}\right),$$

subject to the entrepreneurs’ production function:

$$Y_t = K_t^\alpha (A_t H_t)^{1-\alpha},$$

where $\alpha \in (0, 1)$, $\Lambda_{t,k} \equiv \beta C_t/C_{t+k}$ is the entrepreneur’s discount rate, and $Y_t$ is the real output. Total factor of productivity $A_t$ is assumed to follow the first-order autoregressive process:

$$\log (A_t) = (1 - \rho_a) \log (A) + \rho_a \log (A_{t-1}) + \varepsilon_{at},$$

\textsuperscript{5}This assumption precludes the possibility that the entrepreneurs are fully self-financed so that credit markets play no role in the model.

\textsuperscript{6}This modification does not change the essential dynamics of the model.
\[ \varepsilon_{at} \sim N(0, \sigma_a^2), \]

with \( \rho_a \in (0, 1) \). \( A > 0 \) denotes the steady state level of \( A_t \) and is normalized to unity. \( \varepsilon_{at} \) is serially uncorrelated over time. The first order conditions of the above problem are:

\[ R_t^k = \alpha \frac{Y_t}{K_t} MC_t \]
\[ W_t = (1 - \alpha) \frac{Y_t}{H_t} MC_t \]
\[ Y_t = A_t K_t^\alpha (H_t)^{1-\alpha}, \]

where \( MC_t \) is the Lagrange multiplier of the constraint and plays the role of the real marginal cost of production.

### 2.3 Capital Producers

Capital producers combine investment \( I_t \) and existing depreciated capital stock \((1 - \delta) K_t\) to produce new capital goods sold at the end of period \( t \). They are assumed to face capital adjustment costs, so that the capital accumulation evolves according to:

\[ K_{t+1} = (1 - \delta) K_t + \Phi \left( \frac{I_t}{K_t} \right) K_t, \]

with \( \Phi' \cdot > 0, \Phi'' \cdot < 0 \), and \( \Phi (I/K) = 1 \) where \( I \) and \( K \) without the subscript \( t \) denote their steady state values.

Assuming that the capital producers have to make their investment decision one period in advance, their optimization problem is to maximize the expected profits:

\[ E_t \left[ Q_{t+1} \Phi \left( \frac{I_{t+1}}{K_{t+1}} \right) K_{t+1} - I_{t+1} \right]. \]

Then, the first order condition is:

\[ E_t Q_{t+1} = E_t \left[ \Phi' \left( \frac{I_{t+1}}{K_{t+1}} \right) \right]^{-1}. \]

### 2.4 Retailers

Retailers purchase the wholesale goods at a price that equals to nominal marginal costs \( P_t MC_t \) and differentiate them at no cost. They sell these differentiated retail goods on a monopolistically competitive market. Following Calvo (1983), the retailers are assumed to have an opportunity to change their prices in a given period only with probability \( 1 - \phi \). Each retailer \( j \) chooses his price \( P_t^*(j) \) to maximize expected discounted profits, given by:

\[ E_t \sum_{k=0}^{\infty} \phi^k \left[ \Lambda_{t,k} \frac{P_t^*(j) - P_{t+k} MC_{t+k}}{P_{t+k}} Y_{t+k}^*(j) \right], \]
subject to\footnote{The demand curve that each retailer faces is derived from the definition of aggregate demand as the composite of individual retail goods and the corresponding prices in the framework of Dixit and Stiglitz (1977):}
\[ Y_{t+k}(j) = \left[ \frac{P^*_t(j)}{P_{t+k}} \right]^{-\theta} Y_{t+k}, \]
where $\Lambda_{t,k} \equiv \beta^k C_t/C_{t+k}$ is the retailer’s discount rate. Then, the first order condition is:
\[ P^*_t(j) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} \phi^k \Lambda_{t,k} Y_{t+k}(j) P_{t+k} M C_{t+k}}{E_t \sum_{k=0}^{\infty} \phi^k \Lambda_{t,k} Y_{t+k}(j) P_{t+k}}. \]
The aggregate price is given by:
\[ P_t = \left[ \phi P_{t-1}^{1-\theta} + (1 - \phi) P_t^{*1-\theta} \right]^{1/\theta}. \]
Linear approximation around the steady state of $P_t$ and $P^*_t$ gives New Keynesian Phillips curve as follows:
\[ \pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \beta \phi) (1 - \phi)}{\phi} M C_t + z_t, \]
where the lowercase variables denote percentage deviations from their steady state values and $\pi_t \equiv p_t - p_{t-1}$. Note that $z_t$ is interpreted as a cost shock, which follows the first-order autoregressive process:
\[ z_t = \rho_z z_{t-1} + \varepsilon_{zt}, \]
\[ \varepsilon_{zt} \sim N(0, \sigma^2_z), \]
where $\rho_z \in (0, 1)$ and $\varepsilon_{zt}$ is a serially uncorrelated random disturbance.

\section*{2.5 Government Sector and Monetary Policy Rule}
Since there is no foreign sector, the aggregate expenditures consist of consumption, investment, and government expenditure:
\[ Y_t = C_t + I_t + G_t, \]
where government expenditure $G_t$ is financed by money creation and lump-sum taxes as follows:
\[ G_t = \frac{M_t - M_{t-1} + T_t}{P_t}. \]
\(G_t\) follows the first-order autoregressive process:

\[
\log (G_t) = \rho_g \log (G_{t-1}) + \varepsilon_{gt},
\]

\(\varepsilon_{gt} \sim N(0, \sigma_g^2)\),

where \(\rho_g \in (0, 1)\) and \(\varepsilon_{gt}\) is a serially uncorrelated random disturbance. Note that, in the present model, \(\varepsilon_{gt}\) works as a demand shock to the economy.

The central bank adjusts the nominal interest rate \(R^n_t\) in response to inflation \(\pi_t\) and output \(Y_t\). The monetary policy rule is of the following form:

\[
\log \left( \frac{R^n_t}{R^n_{t-1}} \right) = \rho_r \log \left( \frac{R^n_t}{R^n_{t-1}} \right) + (1 - \rho_r) \left[ \psi_\pi \log \left( \frac{\pi_t}{\pi} \right) + \psi_y \log \left( \frac{Y_t}{Y} \right) \right] + \varepsilon_{rt},
\]

\(\varepsilon_{rt} \sim N(0, \sigma_r^2)\),

with \(\rho_r \in (0, 1)\). \(\varepsilon_{r,t}\) is a serially uncorrelated exogenous policy shock which can be interpreted as an unsystematic component of the monetary policy.

## 3 Sunspot Solution and Sources of Indeterminacy

A distinctive feature of our analysis is that the financial accelerator model is analyzed over the parameter space where sunspot shocks can affect the equilibrium dynamics. In this section, first, a full set of sunspot solution of the linear rational expectations system is presented in a general setting. Next, we show how agents’ expectational beliefs are related with sunspots. In the last subsection, we analyze the sources of indeterminacy in the financial accelerator model.

### 3.1 Sunspot Solution

In solving a rational expectations system, we follow the approach of Lubik and Schorfheide (2003), which provides a full set of non-unique solutions in linear rational expectations models by extending the solution algorithm developed by Sims (2002). To obtain a canonical form of rational expectations model, endogenous forecast errors are defined as follows:

\[
\eta_{t+1} = z_{t+1}^* - E_t z_{t+1}^*,
\]

where \(z_t^*\) is a vector of endogenous variables associated with rational expectations. In the present model, \(z_t^*\) consists of \(c_t, i_t, \pi_t, f_t, r^k_t, \) and \(q_t\). Then, the system can be written in the canonical form:

\[
\Gamma_0 (\theta) z_{t+1} = \Gamma_1 (\theta) z_t + \Psi_0 (\theta) \varepsilon_{t+1} + \Pi_0 (\theta) \eta_{t+1},
\]  

\(8\)Sims’ solution method generalizes the technique in Blanchard and Kahn (1980) and characterizes one particular solution in the case of indeterminacy.

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where \( \Gamma_0, \Gamma_1, \Psi_0 \) and \( \Pi_0 \) are the conformable matrices of coefficients that depend on the structural parameters \( \theta \), \( z_t \) is a stacked vector of endogenous variables and those with expectations at \( t \), and \( \varepsilon_t \) is a vector of fundamental shocks.

According to Lubik and Schorfheide (2003), the full set of sunspot solution is:

\[
z_t = \Gamma_0 (\theta) z_{t-1} + \Psi_0 (\theta, \tilde{M}) \varepsilon_t + \Pi_0 (\theta, M_\zeta) \zeta_t,
\]

(2)

where \( \tilde{M} \) and \( M_\zeta \) are arbitrary matrices, and \( \zeta_t \) is a vector of sunspot shocks, which are non-fundamental stochastic disturbances. If the equilibrium is determinate, the solution (2) is reduced to:

\[
z_t = \Gamma^D (\theta) z_{t-1} + \Psi^D (\theta) \varepsilon_t.
\]

(3)

For derivation of the solution (2) and (3), see Appendix C.

The solution (2) has two important features under indeterminacy. First, business cycle fluctuations are generated not only by fundamental shocks but also by sunspot shocks. Second, the equilibrium representation cannot be unique due to the arbitrary matrices \( \tilde{M} \) and \( M_\zeta \), i.e. the model has multiple solutions and different solutions may exhibit different economic dynamics. Therefore, in order to apply the sunspot solution to an economic analysis, we need to specify \( \tilde{M} \) and \( M_\zeta \) and to select a particular equilibrium path from an infinite number of equilibria; otherwise, any paths can be considered as the equilibrium solution\(^9\). In this paper, following Lubik and Schorfheide (2004), the arbitrary matrices are estimated using Bayesian techniques.

### 3.2 Interpretation of Sunspots: Belief Shocks

The hypothesis tested in this paper is whether the agents’ beliefs have affected the economic fluctuations, in particular, asset price movements. In this subsection, we show how agents’ expectational beliefs are related with sunspots. For this purpose, we introduce belief shocks that lead to a revision of forecasts. Formally, belief shocks are defined as:

\[
z_{t+1}^* = E_t z_{t+1}^* + \zeta_{t+1}^b + \tilde{\eta}_{t+1},
\]

(4)

where \( \zeta_{t+1}^b \) is a vector of belief shocks associated with the endogenous variables \( z_{t+1}^* \), and \( \tilde{\eta}_{t+1} \) are remaining forecast errors after the revision.

Then, the same approach presented above gives the solution with belief shocks:

\[
z_t = \Gamma (\theta) z_{t-1} + \Psi (\theta, \tilde{M}) \varepsilon_t + \Pi^b (\theta, M_\zeta^b) \zeta_t^b.
\]

(5)

See Appendix C for the derivation. Notice that this solution form corresponds to (2) if we normalize the impact of sunspots \( \Pi^b (\theta, M_\zeta^b) \) appropriately. Thus, sunspots can be interpreted as belief shocks to rational expectations.

\(^9\)Some plausible restrictions for the arbitrary parameters are found in Lubik and Schorfheide (2003, 2004).
Although the solution (5) can help us interpret sunspot shocks, the effects of the belief shocks are not clear due to the arbitrary matrix $M_\zeta$. Also, as argued in Lubik and Schorfheide (2003), different realization of the belief shock $\zeta^b_t$ can generate the same equilibrium dynamics since the dimension of $\zeta^b_t$ is usually greater than the degree of indeterminacy. Therefore, we continue our analysis by exploiting the solution form (2) with the dimension of the sunspot shock $\zeta_t$ being one. Then, $\zeta_t$ is considered as a reduced form sunspot shock in a sense that it contains beliefs associated with all the expectational variables.

### 3.3 Sources of Indeterminacy

It is known that the determinacy properties of a prototypical New Keynesian model depend on the monetary policy parameters. In particular, when the monetary policy follows a current inflation targeting rule, the equilibrium is unique if the monetary authority raises the nominal interest rate more than 1 percent in response to a 1 percent increase in the inflation rate\textsuperscript{10}. Recent studies\textsuperscript{11}, however, show that such condition does not always hold if some distortions or externalities in the economy are introduced to the model. In this subsection, it is demonstrated that the parameters associated with financial accelerator mechanism play a crucial role in determining determinacy and indeterminacy of the equilibrium.

The condition for determinacy is related to the number of unstable eigenvalues of the system and the number of endogenous forecast errors. Due to the high dimensionality of our present model, we cannot analytically derive the determinacy condition. Therefore, regions of determinacy in the parameter space are characterized using numerical simulations. To illustrate the relationship between the monetary policy coefficient on inflation $\psi_\pi$ and the parameters associated with financial accelerator effects $\nu$, the policy coefficient on output $\psi_y$ is calibrated to zero and the other parameters than $\psi_\pi$ and $\nu$ are fixed according to the prior mean for the estimation as in Table 1.

Figure 2 depicts the determinacy and indeterminacy region with respect to $\psi_\pi$ and $\nu$. The parameter $\nu$, the elasticity of external finance premium to the firms' leverage, characterizes the financial accelerator effects, which quantifies the importance of credit frictions and the balance sheet channel for investment decision. As a limiting case, if $\nu$ is zero, the external financing cost is always equal to the riskless interest rate and hence the firms' leverage ratio does not play any role in their financial contracts. The figure shows that, for small values of $\nu$, the same determinacy condition as a simple New Keynesian model carries over to the financial accelerator model. But, for high values of $\nu$, the economy always exhibits indeterminacy properties, regardless of any

\textsuperscript{10}See, for instance, Bullard and Mitra (2002) or Woodford (2003).

\textsuperscript{11}These include Benhabib, Schmitt-Grohé, and Uribe (2001), Carlstrom and Fuerst (2001), and Dupor (2001) among others.
aggressive monetary policies against inflation.

What is the intuition for this result? In addition to the standard transmission mechanism in a New Keynesian framework, shocks in the present model have a substantial impact on the firms’ net worth and hence their external financial costs. The change in financial condition affects the firms’ investment plans and their output. Therefore, the financial accelerator mechanism amplifies the response of the economy to shocks. This amplification effect has some similarities with models with increasing returns to scale and externalities, which are considered as major sources of indeterminacy in dynamic general equilibrium models. Suppose that the agents change their expectations and they come to believe that the investment will increase. A boost of the economy triggered by such expectations causes inflation, which leads the monetary authority to raise the interest rate. Without the financial accelerator mechanism, the contractionary monetary policy discourages the further investment and hence the initial expectation turns out to be invalid. However, with sufficiently strong financial accelerator effect, the boom driven by the expectation generates an additional increase in net worth, which lowers the external finance premium for firms. If the decrease in financial cost is dominant over the increase in the riskless interest rate, actual investment will increase and validates the initial expectation.

This explanation is illustrated by Figure 3, which depicts the relationship between $\nu$ and the steady state external finance premium $S$. In this simulation, $\psi_{\pi}$ is calibrated as its prior mean in Table 1. The figure shows that the higher values of $S$ lead to determinacy even if $\nu$ is large. Higher $S$ implies that the external finance is more costly and thus the ratio of capital owned by the firms to output is lower at the steady state. In such a case, the output amplification effects associated with the financial accelerator mechanism become smaller, which leads the equilibrium to be determinate.

4 Estimation Strategy

For our empirical investigation, the distributions of the structural parameters are estimated using Bayesian techniques. One of the important features of our analysis is that the parameters are estimated over the parameter space in which the equilibrium can be both determinate and indeterminate. Following Lubik and Schorfheide (2004), we begin with a review about how the inferences are made in such a parameter region. Next, we describe the data used for estimation and explain the prior distributions of the parameters.

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12See Benhabib and Farmer (1999) for a comprehensive survey.
4.1 Bayesian Estimation Methodology

First, consider the case in which the parameter space $\Theta^D$ is restricted such that the equilibrium is determinate. Let $y_t$ be a vector of observables and $Y^T = \{y_1, ..., y_T\}$. The parameters of the structural model are collected in a vector $\theta$. Then, the data and the model can be related in the following state-space form:

measurement equation: $y_t = A + Bz_t$,

state transition equation: $z_t = C(\theta)z_{t-1} + D(\theta)\varepsilon_t$.

where the state transition equation for $z_t$ corresponds to the solution (3) for the linear rational expectations model. The matrices $A$ and $B$ select and scale the relevant model variables $z_t$ to link them with observed data $y_t$. Assuming that all the shocks are normally distributed and uncorrelated over time, we obtain the likelihood function:

$$L^D(\theta|Y^T) = p(Y^T|\theta) = \prod_{t=1}^{T} p(y_t|Y^{T-1}, \theta),$$

which can be evaluated using the Kalman filter based on the state-space form above.

Next, consider the parameter space $\Theta^I$ in which the model exhibits indeterminacy of equilibria. In such a case, the law of motion for state variables is affected by sunspot shocks, and the solution multiplicity problem arises due to the arbitrary matrices $\tilde{M}$ and $M_\zeta$, as we can see from the solution in (2). Since there is no relative relationship between these two matrices, we impose the normalization such that $M_\zeta = 1$. Then the state transition equation is modified as follows:

$$z_t = F(\theta)z_{t-1} + G(\theta, \tilde{M})\varepsilon_t + H(\theta)\zeta^b_t.$$

The corresponding likelihood function is:

$$L^I(\theta, \tilde{M}|Y^T) = \prod_{t=1}^{T} p(y_t|Y^{T-1}, \theta, \tilde{M}).$$

In the subsequent analysis, the parameters are estimated over the parameter space which allows for both determinacy and indeterminacy. Hence, the overall likelihood function is evaluated by:

$$L(\theta, \tilde{M}|Y^T) = \{\theta \in \Theta^D\} L^D(\theta|Y^T) + \{\theta \in \Theta^I\} L^I(\theta, \tilde{M}|Y^T),$$

where $\{\theta \in \Theta^i\}$ for $i \in \{D, I\}$ is the indicator function that is one if $\theta \in \Theta^i$ and zero otherwise. According to Bayes theorem with a prior distribution $p(\theta, \tilde{M})$, the posterior distribution of $\theta$
is calculated by:

\[
p(\theta, \tilde{M}|Y^T) = \frac{\mathcal{L}(\theta, \tilde{M}|Y^T)p(\theta, \tilde{M})}{p(Y^T)}
\]

\[
= \frac{\mathcal{L}(\theta, \tilde{M}|Y^T)p(\theta, \tilde{M})}{\int \mathcal{L}(\theta, \tilde{M}|Y^T)p(\theta, \tilde{M}) d\theta d\tilde{M}}.
\]

Based on this posterior distribution, we can make inference on the structural parameters. For its computational implementation, see Appendix D.

### 4.2 Data and Priors

The data used for estimation are real GDP, real investment, the nominal interest rate, the inflation rate, and the asset price. Real GDP and investment are seasonally adjusted and are from Cabinet Office’s National Accounts. These two series are in per capita terms divided by the population of 15 years old and over in Labor Force Survey by the Statistics Bureau. The nominal interest rate is the uncollateralized overnight call rate, connected by the collateralized before 1985. The inflation rate is based on the Statistics Bureau’s Consumer Price Index excluding fresh foods. The index is adjusted for consumption tax reforms held in 1989 and 1997. For the asset price, we use Tokyo Stock Price Index (TOPIX). All data are at quarterly frequencies from 1980:2 to 1998:4\(^{13}\) and are detrended using the HP filter.

The prior distributions of the structural parameters are reported in Table 1. The elasticity of the external finance premium with respect to firms leverage \(\nu\) is base on Fuchi, Muto, and Ugai (2005), who estimate a modified version of the BGG model using Japan’s data by means of GMM. The mean values for the steady-state external financial premium \(S\), the capital adjustment cost parameter \(\chi\), and the survival rate of entrepreneurs \(\gamma\) follow from the calibration in BGG. The probability that prices remain unchanged for the next period \(\phi\), so-called Calvo parameter, is in line with Fuchi and Watanabe (2002), who estimate the Calvo parameter for Japan’s inflation with various regression forms. As for the monetary policy parameters, \(\psi_\pi, \psi_y, \rho_r,\) and \(\sigma_r\), OLS estimation results are used for the prior. The mean values for the steady state ratio of the net worth to capital \(N/K\), the exogenous expenditures share in GDP \(G/Y\), the elasticity of labor supply to wages \(\eta\), the depreciation rate of capital \(\delta\), and the capital share \(\alpha\) are based on Fukunaga (2002), who calibrates the BGG model on the basis of historical averages of Japan’s macroeconomic data. The degree of retailers’ monopoly power \(\theta\) is centered at 6

\(^{13}\)While the beginning of the sample period is determined on the basis of data availability, the end is chosen in order not to include the period during which zero nominal interest rate policy is adopted by the Bank of Japan. This is because there should be the least relationship between the nominal interest rate and the other variables during the period.
following Christensen and Dib (2006), which implies that the gross steady-state markup is 1.2.
The model is parameterized in terms of the steady state real interest $R$ instead of the discount
factor $\beta$ so that the conversion is $\beta = \left[ \exp\left(\frac{R}{400}\right) \right]^{-1}$. The mean of $R$ is set according to the
historical average of the call rate in real terms.

For the prior distribution of autoregressive coefficients of the shocks are centered at 0.7 and
the variances of the shocks are around 4.8. Since we have no references on these parameters, we
set wide confidence intervals for them. The priors for the components of the arbitrary matrix
$\tilde{M}$ are normally distributed around zero, based on the fact that most of the previous studies
typically ignore the solution multiplicity under indeterminacy by setting $\tilde{M}$ as zero.

5 Estimation Results

In this section, posterior distribution of the structural parameters, impulse responses, and variance
decompositions are presented. Based on the estimation results, characteristics of Japan’s
economy are revealed, and we can assess whether and to what extent sunspot shocks have
affected macroeconomic fluctuations in Japan.

5.1 Posterior Distribution of the Structural Parameters

The estimation results of the parameters are reported in Table 2. The elasticity of the external
finance premium with respect to firms leverage $\nu$ is centered at 0.08. This parameter character-
izes the existence of a financial accelerator effects in this model. The value is larger than the
estimates in Fuchi, Muto, and Ugai (2005) and the calibration in BGG. The mean value of the
steady state external finance premium $S$ is 4 percent (annually 16 percent), which is smaller
than usual calibration of 5 percent (annually 20 percent) in the BGG model. This estimator is
close to the Japan’s historical average of the spread between the average short term contracted
lending rate and the official discount rate. These results illustrate importance of our estimation
strategy that allows for indeterminacy of the equilibrium. According to the discussion on sources
of indeterminacy in Section 3, high value of $\nu$ and low value of $S$ generate indeterminacy. Hence,
if we restrict the parameter space that leads only to determinacy, we might underestimate $\nu$ or
overestimate $S$.

The posterior mean of capital adjustment cost parameter $\chi$ is 1.8, which is remarkably higher
than the value 0.25 used by BGG. This high value is, however, consistent with the estimates for
$\phi$, the probability that prices remain unchanged, is lower than that in Fuchi, Muto, and Ugai
(2005) for Japan’s economy, but almost the same as in Christensen and Dib (2006). The mean
value 0.55 implies that retailers set prices approximately every two quarters.
The monetary policy parameters imply that the Bank of Japan had strongly responded to inflation, but not so much to output. The coefficient on inflation $\psi_\pi$ is considerably higher than the prior mean, i.e. the preliminary OLS estimate. But it lies close to the estimator for U.S. monetary policy in a post-1982 sample reported in Lubik and Schorfheide (2004).

The autoregressive coefficients for the structural shocks, which represent persistency of the shocks, take fairly high values except for the parameter on the net worth shock. The variances of the shocks associated with demand, net worth and sunspots are relatively higher than those of the other shocks. As for the arbitrary matrix $\tilde{M}$, some of the components are different from zero. The last finding casts doubt on the common practice that overlooks the solution multiplicity inherent under indeterminacy.

5.2 Impulse Responses

In this subsection, we investigate the dynamic property of the estimated model. Figure 4 depicts impulse responses of output, investment, the nominal interest rate, inflation, and the asset price to the following six shocks: policy, technology, demand, marginal cost, net worth, and sunspot shock.

Since the model is based on a New Keynesian monetary DSGE framework with capital stock, the impulse responses on the monetary, technology, demand, and cost shock are standard. The positive monetary policy shock has a contractionary effect on output and inflation, and so does on the asset price. The positive technology shock has a negative effect on marginal cost and hence on inflation. Responding to inflation, the central bank lower the interest rate. Investment and output boost because of the high productivity and the low interest rate. The demand shock (specifically, the government expenditure shock) increases output but has a negative crowding-out effect on investment. Corresponding to the lower demand for capital, the asset price declines. The cost shock causes the monetary authority to raise the interest rate. The contractionary monetary policy depresses the economy.

The net worth shock and the sunspot shock are specific to our analysis. The net worth shock improves firms’ financial positions and lower the external financial premium. The low borrowing cost stimulates the investment and raises the asset price. Since the investment decisions are made one period in advance, the shock has negative effects on the economy at the first period. The sunspot shock has a positive effect on output and investment, which results in increase in the inflation rate and the interest rate while the asset price is dropped. The interpretation of these responses requires some caution. Since the shock is a reduced form sunspot as mentioned in Section 3, the dynamics are considered to be driven by beliefs for all the expectational variables. However, judging from the impulse responses, the dominant source of the sunspot seems to be a belief on investment; since the asset price is decreasing in a flow of investment due to the
capital adjustment costs under the capital stock being predetermined and fixed, the self-fulfilling prophecies on investment validates the response of the asset price in the figure.

5.3 Variance Decompositions

Table 3 reports variance decompositions of the model. According to the estimation results, Japan’s economy had been affected by sunspots but the contribution of the sunspot shock to the asset price had been negligible. Rather, the asset price fluctuation had been mainly driven by the shock on the firms’ net worth. The Kalman filter enables us to infer a series of the net worth implied by the model although such a series is not available in the actual data. Figure 5 depicts a smoothed estimate of the net worth computed from the structural parameters at the posterior means. The graph shows rapid expansion of the net worth in the latter half of the 1980s and its sharp drop in the early 1990s. During the former period, a positive net worth shock improved firms’ asset position and lowered the external financial premium. Decrease in cost of borrowing stimulated demand for capital, which led the asset price to rise consequently. On the other hand, in the early 1990s, a negative shock on the net worth is considered to have caused a sharp drop in asset prices with the above mechanism working in the opposite direction.

Another major source of asset price fluctuation is the cost shock. This finding can be explained in the following logic. In the late 1980s, inflation had been stable although the real economic activity had been boomed, as shown in Figure 1. It is often pointed out that one reason for stable inflation during the period was the increase in imports from NIEs under the yen’s appreciation after the Plaza Agreement in 1985. The model captures these phenomena as a negative cost shock. Since the inflation rate was not regarded as particularly high compared with that before the expansionary period, the interest rate had not been raised. The low interest rate had stimulated the investment activity during the period, and hence the demand for capital was increased, which led to the rise in asset prices. Also, we can see from Figure 1 that, even after the sharp drop in asset prices, the inflation rate had remained as high as in the boom. Hence, our results suggest that a contractionary monetary policy responding to higher inflation due to a positive cost shock had caused a severe recession in 1990s.

The variance decomposition of investment enables us to confirm the explanation above. The cost shock and the net worth shock had strikingly contributed to investment fluctuations. Another finding is that investment had been driven by sunspots in substantial degree. This finding provides a novel view of investment fluctuations since the previous studies have attempted to explain the volatility of investment only by fundamental factors.

Fluctuations in Output, inflation, and nominal interest rate are mainly ascribed to the cost shock and the productivity shock. The mechanism for high contributions of the cost shock is clear from the discussion in the preceding paragraphs. The variances due to the productivity
shock are relatively small compared with the standard real business cycle research.

6 Robustness Analysis

Our empirical results have shown that Japan’s economy had exhibited indeterminacy of the equilibrium. In this section, we investigate to what extent the data favors indeterminacy against determinacy. Also, we check robustness of our estimation results under different priors and sub-samples. The sub-sample analysis illustrates the regime change before and after the bust of asset prices in Japan.

6.1 Determinacy vs. Indeterminacy

While our estimation procedure takes into account the both possibility of determinacy and indeterminacy, the posterior probabilities for the determinacy and indeterminacy region of the parameter space are computed from the following marginal data densities:

\[ p^i (Y^T) = \int \{ \theta \in \Theta^i \} \mathcal{L} (\theta, \tilde{M}|Y^T) p (\theta, \tilde{M}) d\theta dM, \]

for \( i \in \{ D, I \} \). Then, the posterior probability of indeterminacy is given by

\[ \pi^I = \frac{p^I (Y^T)}{p^I (Y^T) + p^D (Y^T)}. \]

The resulting data densities\(^{14}\) are \( \ln p^D (Y^T) = -987 \) and \( \ln p^I (Y^T) = -890 \). Thus, the posterior probability of indeterminacy is almost 1 although the prior probability is 0.49 as reported in Table 1. Therefore, the data strongly prefers indeterminacy that generates sunspot fluctuations. Based on our discussion about sources of indeterminacy in Section 3, this finding suggests that Japan’s economy had exhibited strong financial accelerator effects and hence allowed for self-fulfilling prophecies. Also, the result reflects persistency of the data since a system under indeterminacy generates richer dynamics due to fewer autoregressive roots being suppressed.

6.2 Estimation under Different Priors

As discussed in Section 3, the components of the matrix \( \tilde{M} \) in the solution under indeterminacy are purely arbitrary. Lubik and Schorfheide (2004) have reported that their parameter estimation results and the resulting impulse response functions can be altered depending on the priors for the arbitrary matrix. Table 4 reports the estimation results of the structural parameters under different priors for the components of the matrix \( \tilde{M} \). While our baseline prior for \( M_{\epsilon \zeta}, M_{\alpha \zeta}, \)

\(^{14}\)The log marginal data densities are approximated using the harmonic mean estimator proposed by Geweke (1999).
$M_{g\zeta}, M_{z\zeta}$ and $M_{n\zeta}$ are the standard normal distribution, the prior 2 and 3 set them centered at 1 and -1 respectively with the same variance as the baseline. The results show that the posterior distributions of the parameters other than the components of the matrix $\tilde{M}$ do not remarkably change. Also, since the changes in the arbitrary coefficients are marginal, the impulse responses are almost the same as our baseline results.\footnote{The impulse responses under prior 2 and 3 are available from the author upon request.}

6.3 Sub-sample Estimation

Our analysis so far assumes that economic properties in the 1980s, during which the economy and asset prices boomed, are symmetric with those since 1990, when asset prices had collapsed and the economy had been extremely weak. However, economic regimes may have been changed before and after the bust of stock markets. In order to investigate the possibility of such regime change, we re-estimate the model for the period of 1980:2-1989:4 and 1990:1-1998:4 separately.\footnote{The sub-samples were chosen to separate the periods before and after Tokyo Stock Price Index (TOPIX) recorded the highest point.}

Table 5 shows the estimation results for structural parameters. Notable differences are found in the autoregressive parameters for the fundamental shocks; that is, the estimates are higher after the collapse of asset prices. These results come from the fact that Japan had experienced a long-lasting severe recession after the decline in asset prices. As for the monetary policy parameters, the estimates reveal that the Bank of Japan had placed more weight on inflation rate and less on output in the latter period.

Table 6 reports the variance decompositions for each sub-sample. For all the variables, the contributions of the net worth shock are larger during the boom and those of the cost shock are dominant after the bust of the stock markets. From these observations, it can be concluded that the main source of the rapid expansion in asset prices was excess net worth for firms, which lowered the external finance premium and stimulated demand for capital leading a rapid rise in asset prices. After the collapse of asset prices, inflation was still higher than its normal level although the economic activities were sharply deteriorated. Thus, the interest rate remained high, and it had an additional contractionary effect to the economy. Since the shock had a persistent effect, it caused a long-term economic slowdown in 1990s.

7 Conclusion

We have estimated the financial accelerator model over the parameter space where the equilibrium can be both determinate and indeterminate. This estimation strategy is vital for our analysis since the financial accelerator mechanism is likely to generate indeterminacy and the
solution under indeterminacy gives an explicit formulation for the agents’ beliefs. The estimation results show that the asset price fluctuation in Japan had been mainly driven by shocks on firm’s net worth and cost shocks. Also, the Japan’s economy had exhibited a strong financial accelerator effect and hence sunspots had affected equilibrium dynamics, in particular, investment spending.

One of the remaining questions is what is the background of the net worth and cost shocks. Although some answers have been provided anecdotally, which address, for instance, behavioral changes of firms and banks or variability of import prices, the model needs to be extended so that we can evaluate such possible sources formally.

Another avenue for future research is to explore the possibility of eliminating sunspots by other policy rules. From a policy maker’s point of view, a monetary authority has an incentive to choose a policy that leads the economy to determinacy in order to avoid unexpected volatility due to sunspots. Based on the policy specification in this paper, a significant financial accelerator effect always generates indeterminacy. But, with other policy rules, the possibility of indeterminacy might disappear.

Appendix

A Steady State Equilibrium

\[
\pi = Q = 1 \quad (7)
\]
\[
R = R^n = \frac{1}{\beta} \quad (8)
\]
\[
F = (1 + S) R \quad (9)
\]
\[
R^k = F + \delta - 1 \quad (10)
\]
\[
MC = \frac{\theta - 1}{\theta} \quad (11)
\]
\[
\frac{K}{Y} = \frac{\alpha MC}{R^k} \quad (12)
\]
\[
\frac{C}{Y} = 1 - \delta \frac{K}{Y} - \frac{G}{Y} \quad (13)
\]
\[
WH \frac{1}{C} = \frac{(1 - \alpha) MC}{C} \quad (14)
\]
\[
H = \frac{WH \frac{1}{C}}{\eta + WH \frac{1}{C}} \quad (15)
\]
\[
A = 1 \quad (16)
\]
\[ Y = AH \left( \frac{K}{Y} \right)^{\alpha_0} \]  
(17)

\[ K = Y \times \frac{K}{Y} \]  
(18)

\[ I = \delta K \]  
(19)

### B Log-linearized Equations

The steady state levels are written in capital letters without any subscripts, while log-deviations from the steady state are represented by small letters with time subscripts.

\[ E_{t+1} = c_t + r^n_t - E_{t+1} \pi \]  
(20)

\[ \frac{H}{1-H} h_t - w_t = -c_t \]  
(21)

\[ w_t = y_t + mc_t - h_t \]  
(22)

\[ r^k_t = y_t + mc_t - k_t \]  
(23)

\[ y_t = (1 - \alpha) a_t + \alpha k_t + (1 - \alpha) h_t \]  
(24)

\[ y_t = \frac{C}{Y} c_t + \frac{I}{Y} i_t + (1 - \frac{C}{Y} - \frac{I}{Y}) g_t \]  
(25)

\[ r^{n}_{t} = \rho r^{n}_{t-1} + (1 - \rho_r) (\psi_{\pi} \pi_t + \psi_y y_t) + \varepsilon_r \]  
(26)

\[ E_t q_t = \chi (E_t i_t - k_t) \]  
(27)

\[ \pi_t = \beta E_t \pi_{t+1} + \left( \frac{1 - \beta \phi}{\phi} \right) mc_t + z_t \]  
(28)

\[ r^n_t = r^n_t - r_t \]  
(29)

\[ k_{t+1} = \delta i_t + (1 - \delta) k_t \]  
(30)

\[ E_t f_{t+1} + \nu E_t n_{t+1} - \nu k_{t+1} = r_t + \nu q_t \]  
(31)

\[ f_t + q_{t-1} = \frac{R^k}{F} r^k_t + \frac{1 - \delta}{F} q_t \]  
(32)

\[ \frac{n_{t+1}}{\gamma F} = K \frac{N}{f_t} - \left( \frac{K}{N} - 1 \right) E_{t-1} f_t + n_t + nx_t \]  
(33)

\[ a_t = \rho_a a_{t-1} + \varepsilon_{at} \]  
(34)

\[ g_t = \rho_g g_{t-1} + \varepsilon_{gt} \]  
(35)

\[ z_t = \rho_z z_{t-1} + \varepsilon_{zt} \]  
(36)

\[ nx_t = \rho_n nx_{t-1} + \varepsilon_{nt} \]  
(37)
C Derivation of the Full Set of Sunspot Solution

The canonical system (1) can be written as the following reduced form\textsuperscript{17}:

\[ z_{t+1} = \Gamma^* z_t + \Psi^* \varepsilon_{t+1} + \Pi^* \eta_{t+1}. \]  

(38)

where \( \Gamma^* \equiv \Gamma_0^{-1} \Gamma_1 \), \( \Psi^* \equiv \Gamma_0^{-1} \Psi_0 \) and \( \Pi^* \equiv \Gamma_0^{-1} \Pi_0 \). Replacing \( \Gamma^* \) by its Jordan decomposition \( \Gamma^* \equiv \Gamma^0 - 1 \Gamma^1 > 0 \), \( \Psi^* \equiv \Gamma^0 - 1 \Psi_0 > 0 \) and \( \Pi^* \equiv \Gamma^0 - 1 \Pi_0 > 0 \). Replacing \( \Gamma^* \) by its Jordan decomposition \( P \Lambda P^{-1} \), the decoupled system is:

\[ P^{-1} z_{t+1} = \Lambda P^{-1} z_t + P^{-1} \Psi^* \varepsilon_{t+1} + P^{-1} \Pi^* \eta_{t+1} \]

\( \Leftrightarrow w_{t+1} = \Lambda w_t + P^{-1} \Psi^* \varepsilon_{t+1} + P^{-1} \Pi^* \eta_{t+1}, \]

(39)

where \( w_t \equiv P^{-1} z_t \). Rewrite (39) as

\[ \begin{bmatrix} w_{U,t} \\ w_{S,t} \end{bmatrix} = \begin{bmatrix} \Lambda_U & 0 \\ 0 & \Lambda_S \end{bmatrix} \begin{bmatrix} w_{U,t-1} \\ w_{S,t-1} \end{bmatrix} + \begin{bmatrix} P_{U,1}^{-1} \\ P_{S,1}^{-1} \end{bmatrix} \Psi^* \varepsilon_t + \begin{bmatrix} P_{U,2}^{-1} \\ P_{S,2}^{-1} \end{bmatrix} \Pi^* \eta_t, \]

where subscript \( U \) and \( S \) represent ‘unstable’ and ‘stable’ parts of the matrices which correspond to eigenvalues that are greater than unity and less than unity respectively.

For the solution for \( w_t \) to be stationary, we need to have:

\[ w_{U,t} = 0 \ \forall t, \]

which is obtained if \( w_{U,0} = 0 \) or \( \Lambda_U = 0 \) and

\[ P_{U,1}^{-1} \Psi^* \varepsilon_t + P_{U,2}^{-1} \Pi^* \eta_t = 0 \ \forall t. \]  

(40)

If (40) has a unique solution, the equilibrium is determinate since the expectation errors \( \eta_t \) are uniquely determined as functions of the structural shock \( \varepsilon_t \); otherwise, it is indeterminate. In other words, the equilibrium is indeterminate if the number of unstable roots is less than the number of forecast errors.

Assuming at least one solution to (40) exists, a singular value decomposition of the matrix \( P_{U,1}^{-1} \Pi^* \) gives:

\[ P_{U,1}^{-1} \Pi^* = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} D_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V'_{11} \\ V'_{21} \end{bmatrix} = UDV' = U_1 D_{11} V'_{1}, \]

where \( D_{11} \) is an \( r \times r \) diagonal matrix, \( U \) and \( V \) are orthonormal matrices, \( U_1 \) is an \( m \times r \) submatrix of \( U \), and \( V'_{1} \) is an \( r \times k \) submatrix of \( V' \). Here, \( m \) denotes the number of unstable eigenvalues and \( r \) is the number of non-zero singular values of \( P_{U,1}^{-1} \Pi^* \) and \( k \) is the dimension

\textsuperscript{17}For simplicity, we assume non-singularity of \( \Gamma_0 \). If \( \Gamma_0 \) is singular, the canonical system can be transformed through generalized Schur decomposition as in Sims (2002).
of the vector of rational expectation forecast errors $\eta_t$. Following the proposition in Lubik and Schorfheide (2003), the expectation forecast errors can be expressed as:

$$\eta_t = \left( -V_1D_1^{-1}U_1P_U^{-1}\Psi^* + V_2\tilde{M} \right) \epsilon_t + V_2M_\zeta \zeta_t,$$

(41)

where $\tilde{M}$ is a $(k - r) \times l$ matrix, $M_\zeta$ is a $(k - r) \times p$ matrix, and $V_2$ is $k \times (k - r)$ matrix. $l$ denotes the number of exogenous fundamental shocks and that $p$ is the dimension of the sunspot shock $\zeta_t$. Notice that solution multiplicity arises since the matrices $\tilde{M}$ and $M_\zeta$ are arbitrary.

Substituting the representation for the rational expectations forecast errors (41) into the original system (38), we have the solution as in (2). Note that $V_2 = 0$ if the equilibrium is determinate and that sunspot shock does not affect the equilibrium dynamics. In such a determinate case, the solution is reduced to (3).

Under the definition of the belief shock in (4), the stability condition (40) can be written as:

$$\left[ P_U^{-1}\Psi^* P_U^{-1}\Pi^* \right] \left[ \begin{array}{c} \epsilon_t \\ \zeta_t^b \end{array} \right] + P_U^{-1}\Pi^* \tilde{\eta}_t = 0.$$

Hence, we have the following expression for $\tilde{\eta}_t$:

$$\tilde{\eta}_t = -V_1D_1 U_1' \left[ P_U^{-1}\Psi^* P_U^{-1}\Pi^* \right] \left[ \begin{array}{c} \epsilon_t \\ \zeta_t^b \end{array} \right] + V_2 \left[ \begin{array}{c} \tilde{M} \epsilon_t \\ M_\zeta \zeta_t^b \end{array} \right].$$

Since $P_U^{-1}\Pi^* = U_1D_1 V_1'$ and orthogonality of $V$ implies $I - V_1V_1' = V_2V_2'$, the overall forecast errors are:

$$\eta_t = \tilde{\eta}_t + \zeta_t^b = \left( -V_1D_1 U_1' P_U^{-1}\Psi^* + V_2\tilde{M} \right) \epsilon_t + V_2 \left( V_2' + M_\zeta \right) \zeta_t^b.$$

Then, the reduced form of the system with belief shocks is given by the form in (5).

### D Computation of Posterior Distribution

For notational simplicity, redefine $\theta$ as a parameter vector which stacks $\theta$ and $\tilde{M}$ in (6). Then, the posterior distribution is expressed as $p(\theta|Y_T) = \frac{\mathcal{L}(\theta|Y_T)p(\theta)}{\int \mathcal{L}(\theta|Y_T)p(\theta)d\theta}$.

We exploit numerical optimization routine to find the mode $\tilde{\theta}$ of the posterior density and inverse Hessian $\tilde{\Sigma}$ at its mode. In order to evaluate the estimated parameter by its mean, variance or confidence interval from its posterior distribution $p(\theta|Y_T)$, we generate draws from the distribution. Markov Chain Monte Carlo (MCMC) method is used to generate the draws. MCMC is a simulation-based sampling method to construct a stochastic process which converges to the target distribution that we want to sample, to simulates the stochastic process, and then to use such samples for inference.

For implementation of MCMC, Metropolis-Hastings algorithm is used. This algorithm works in the following way.
1. Draw candidate parameter vector $\vartheta$ from a jumping distribution $J_s(\vartheta|\theta^{(s-1)})$, where, for example, $J_s \sim N(\theta^{(s-1)}, c^2\tilde{\Sigma})$.

2. The jump from $\theta^{(s-1)}$ is accepted ($\theta^{(s)} = \vartheta$) with probability $\min(r, 1)$ and rejected ($\theta^{(s)} = \theta^{(s-1)}$), where $r = \frac{L(\vartheta|Y^T)p(\vartheta)}{L(\theta^{(s-1)}|Y^T)p(\theta^{(s-1)})}$.

3. Repeat the processes 1 and 2, and we have a sequence of draws $\{\theta^{(s)}\}$.

4. Calculate $E[g(\theta)|Y^T] = \frac{1}{n_s} \sum_{s=1}^{n_s} g(\theta^{(s)})$ for its mean (if $g(\theta) = \theta$), variance, and impulse response functions.

It is known that the Metropolis chain has stationary distribution which is identical to the target distribution $p(\theta|Y^T)$ and that distribution of $\theta^{(s)}$ converges to the target distribution. Hence, although the samples it yields are typically not independent draws from the target distribution, each draw can be considered to come from the target distribution by observing the stochastic process for sufficiently long.

References


\footnote{For our analyses in Section 5 and 6, 500,000 draws are generated with the algorithm, and the first 50,000 draws are discarded.}


Table 1: Prior Distributions

<table>
<thead>
<tr>
<th>Name</th>
<th>Range</th>
<th>Density</th>
<th>Mean</th>
<th>90% Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
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<td>Beta</td>
<td>0.04</td>
<td>[0.01, 0.07]</td>
</tr>
<tr>
<td>$S$</td>
<td>[0, 1)</td>
<td>Beta</td>
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<tr>
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<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
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<td>[0.09, 0.40]</td>
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<td>Beta</td>
<td>0.83</td>
<td>[0.78, 0.88]</td>
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<tr>
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<td>Gamma</td>
<td>1.23</td>
<td>[0.81, 1.63]</td>
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<tr>
<td>$\psi_y$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>0.26</td>
<td>[0.06, 0.45]</td>
</tr>
<tr>
<td>$\rho_r$</td>
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<td>Beta</td>
<td>0.73</td>
<td>[0.65, 0.81]</td>
</tr>
<tr>
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<td>Beta</td>
<td>0.97</td>
<td>[0.95, 1.00]</td>
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<tr>
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<td>Beta</td>
<td>0.50</td>
<td>[0.38, 0.61]</td>
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<tr>
<td>$G/Y$</td>
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<td>Beta</td>
<td>0.30</td>
<td>[0.26, 0.33]</td>
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<tr>
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<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>2.74</td>
<td>[2.41, 3.07]</td>
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<tr>
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<td>Beta</td>
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<td>[0.02, 0.03]</td>
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<td>Beta</td>
<td>0.36</td>
<td>[0.31, 0.41]</td>
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<td>Gamma</td>
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<td>[5.35, 6.66]</td>
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<td>Gamma</td>
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<td>[2.41, 4.04]</td>
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<td>Beta</td>
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<td>[0.47, 0.94]</td>
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<tr>
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<td>[0.20, 0.99]</td>
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<td>InvGamma</td>
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<td>[1.67, 8.05]</td>
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Note: The fraction of the parameter space which leads to indeterminacy is 49%.
Table 2: Parameter Estimation Results

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<tr>
<th>Name</th>
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<th>Posterior Distributions</th>
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<tr>
<td></td>
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<td>Mean 90% Interval</td>
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<tr>
<td>( \nu )</td>
<td>0.04 [ 0.01, 0.07]</td>
<td>0.08 [ 0.04, 0.12]</td>
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<tr>
<td>( S )</td>
<td>0.05 [ 0.04, 0.05]</td>
<td>0.04 [ 0.03, 0.05]</td>
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<tr>
<td>( \chi )</td>
<td>0.25 [ 0.09, 0.40]</td>
<td>1.84 [ 1.51, 2.19]</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.83 [ 0.78, 0.88]</td>
<td>0.55 [ 0.50, 0.59]</td>
</tr>
<tr>
<td>( \psi )</td>
<td>1.23 [ 0.81, 1.63]</td>
<td>1.98 [ 1.65, 2.30]</td>
</tr>
<tr>
<td>( \psi_y )</td>
<td>0.26 [ 0.06, 0.45]</td>
<td>0.05 [ 0.01, 0.08]</td>
</tr>
<tr>
<td>( \rho_r )</td>
<td>0.73 [ 0.65, 0.81]</td>
<td>0.71 [ 0.66, 0.76]</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.97 [ 0.95, 1.00]</td>
<td>0.89 [ 0.83, 0.95]</td>
</tr>
<tr>
<td>( N/K )</td>
<td>0.50 [ 0.38, 0.61]</td>
<td>0.50 [ 0.39, 0.61]</td>
</tr>
<tr>
<td>( G/Y )</td>
<td>0.30 [ 0.26, 0.33]</td>
<td>0.31 [ 0.27, 0.34]</td>
</tr>
<tr>
<td>( \eta )</td>
<td>2.74 [ 2.41, 3.07]</td>
<td>2.67 [ 2.35, 3.00]</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.02 [ 0.02, 0.03]</td>
<td>0.02 [ 0.02, 0.03]</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.36 [ 0.31, 0.41]</td>
<td>0.44 [ 0.39, 0.49]</td>
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<tr>
<td>( \theta )</td>
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<td>6.07 [ 5.43, 6.73]</td>
</tr>
<tr>
<td>( R )</td>
<td>3.24 [ 2.41, 4.04]</td>
<td>3.15 [ 2.35, 3.91]</td>
</tr>
<tr>
<td>( \rho_a )</td>
<td>0.70 [ 0.47, 0.94]</td>
<td>0.90 [ 0.81, 0.99]</td>
</tr>
<tr>
<td>( \rho_g )</td>
<td>0.70 [ 0.47, 0.94]</td>
<td>0.79 [ 0.68, 0.91]</td>
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<tr>
<td>( \rho_z )</td>
<td>0.70 [ 0.47, 0.94]</td>
<td>0.95 [ 0.92, 0.99]</td>
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<td>( \rho_n )</td>
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<td>0.00 [-1.65, 1.67]</td>
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<td>-0.20 [-0.63, 0.24]</td>
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<td>1.40 [ 1.11, 1.69]</td>
</tr>
<tr>
<td>( \sigma_g )</td>
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<td>2.27 [ 1.70, 2.82]</td>
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<td>1.02 [ 0.84, 1.20]</td>
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<td>6.25 [ 2.38, 10.29]</td>
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<td>8.27 [ 6.76, 9.91]</td>
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Table 3: Variance Decompositions

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<th>Mean</th>
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<td><strong>Investment</strong></td>
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<td>[0.00, 0.00]</td>
<td>Policy</td>
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<td>[0.00, 0.00]</td>
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<td>[0.00, 0.02]</td>
<td>Productivity</td>
<td>0.02</td>
<td>[0.00, 0.03]</td>
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<td>Demand</td>
<td>0.01</td>
<td>[0.00, 0.03]</td>
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<tr>
<td>Cost</td>
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<td>[0.09, 0.40]</td>
<td>Cost</td>
<td>0.24</td>
<td>[0.05, 0.43]</td>
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<tr>
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<td>0.45</td>
<td>[0.24, 0.67]</td>
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<td>[0.00, 0.01]</td>
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<td>[0.09, 0.47]</td>
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<td></td>
<td><strong>Inflation</strong></td>
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<td>[0.03, 0.14]</td>
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<td><strong>Interest</strong></td>
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<td>[0.12, 0.37]</td>
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<td>[0.01, 0.08]</td>
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<td>[0.03, 0.26]</td>
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<tr>
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<td>[0.01, 0.09]</td>
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Table 4: Parameter Estimation Results under Different Priors

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<th>Mean</th>
<th>90% Interval</th>
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<td>0.09</td>
<td>[0.05, 0.13]</td>
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<td>[0.03, 0.05]</td>
<td>0.04</td>
<td>[0.03, 0.05]</td>
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<td>1.82</td>
<td>[1.49, 2.15]</td>
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<tr>
<td>$\phi$</td>
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<td>[0.51, 0.60]</td>
<td>0.54</td>
<td>[0.50, 0.59]</td>
</tr>
<tr>
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<td>1.98</td>
<td>[1.65, 2.32]</td>
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<tr>
<td>$\psi_y$</td>
<td>0.05</td>
<td>[0.02, 0.08]</td>
<td>0.05</td>
<td>[0.01, 0.08]</td>
</tr>
<tr>
<td>$\rho_r$</td>
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<td>[0.66, 0.77]</td>
<td>0.71</td>
<td>[0.66, 0.77]</td>
</tr>
<tr>
<td>$\gamma$</td>
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<td>[0.85, 0.95]</td>
<td>0.89</td>
<td>[0.83, 0.94]</td>
</tr>
<tr>
<td>$N/K$</td>
<td>0.50</td>
<td>[0.40, 0.61]</td>
<td>0.50</td>
<td>[0.39, 0.62]</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>0.31</td>
<td>[0.27, 0.34]</td>
<td>0.31</td>
<td>[0.27, 0.34]</td>
</tr>
<tr>
<td>$\eta$</td>
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<td>[2.34, 3.00]</td>
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<td>[2.36, 3.01]</td>
</tr>
<tr>
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<td>[0.02, 0.03]</td>
<td>0.02</td>
<td>[0.02, 0.03]</td>
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<tr>
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<td>[0.39, 0.49]</td>
</tr>
<tr>
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<td>6.08</td>
<td>[5.44, 6.75]</td>
</tr>
<tr>
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<td>[0.77, 0.98]</td>
<td>0.92</td>
<td>[0.85, 0.99]</td>
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<tr>
<td>$\rho_g$</td>
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<tr>
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<td>0.95</td>
<td>[0.91, 0.99]</td>
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<tr>
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<tr>
<td>$M_{g\zeta}$</td>
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<td>[0.08, 3.22]</td>
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<tr>
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<td>[-0.44, 0.53]</td>
<td>-0.42</td>
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<td>0.17</td>
<td>[0.14, 0.19]</td>
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<td>$\sigma_a$</td>
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<td>[1.07, 1.74]</td>
<td>1.45</td>
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<td>1.02</td>
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<tr>
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<td>[2.28, 8.72]</td>
<td>5.96</td>
<td>[2.58, 9.26]</td>
</tr>
<tr>
<td>$\sigma_\zeta$</td>
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<td>[4.94, 9.75]</td>
<td>7.99</td>
<td>[6.16, 10.08]</td>
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</table>
Table 5: Parameter Estimation Results Before and After Collapse of Asset Prices

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<td>0.06 [0.03, 0.09]</td>
</tr>
<tr>
<td>$S$</td>
<td>0.04 [0.03, 0.05]</td>
<td>0.04 [0.04, 0.05]</td>
</tr>
<tr>
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<td>1.69 [1.33, 2.03]</td>
<td>1.15 [0.86, 1.43]</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.79 [0.73, 0.84]</td>
<td>0.62 [0.58, 0.66]</td>
</tr>
<tr>
<td>$\psi_\pi$</td>
<td>1.62 [1.26, 1.97]</td>
<td>2.10 [1.73, 2.44]</td>
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<td>0.16 [0.05, 0.27]</td>
<td>0.03 [0.01, 0.06]</td>
</tr>
<tr>
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<td>0.72 [0.65, 0.78]</td>
<td>0.70 [0.64, 0.76]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.92 [0.88, 0.96]</td>
<td>0.93 [0.90, 0.97]</td>
</tr>
<tr>
<td>$N/K$</td>
<td>0.49 [0.38, 0.61]</td>
<td>0.49 [0.38, 0.60]</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>0.30 [0.27, 0.33]</td>
<td>0.30 [0.27, 0.33]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2.73 [2.40, 3.04]</td>
<td>2.69 [2.36, 3.02]</td>
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<td>0.02 [0.02, 0.03]</td>
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<td>0.39 [0.34, 0.44]</td>
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<td>6.04 [5.39, 6.69]</td>
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<tr>
<td>$R$</td>
<td>3.20 [2.34, 4.01]</td>
<td>3.21 [2.40, 4.01]</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.53 [0.29, 0.76]</td>
<td>0.95 [0.90, 0.99]</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.68 [0.49, 0.86]</td>
<td>0.86 [0.75, 0.97]</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.20 [0.08, 0.32]</td>
<td>0.97 [0.95, 0.99]</td>
</tr>
<tr>
<td>$\rho_n$</td>
<td>0.68 [0.49, 0.88]</td>
<td>0.67 [0.46, 0.89]</td>
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<tr>
<td>$M_{r\zeta}$</td>
<td>0.02 [-1.53, 1.60]</td>
<td>-0.10 [-1.75, 1.49]</td>
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<tr>
<td>$M_{a\zeta}$</td>
<td>0.43 [-1.65, 2.26]</td>
<td>0.26 [-1.51, 2.00]</td>
</tr>
<tr>
<td>$M_{g\zeta}$</td>
<td>-0.12 [-0.98, 0.67]</td>
<td>-0.77 [-2.03, 0.51]</td>
</tr>
<tr>
<td>$M_{z\zeta}$</td>
<td>0.12 [-1.63, 1.82]</td>
<td>0.10 [-1.56, 1.78]</td>
</tr>
<tr>
<td>$M_{n\zeta}$</td>
<td>0.03 [-0.63, 0.72]</td>
<td>0.30 [-0.58, 1.23]</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.18 [0.15, 0.22]</td>
<td>0.17 [0.13, 0.20]</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>2.77 [1.63, 3.93]</td>
<td>1.94 [1.33, 2.55]</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>2.80 [2.06, 3.51]</td>
<td>2.54 [1.66, 3.42]</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>1.07 [0.86, 1.27]</td>
<td>1.17 [0.93, 1.40]</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>3.92 [1.96, 5.87]</td>
<td>4.39 [2.04, 6.83]</td>
</tr>
<tr>
<td>$\sigma_\zeta$</td>
<td>6.66 [3.14, 9.52]</td>
<td>11.46 [8.27, 14.85]</td>
</tr>
</tbody>
</table>
Table 6: Variance Decompositions Before and After Collapse of Asset Prices

<table>
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<tr>
<th>Shock</th>
<th>Mean 90% Interval</th>
<th>Mean 90% Interval</th>
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<tbody>
<tr>
<td></td>
<td>Asset Price</td>
<td>Asset Price</td>
</tr>
<tr>
<td>Policy</td>
<td>0.01 [0.00, 0.01]</td>
<td>0.00 [0.00, 0.00]</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.00 [0.00, 0.01]</td>
<td>0.01 [0.00, 0.02]</td>
</tr>
<tr>
<td>Demand</td>
<td>0.00 [0.00, 0.00]</td>
<td>0.00 [0.00, 0.00]</td>
</tr>
<tr>
<td>Cost</td>
<td>0.07 [0.01, 0.13]</td>
<td>0.54 [0.28, 0.81]</td>
</tr>
<tr>
<td>Net Worth</td>
<td>0.92 [0.85, 0.99]</td>
<td>0.44 [0.18, 0.71]</td>
</tr>
<tr>
<td>Sunspot</td>
<td>0.00 [0.00, 0.01]</td>
<td>0.00 [0.00, 0.00]</td>
</tr>
<tr>
<td></td>
<td>Investment</td>
<td>Investment</td>
</tr>
<tr>
<td>Policy</td>
<td>0.00 [0.00, 0.01]</td>
<td>0.00 [0.00, 0.00]</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.06 [0.00, 0.16]</td>
<td>0.06 [0.00, 0.03]</td>
</tr>
<tr>
<td>Demand</td>
<td>0.01 [0.00, 0.02]</td>
<td>0.01 [0.00, 0.01]</td>
</tr>
<tr>
<td>Cost</td>
<td>0.04 [0.00, 0.08]</td>
<td>0.54 [0.25, 0.83]</td>
</tr>
<tr>
<td>Net Worth</td>
<td>0.73 [0.53, 0.93]</td>
<td>0.37 [0.10, 0.64]</td>
</tr>
<tr>
<td>Sunspot</td>
<td>0.17 [0.00, 0.33]</td>
<td>0.07 [0.00, 0.14]</td>
</tr>
<tr>
<td></td>
<td>Output</td>
<td>Output</td>
</tr>
<tr>
<td>Policy</td>
<td>0.01 [0.00, 0.02]</td>
<td>0.00 [0.00, 0.00]</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.04 [0.00, 0.09]</td>
<td>0.06 [0.00, 0.14]</td>
</tr>
<tr>
<td>Demand</td>
<td>0.05 [0.00, 0.12]</td>
<td>0.01 [0.00, 0.01]</td>
</tr>
<tr>
<td>Cost</td>
<td>0.12 [0.01, 0.22]</td>
<td>0.74 [0.49, 0.97]</td>
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<tr>
<td>Net Worth</td>
<td>0.75 [0.52, 0.98]</td>
<td>0.19 [0.01, 0.42]</td>
</tr>
<tr>
<td>Sunspot</td>
<td>0.04 [0.00, 0.08]</td>
<td>0.00 [0.00, 0.01]</td>
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<td>Inflation</td>
<td>Inflation</td>
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<td>Policy</td>
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<td>0.04 [0.00, 0.07]</td>
</tr>
<tr>
<td>Productivity</td>
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<td>0.13 [0.02, 0.22]</td>
</tr>
<tr>
<td>Demand</td>
<td>0.00 [0.00, 0.00]</td>
<td>0.01 [0.00, 0.02]</td>
</tr>
<tr>
<td>Cost</td>
<td>0.52 [0.18, 0.87]</td>
<td>0.61 [0.36, 0.88]</td>
</tr>
<tr>
<td>Net Worth</td>
<td>0.41 [0.04, 0.78]</td>
<td>0.15 [0.01, 0.30]</td>
</tr>
<tr>
<td>Sunspot</td>
<td>0.00 [0.00, 0.01]</td>
<td>0.07 [0.00, 0.14]</td>
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<tr>
<td></td>
<td>Interest</td>
<td>Interest</td>
</tr>
<tr>
<td>Policy</td>
<td>0.03 [0.00, 0.05]</td>
<td>0.01 [0.00, 0.01]</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.05 [0.00, 0.12]</td>
<td>0.16 [0.03, 0.28]</td>
</tr>
<tr>
<td>Demand</td>
<td>0.01 [0.00, 0.02]</td>
<td>0.02 [0.00, 0.04]</td>
</tr>
<tr>
<td>Cost</td>
<td>0.31 [0.02, 0.60]</td>
<td>0.59 [0.34, 0.86]</td>
</tr>
<tr>
<td>Net Worth</td>
<td>0.59 [0.21, 0.96]</td>
<td>0.21 [0.02, 0.40]</td>
</tr>
<tr>
<td>Sunspot</td>
<td>0.01 [0.00, 0.02]</td>
<td>0.02 [0.00, 0.03]</td>
</tr>
</tbody>
</table>
Figure 1: Asset Price, Output growth, and Inflation

Notes: Asset price is the Tokyo Stock Price Index. Inflation is based on the Statistics Bureau’s Consumer Price Index excluding fresh foods, adjusted for consumption tax reforms. Real GDP is from Cabinet Office’s National Accounts. Output growth and inflation are changes from a year earlier at quarterly frequencies.
Figure 2: Determinacy and Indeterminacy Region I

Figure 3: Determinacy and Indeterminacy Region II
Note: The Figure depicts posterior means (solid lines) and pointwise 90% posterior probability intervals (dashed lines) for the impulse responses to one-standard deviation shocks in terms of percentage deviation from the steady state.
Notes: The graph shows the percentage deviation of the Kalman smoothed estimate of the net worth from the steady state. The sequence is computed from the parameters at the posterior means.