R&D Contests with Spillovers

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Abstract

A tournament framework is used to study R&D competition. We generalize the tournament framework to competition among more than two contestants. Our approach of modeling R&D uncertainty is more general than the literature. The relationship between market structure and R&D spending is shown to be sensitive to the form of uncertainty that characterizes the R&D process. In a “winner-take-all” contest, R&D spending of each contestant may increase with the degree of spillovers. Our results are supported by empirical evidence. Our framework is also useful in studying other kinds of contests, such as rent seeking.

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I. Introduction

Some of the empirical work on R&D spending is inconsistent with the theoretical work on this issue. When a firm conducts R&D, it is often in competition with other firms. Loury (1979) shows that a firm’s R&D spending decreases with the number of rival firms.\(^1\) However, empirical evidence does not demonstrate a monotonic relationship between a firm’s R&D spending and the number of firms.\(^2\) Spillovers imply that a research agent may benefit from the research of other agents. Traditional wisdom holds that R&D spending decreases with the degree of spillovers. Levin (1988), however, finds that industries with the highest level of spillovers also rank high in R&D intensity.

Two key features of R&D are uncertainty and spillovers. Uncertainty in R&D is well recognized in the literature. Spillovers are also frequently invoked in empirical and theoretical studies related to R&D. In his survey of empirical studies, Griliches (1992) concludes that R&D spillovers are both prevalent and important. And R&D spillover is a key assumption of Romer's (1990) influential endogenous growth model.

In this paper, we use a tournament approach to study the influence of uncertainty and spillovers in R&D competition in a unified framework and we are able to explain the above empirical results. In a tournament, what affects a contestant’s payoff is a contestant’s relative rank, rather than absolute performance. In our model, a contestant’s R&D output is affected by spillovers from her rivals. In addition, it stochastically depends on R&D spending in a more general fashion. First, we present a model where the size of the prize to the winner is exogenously given. Depending on the form of uncertainty, a contestant’s R&D spending may decrease, remain the same, or increase with the number of contestants in an industry. When prizes are predetermined, we show that an increase in reward to the winner may decrease each contestant’s expected profit. The reason is that an increase in reward to the winner causes an increase in each contestant’s R&D spending. Second, we study the situation where size of the prize to the winner is endogenously determined by R&D spending. We find that each contestant’s R&D spending may increase with the degree of spillovers. There are two effects when

\(^1\) Pioneering on applying game theory to study patent races, Loury’s paper sparked a large volume of papers. Examples include Lee and Wilde (1980), Delbono and Denicolo (1991).
\(^2\) See Scherer and Ross (1990) for a survey on the relationship between market structure and R&D spending.
the spillover rate increases. On the one hand, at a given level of R&D spending, a contestant’s probability of winning decreases as a higher spillover rate helps rivals more. This decreases a contestant’s incentive to spend on R&D. On the other hand, the expected prize increases when the winner takes all. The latter effect increases a firm’s incentive to spend on R&D. This effect has not been explored in the literature. Under some parameter values, the latter effect dominates and R&D spending increases with the degree of spillovers. Our model is also very useful in analyzing other issues related to R&D, such as contestants’ incentives to increase the efficiency of R&D.

In the literature, there are studies on contests similar to R&D competition, such as rent seeking and labor tournament. A common element about those contests is that rank order rather than absolute difference in performance is more important in affecting contestants’ payoffs. Our approach and many results in our paper may also be applicable to the above kinds of contests. We can reinterpret R&D spending as lobbying effort in rent seeking or effort in tournament and the above result will apply.

In the literature on the relationship between market structure and firms’ incentives to invest in R&D, Schumpeter (1950) argues that some concentration in an industry is necessary to provide firms with sufficient incentives to invest in R&D. Schumpeter’s argument sparked a large volume of empirical and theoretical studies. In a seminal paper, Loury (1979) establishes the patent race framework to study the relationship between market structure and R&D spending. This approach is followed by Lee and Wilde (1980) and Delbono and Denicolo (1991). Yi (1999) does not adopt the patent race approach. However, there is only one firm that is capable of conducting R&D in his study of process innovation.

The patent race paradigm is not appropriate in some circumstances. For example, Super Efficient Refrigerator Program Inc., a coalition of 25 private and public electric utilities, offered 30 million dollars for the developer of a refrigerator that can save energy significantly over current models. Frididaire Co. and Whirlpool Corp. were selected as the two finalists from 14 contestants to compete for this reward (The Washington Post, December 20, 1992). Whirlpool won this contest. In the above example, firms compete to
produce the best innovation on a specific date, rather than to discover a new product of a
given quality, as in patent race models.\textsuperscript{3}

There are some limitations of traditional patent race models. First, the patent race
literature relies on exponential distribution to model uncertainty. The reason is that for
exponentially distributed independent random variables, the lowest order statistics of
those random variables also has an exponential density function. The economic
interpretation of the above mathematical result is that a firm's invention date and its
rivals' earliest invention date have the \textit{same} distribution. Without this property, it will be
impossible to integrate out a firm's expected profit and solve the model. However, some
of the results in the patent race literature depend on the assumption that random variables
are exponentially distributed. For example, Loury’s result that a firm’s R&D spending
decreases with the number of rivals depends on this assumption. The exponential
distribution has a monotonically decreasing density function. If the form of uncertainty
does not have this monotonicity property, the relationship between market structure and
R&D spending will be more complex. Second, the patent race models do not directly
accommodate how the value of a patent changes with R&D spending.\textsuperscript{4} In real world
situations, higher R&D spending may give rise to a product with a higher value. In
Loury’s model, the value of the patent is a fixed amount, which does not change with
firms’ R&D spending. Finally, in the patent race literature, a firm is typically choosing
the mean and the variance of the stochastic time of invention by controlling R&D
spending. Higher spending on R&D not only decreases the mean time of invention, but
also decreases the variance of invention date. If we interpret a decrease of variance as a
decrease of uncertainty, whether the uncertainty of a R&D project is controllable is not
clear-cut. Uncertainty is an essential feature of R&D and how it affects a firm’s R&D
spending is not studied in the patent race models.

We analyze R&D competition in a tournament framework. For the literature on
labor tournaments, see the seminal papers by Lazear and Rosen (1981) and Nalebuff and
Stiglitz (1983). There are some differences between their research and our model. First,
Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983) mainly focus on tournaments

\textsuperscript{3} See also Taylor (1995).
\textsuperscript{4} In the literature on patent races, an earlier invention is more profitable as future profits are discounted.
between two contestants. In our model, the number of contestants may be more than two. Second, in the labor tournament literature, random effects among different agents usually have some common components. Common shocks justify the principal to base agents' pay on rank order performance rather than to use individual payment schemes. In our model, random effects among different contestants are independently distributed. Finally, in the labor tournament literature, positive spillovers are not considered.

Taylor (1995) and Fullerton and McAfee (1999) also study R&D tournaments. However, their main interests and approaches are totally different from ours. Taylor (1995) uses a sequential search approach to model R&D competition. The sponsor of a tournament chooses the number of contestants, an entry fee, and the prize to the winner to maximize his own profit. Fullerton and McAfee (1999) are mainly interested in finding a mechanism to select top contestants from a group of heterogeneous contestants. Neither of the above papers studies spillovers in R&D.

For the literature on R&D spillovers, see Spence (1984), d’Aspremont and Jacquemin (1988), and Qiu (1997). Uncertainty is not considered in those papers. In this line of literature, firms conduct R&D in the first stage and engage in price or quantity competition in the second stage. This is different from our model as we assume that the winner takes all. The winner of the contest has the monopoly power to develop the product and there is no product market competition. This difference also leads to different social welfare results. The “winner-take-all” assumption is also adopted in Loury (1979).

The paper is organized as follows. Section II sets up the basic model. Section III focuses on contests when prizes are optimally chosen by a principal. In Section IV, we analyze contestants' R&D spending when size of prizes are endogenously determined by R&D spending. In Section V, we evaluate market performance for a given number and an endogenously determined number of contestants. Section VI summarizes the results of this paper and discusses some possible extensions.

II. The Model
There are \( n \) risk neutral contestants conducting research to produce the best R&D output, \( n \geq 2 \). The meaning of contestants may be interpreted in different ways. In one case, contestants can be different research teams in the same company. In another case,
contestants may be different companies competing to sell to the same customer. For example, in national defense procurement, firms conduct R&D to develop a new weapon. Suppose that weapons designed by different firms differ only in their reliability. The Department of Defense will procure a product from the firm providing the highest reliability. For another example, contestants may be startup companies specializing in R&D. Those firms do not have the necessary marketing expertise and they have to sell their R&D products to an established firm with strong marketing skills. The established firm only buys products from the firm providing the best product.

Suppose that the contestant who produces the best R&D output wins the contest and gets a reward of $W_1$. All other contestants lose and each loser gets a payoff of $W_2$. Define the prize spread by $\Delta W = W_1 - W_2$. The society only benefits from the best R&D output.

Three things deserve some explanations. First, the meaning of “best output” may be interpreted in different ways. It may mean the first to innovate in a patent race. In the research on missiles, best output may mean the product with the highest probability of hitting a target. In product design, best output may mean a product with the most appealing feature, like color. Second, the society only benefits from the best R&D output. This is different from the labor tournament literature, where the social output includes all contestants’ output. Third, if the highest realized R&D output is $q$, the benefit to society is $v(q)$. To simplify the analysis, we assume that the return to society is a linear function of the highest realized R&D output and $v(q) = q$. Our main results (Propositions 1 and 4) do not depend on this assumption.

Let $\mu_i$ denote a representative contestant $i$’s R&D input, $\mu_j$ denote the R&D input of one of her rivals, $i, j \in \{1, 2, \ldots n\}$. Let $\varepsilon_i$ and $\varepsilon_j$ denote some random variables. Those random variables are identically and independently distributed and their first and second moments exist and are finite. Let the distribution functions be $F$, which is assumed to be continuous and twice differentiable. The corresponding density function is $f$. The effects of spillovers are modeled by a coefficient $\beta$, which is exogenously

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given and satisfies \( 0 \leq \beta \leq 1 \). The realized R&D output \( q_i \) of contestant \( i \) is defined by the following equation\(^6\)

\[
q_i = \mu_i + \beta \sum_{j \neq i} \mu_j + \varepsilon_i.
\]

Except for the random term, (1) is commonly used in the R&D spillover literature. Examples include d’Aspremont and Jacquemin (1988) and Qiu (1997).\(^7\) When the spillover coefficient is equal to zero, (1) is similar to the formula used in the labor tournament literature. Examples include Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983).

In our model, a contestant has no control over the random effects. The variance is exogenously given and we can explicitly analyze how the variance of random effects affects contestants’ R&D spending.

Let \( C(\mu_i) \) denote contestant \( i \)’s R&D cost when her R&D input is \( \mu_i \). It is assumed that \( C' > 0 \) and \( C'' > 0 \). Let \( P_i \) denote the probability that contestant \( i \) wins. Then contestant \( i \)’s expected payoff is

\[
P_i W_1 + (1 - P_i) W_2 - C(\mu_i).
\]

For \( q_i \) to be the highest, it must be sure that

\[
\mu_i + \beta \sum_{j \neq i} \mu_j + \varepsilon_i > \mu_k + \beta \sum_{l \neq k} \mu_l + \varepsilon_k \quad \text{for all } k \neq i.
\]

From the above expression, we get

\[
(1 - \beta)(\mu_i - \mu_k) + \varepsilon_i > \varepsilon_k \quad \text{for all } k \neq i.
\]

Assume there exists a symmetric equilibrium level of R&D spending.\(^8\) In a symmetric equilibrium, all rivals’ R&D spending is the same. Let this level be denoted

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\(^6\) In equation (1), a contestant’s realized R&D output may be negative. We may impose some restrictions on the support of random variables to make sure that R&D output is always positive. The main results will not change with those restrictions.

\(^7\) d’Aspremont and Jacquemin (1988)’s approach is generalized by Kamien et al. (1992), Suzumura (1992) and others. In d’Aspremont and Jacquemin (1988), the spillover comes from R&D output spillover. In Kamien et al. (1992), the spillover comes from input spillovers. Amir (2000) shows the two approaches are equivalent when the spillover rate is not very large. In Kamien and Zang (2000), the spillover rate depends on a firm’s R&D spending. This formulation is not very tractable and they restrict their attention to duopolistic firms only.

\(^8\) A symmetric pure strategy equilibrium may not exist. When the variance of random variables is larger, a symmetric equilibrium is more likely to exist. Also asymmetric equilibrium may exist. See Nalebuff and Stiglitz (1983) for more discussion on the existence of symmetric equilibrium in tournaments.
by $\mu$. Given the R&D spending of her rivals, contestant $i$’s probability of producing the best R&D output is $F^{n-1}[(1 - \beta)(\mu_i - \mu) + \varepsilon_i]$ for a given $\varepsilon_i$. Integrating over all possible realization of $\varepsilon_i$, contestant $i$’s expected probability of winning the contest is

$$\int_{-\infty}^{+\infty} F^{n-1}[(1 - \beta)(\mu_i - \mu) + \varepsilon_i] f(\varepsilon_i) d\varepsilon_i.$$ 

Thus her expected payoff is

$$W_i \int_{-\infty}^{+\infty} F^{n-1}[(1 - \beta)(\mu_i - \mu) + \varepsilon_i] f(\varepsilon_i) d\varepsilon_i + W_2 \{1 - \int_{-\infty}^{+\infty} F^{n-1}[(1 - \beta)(\mu_i - \mu) + \varepsilon_i] f(\varepsilon_i) d\varepsilon_i\} - C(\mu_i).$$

Contestant $i$ will choose $\mu_i$ to maximize expected payoff (4). Whether we will get a corner solution or an interior solution depends on the values of parameters. Assuming an interior solution, we get

$$(1 - \beta)(W_1 - W_2) \int_{-\infty}^{+\infty} (n-1)f[(1 - \beta)(\mu_i - \mu) + \varepsilon_i] F^{n-2}[(1 - \beta)(\mu_i - \mu) + \varepsilon_i] f(\varepsilon_i) d\varepsilon_i - C'(\mu_i) = 0.$$

In a symmetric equilibrium, $\mu_i = \mu$ and the above equation reduces to

$$(1 - \beta)(W_1 - W_2) \int_{-\infty}^{+\infty} (n-1)f^2(\varepsilon_i) F^{n-2}(\varepsilon_i) d\varepsilon_i - C'(\mu) = 0.$$

Let $\varphi \equiv \int_{-\infty}^{+\infty} (n-1)f^2 F^{n-2} d\varepsilon$. The following example illustrates the economic interpretation of $\varphi$. Suppose there are only two contestants $i$ and $j$. Suppose $\varepsilon_i$ has a discrete distribution and the probability that $\varepsilon_i = 0$ is 0.2 and the probability that $\varepsilon_i = 1$ is 0.8. Similarly, $\varepsilon_j$ has a discrete density function and the probabilities that $\varepsilon_j = 0$ and $\varepsilon_j = 1$ are 0.4 and 0.6 respectively. Suppose that initially $\mu_i = \mu_j$. If contestant $i$ increases R&D spending slightly, it will increase her chance of winning by $0.2 \times 0.4 + 0.8 \times 0.6 = 0.56$. In $\varphi$, $f$ is the density function of contestant $i$, and $(n-1)fF^{n-2}$ is the density function of her best rival. Integration in $\varphi$ is the same as the summation in the discrete example. Scaling up by a constant $(1 - \beta)(W_1 - W_2)$,

\[9\] If $\partial P_i / \partial \mu_i$ is very small, $(W_1 - W_2) \partial P_i / \partial \mu_i$ will be very small as $\Delta W$ is bounded. In the limiting case of full spillovers ($\beta = 1$), $\partial P_i / \partial \mu_i = 0$. For a positive $C(\mu)$, no contestant will spend on R&D. In this case, we will get a corner solution rather than an interior one. Thus rank order tournaments may not be used when spillovers are large.
(1 - \beta)(W_1 - W_2)\varphi \text{ measures the marginal benefit of increasing R&D spending in a symmetric equilibrium. Let } \omega \equiv \int_{-\infty}^{+\infty} (n-1)f^2 F^{n-2} d\epsilon, \text{ then } \omega \text{ measures the expected marginal benefit from random factors in a symmetric equilibrium.}

We now study how a contestant’s R&D spending changes with the number of contestants.

**Proposition 1:** When contestants compete for a fixed prize, \( f' < 0 \) is a sufficient condition under which a contestant’s R&D spending decreases with the number of rivals; \( f' = 0 \) is a sufficient condition under which a contestant’s R&D spending does not change with the number of rivals; \( f' > 0 \) is a sufficient condition under which a contestant’s R&D spending increases with the number of rivals.

**Proof:** Integration by parts, we get

\[ \varphi = f(+\infty) - \int_{-\infty}^{+\infty} F^{n-1} f' d\epsilon. \]

In (6), \( f(+\infty) \) does not depend on \( n \). As a result, we get

\[ \frac{d\varphi}{dn} = \int_{-\infty}^{+\infty} (-\ln F) F^{n-1} f' d\epsilon. \]

From (5) and (7), when \( \Delta W \) is fixed, the sign of \( \frac{d\mu}{dn} \) is the same as the sign of \( f' \). QED

The restriction on the monotonicity of the density function is not standard. How \( \varphi \) changes with \( n \) cannot be signed in general. The following are three examples that R&D spending may decrease, remain the same, or increase with the number of contestants. In the first example, random variables are exponentially distributed. This is the assumption used in Loury (1979). The exponential distribution satisfies the condition \( f' < 0 \) and Loury finds that a firm’s R&D spending decreases with the number of firms in the race.\(^{10}\) Let \( \alpha \) be the parameter of the exponential distribution, then each contestant will maximize \( (W_1 - W_2)e^{\alpha(1-\beta)(\mu_i - \mu_{-i})}/n + W_2 - C(\mu_i) \) in our model. In a symmetric

\(^{10}\) One difference between our model and Loury’s is that there is time discounting of future income in his model. This difference is not essential as his main result will not change when there is no discount.
equilibrium, the first order condition for maximization is \( \alpha(1-\beta)(W_1 - W_2)/n - C'(\mu_i) = 0 \). From the first order condition, it can easily be checked that each contestant’s R&D spending decreases with the number of rivals. Thus we get the same result as the patent race literature if we assume that random variables are exponentially distributed. In the second example, random variables are uniformly distributed. For the uniform distribution, \( f' = 0 \), and a contestant’s R&D spending does not change with the number of rivals. In the third example, random variables have a power function distribution. For the power function distribution \( F(\varepsilon) = (\varepsilon/\gamma)^\theta \), \( 0 < \varepsilon < \gamma \), \( \theta > 0 \), a contestant’s R&D spending increases with the number of contestants as \( f' > 0 \)\(^{11}\).

Proposition 1 may also be used to study labor tournament and rent seeking because a common feature of those contests is that rank order rather than the absolute difference in performance is more important in affecting contestants’ payoffs. In the labor tournament literature, Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983) mainly focus on competition between two contestants. Here we generalize the tournament framework to competition among more than two contestants. Tullock (1980) analyzes a rent-seeking model, which has a similar payoff structure as the patent race models.\(^{12}\) We may reinterpret a contestant’s R&D spending as lobbying effort and there is some uncertainty regarding who will capture the rent. If we use the power function distribution to model the uncertainty in rent seeking, from Proposition 1, each rent seeker will increase lobbying effort to fight for a \textit{fixed} rent when the number of rent seekers increases!

Proposition 1 is supported by empirical evidence, which does not show a monotonic relationship between market structure and R&D spending. The “inverted-U” relationship is frequently observed in the empirical literature about the relationship between market structure and R&D spending. That is, R&D spending initially increases

\(^{11}\) Suppose each contestant conducts \( K \) experiments and only the best result is kept. We can interpret \( K f^{K-1} \) as the new density function. Its derivative with respect to \( \varepsilon \) will be \( K F^{K-2}[(K-1)f^2 + f'F] \). As \( K \) goes to infinity, \((K-1)f^2 \) dominates \( f'F \) and we get a monotonic increasing density function.

\(^{12}\) When there is no discount of future profits, the payoff structure in Loury (1979) is the same as that in Tullock (1980).

In addition to the results in Proposition 1, equation (5) provides a framework that is useful in analyzing other issues related to R&D. First, the empirical literature uses broadly defined “technological opportunities” to explain why industries differ significantly in R&D spending. See Cohen and Levin (1989) for a survey of this line of literature. Equation (5) provides two kinds of interpretations for technological opportunities. The first one is that different industries have different marginal cost in conducting R&D. Fixing the number of firms in an industry, an increase in marginal cost decreases R&D spending. The second one is that different industries have different kinds of uncertainties \( f \). The influence of uncertainty on R&D spending is studied in Proposition 1. Second, \( \Delta W \) may be interpreted as demand influence and \( f' \) may be interpreted as supply influence on R&D spending. In the literature, there is debate about whether demand or supply factors are more important in eliciting invention, see Dosi (1988) for a detailed discussion. Both demand and supply factors are important in equation (5). Third, from (5), when prizes are fixed, a larger spillover always decreases a contestant’s incentive to spend on R&D. Fourth, Reinganum (1989) shows that increasing the reward to an innovator in a patent race will increase a firm’s R&D spending and increasing the payoff to an imitator will decrease a firm’s R&D spending. Those changes can be interpreted as changes in \( W_1 \) and \( W_2 \). Finally, Loury (1979) shows that social efficiency can be established by a subsidy (tax) to firms, which is equal to changing \( \Delta W \) here.

From here on we assume that the R&D cost function has the quadratic form,

\[
C(\mu) = k\mu^2 / 2,
\]

where \( k \) is a positive constant. As \( k \) increases, the cost of implementing a given level of R&D increases. One interpretation of \( k \) is the efficiency of R&D. An increase in \( k \) can be interpreted as a decrease in R&D efficiency.
In a symmetric equilibrium, each contestant has an equal chance of winning, which is equal to $1/n$. Thus a contestant’s expected payoff $\Pi$ will be

$$
\frac{W_1}{n} + \frac{n-1}{n} W_2 - \frac{k}{2} \mu^2 = \frac{W_1 + (n-1)W_2}{n} - \frac{(1-\beta)(W_1 - W_2)^2}{2k}.
$$

We now study the relationship between the size of reward and a contestant’s expected profit.

**Proposition 2:** Suppose $C(\mu) = k\mu^2 / 2$. When contestants compete for fixed prizes, an increase in the prize to the winner will decrease each contestant's expected profit if and only if $k < n(1-\beta)^2(W_1 - W_2)^2$.

Proof: From (8), we get

$$
\frac{d\Pi}{dW_1} = \frac{1}{n} - \frac{(1-\beta)^2(W_1 - W_2)^2}{k}.
$$

From (9), $d\Pi / dW_1 < 0$ if and only if $k < n(1-\beta)^2(W_1 - W_2)^2$. QED

The following is an example that an increase in the prize to the winner decreases each contestant’s expected profit. Let $n = 2$, $\beta = 0$, $k = 1$, $W_1 = 3/2$, $W_2 = 0$ and $f(\varepsilon) = 1$ for $\varepsilon \in [-1/2,1/2]$.$^{13}$ In this case, $\mu = 3/4$ and $\Pi = 3/16$. Since a contestant’s expected profit is positive, the individual rationality constraint is satisfied. If $W_2$ is held constant, increase $W_1$ a little will decrease each contestant’s expected profit since $\partial \Pi / \partial W_1 = -1/4$.

The result that an increase in the prize to the winner may decrease a contestant’s expected payoff is counterintuitive. The explanation for Proposition 2 is as follows. Increasing the prize to the winner by one unit has two effects on a contestant’s expected payoff. First, it increases each contestant's expected profit by $1/n$ unit. Second, each contestant now spends more on R&D, but has no greater chance of winning the prize. The second effect may dominate the first one and each contestant will be harmed by an

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$^{13}$ For those parameter values, it can be verified that the second order condition for a contestant’s profit maximization is also satisfied.
increase of the prize to the winner. If we interpret contestants as rent seekers, there is over-dissipation of rents in the margin.

When the prize to the loser increases, each contestant decreases R&D spending and each of them benefits from an increase in $W_2$.

We have assumed that all contestants get the same reward when they win. One possible situation is that the payoff when contestant $i$ wins is $W_i^h$, while when contestant $j$ wins is $W_i^l$, and $W_i^h > W_i^l$. This may come from the fact that contestant $i$ is an incumbent monopolist and contestant $j$ is a potential entrant and that the monopoly profit is larger than the sum of two duopolists' profit. In this case, if $W_i^h$ increases a little, whether contestant $i$ will benefit or not is ambiguous. When $W_i^h$ increases, there are three effects on contestant $i$’s profit $\Pi_i$. The first effect is that contestant $i$’s R&D spending changes. From the Envelope Theorem, this effect equals zero in equilibrium. The second effect is that $\mu_j$ also changes. The third effect comes from $\partial \Pi_i / \partial W_i^h$, which is positive. Thus, a sufficient condition for $d\Pi_i / dW_i^h > 0$ is that the two contestants’ R&D spending are strategic substitutes. That is, an increase in $\mu_i$ causes a decrease in $\mu_j$.

III. R&D Contests with Optimally Chosen Prizes

Suppose there is a social planner (the principal). The social planner’s objective is to choose prizes to maximize social welfare. Social welfare here is defined as contestants’ expected payoff, which is the difference between social R&D output and total R&D spending. Here we are considering a second best solution as the social planner has no control over the number of contestants. In this paper, we assume that the owner of the highest R&D output has the monopoly power to develop the product and gets all the

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14 The principal can be the CEO of a company and contestants are different research teams of this company. Another interpretation is that the principal is the authority who designs the breadth of patents and contestants are different companies. By changing the breadth of patents, the principal will affect payoffs to winners and losers of patent races.

15 The R&D spending of each contestant will be the same when the principal maximizes his own profit, subject to the constraint that contestants get their reservation utilities. Compared to the case that contestants’ profits are maximized, prizes are lower now but the prize spread is the same.

16 Moldovanu and Sela (2001) also consider optimal prizes in contests. Their setup is very different from ours. In their model, each contestant has private information about her ability. There are no spillovers among contestants and the designer of the contest tries to maximize contestants’ total effort.
social surplus (this may be achieved through perfect price discrimination). Consumer surplus is not considered here. This is consistent with Loury (1979). Without loss of generality, suppose that there are only two different prizes.  

The social planner is constrained by the fact that he cannot pay more to the contestants than they produce. Since the society only benefits from the highest R&D output, the social planner’s expected revenue is the expected value of \( \max(q_i) \), for \( i \in \{1, \ldots, n\} \). His cost is the prizes awarded to the \( n \) contestants. Let \( E \) denote the expectation operator, the balanced budget constraint is

\[
E[\max(q_i)] = W_1 + (n-1)W_2.
\]

Let \( e_n \) denote the expected value of the highest order statistics of \( n \) random variables. Let \( \mu_s \) denote the equilibrium R&D spending in a symmetric equilibrium. Thus the balanced budget constraint requires

\[
(1 + (n-1)\beta)\mu_s + e_n = W_1 + (n-1)W_2.
\]

From (5), we get

\[
\mu_s = \frac{(1-\beta)(W_1-W_2)\phi}{k}.
\]

As a result, we get

\[
W_1 + (n-1)W_2 = \frac{(1-\beta)(1 + (n-1)\beta)(W_1-W_2)\phi}{k} + e_n.
\]

In a symmetric equilibrium, the number of different prizes does not affect contestants’ expected payoffs when they are risk neutral. For a given number of contestants, suppose there is a unique efficient level of R&D spending \( \mu^* \), which maximize \( \{[1+(n-1)\beta]/n]\mu-C(\mu) \). The principal implements this level of R&D by choosing the prizes optimally. Suppose there are \( m \) different prizes and \( m \leq n \). The contestant with the best R&D output gets \( W_1 \), the second highest output gets \( W_2 \), ..., and the \( m-1 \) highest output gets \( W_{m-1} \). Each of the other \( n-m \) contestants gets \( W_m \). Let \( P_1 \) denote a contestant’s probability of winning \( W_1 \), \( P_2 \) denotes the probability of winning \( W_2 \), and so on. As a result, a contestant’s expected payoff is

\[
\sum_{k=1}^{m} P_k W_k - C(\mu^*).
\]

Let \( e_n \) denote the expected value of the highest order statistics of \( n \) random variables.

From the balanced budget constraint, \( n \sum_{k=1}^{m} P_k W_k = [1+(n-1)\beta]\mu^* + e_n \). Thus a contestant’s expected payoff is

\[
\{[1+(n-1)\beta]\mu^* + e_n \}/n - C(\mu^*),
\]

which does not depend on \( m \).
In equilibrium, a contestant’s probability of winning $W_1$ is $1/n$ and her probability of getting $W_2$ is $(n-1)/n$. From (11), a contestant’s expected profit will be

$$\frac{W_1 + (n-1)W_2}{n} = \frac{(1 - \beta)(W_1 - W_2)\varphi^2}{2k}.$$ 

The principal will select prizes to maximize the above expected profit, subject to the constraint (12).

Let $\lambda$ be the Lagrange multiplier associated with the budget constraint. The first order conditions are

$$\frac{1}{n} - \frac{(1 - \beta)^2(W_1 - W_2)\varphi^2}{k} + \frac{\lambda}{k} = 0,$$

$$\frac{n-1}{n} + \frac{(1 - \beta)^2(W_1 - W_2)\varphi^2}{k} + \lambda(n-1) + \frac{\lambda(1 - \beta)(1 + (n-1)\beta)\varphi}{k} = 0.$$ 

From (12) and (13), we get

$$W_1 = \frac{[1 + (n-1)\beta]^2}{n^2k} + \frac{e_n}{n} + \frac{(n-1)[1 + (n-1)\beta]}{(1 - \beta)n^2\varphi},$$

$$W_2 = \frac{[1 + (n-1)\beta]^2}{n^2k} + \frac{e_n}{n} - \frac{1 + (n-1)\beta}{(1 - \beta)n^2\varphi},$$

$$\lambda = -1/n.$$ 

From (11), (14a) and (14b), a contestant’s R&D spending for optimally chosen prizes is defined by

$$\mu_s = \frac{1 + (n-1)\beta}{nk}.$$ 

The intuition for (15) is the following. The principal selects the prizes to achieve efficiency. Taking the externality from spillovers into account, an additional unit of R&D spending increases “raw” R&D activity by $1 + (n-1)\beta$. Since the society only values the best R&D output and each contestant has an equal chance of producing the highest R&D output, the social benefit is $(1 + (n-1)\beta)/n$, which should be equal to marginal social cost, $k\mu$. Because the social planner does not care which realization of $n$ random variables is the highest, those random effects do not affect the equilibrium level of R&D spending.
Suppose a contestant’s reservation utility is zero. For a contestant’s individual rationality constraint to be satisfied, her expected equilibrium payoff should be larger than zero. We now demonstrate that this is satisfied. From (8), (14a), and (14b), a contestant’s expected payoff is
\[ \frac{1 + (n - 1)\beta}{2kn} + \frac{e_n}{n}, \]
which is strictly positive.

From (14a) and (14b), the optimal prize spread is given by
\[ \Delta W = \frac{1 + (n - 1)\beta}{(1 - \beta)n\varphi}. \]  

Differentiate (16) with respect to \( \beta \), we get
\[ \frac{d\Delta W}{d\beta} = \frac{1}{(1 - \beta)^2 n\varphi}. \]

Since \( \varphi \) is always positive, (17) is positive. Thus the optimal prize spread increases with the degree of spillovers. This is not surprising. When a contestant conducts R&D, she increases the social R&D output by increasing her own and her rivals’. The difference between a contestant’s private benefit and social benefit comes from the externality of spillovers. The optimal prize structure recognizes this externality and increases the reward to the winner accordingly.\(^{18}\)

From (16), \( \Delta W \) does not depend on \( k \). We may have thought that a harder project demands a higher reward for the winner. However, when \( k \) increases, marginal cost of R&D increases and less R&D should be encouraged. These two factors cancel out. As a result, the reward does not change with \( k \).

We now characterize individual and total R&D spending when prizes are optimally chosen.

PROPOSITION 3: Suppose that prizes are chosen optimally. (i) A contestant’s R&D spending increases with the degree of spillovers and the efficiency of R&D, and decreases with the number of contestants. (ii) Total R&D spending increases with the number of contestants, the efficiency of R&D, and the degree of spillovers.

\(^{18}\) When the spillover rate is very large, losers may get a negative expected prize in equilibrium. In reality, contestants may be protected by limited liability. This may affect the implementability of the prizes.
Proof: (i) Differentiate (15), we get \( \frac{d\mu_s}{d\beta} > 0, \frac{d\mu_s}{dk} < 0, \frac{d\mu_s}{dn} < 0 \).

(ii) From (15), we get the total expenditure of \( n \) contestants

\[
n\mu_s = \frac{1 + (n-1)\beta}{k}.
\]

From (18),\( \frac{d(n\mu_s)}{d\beta} > 0, \frac{d(n\mu_s)}{dk} < 0, \frac{d(n\mu_s)}{dn} > 0 \). \[\text{QED}\]

Given R&D spending, the expected social surplus increases when the spillover rate increases.\(^{19}\) As a result, social optimal R&D spending increases with the degree of spillovers.

Should the social planner increase the reward to the winner when the number of contestants increases? From (16), when the number of contestants increases, there are two effects affecting the optimal prize spread. The first effect works through the R&D level and the second one works through \( \varphi \), which is the marginal benefit of increasing R&D. For the first effect, when the number of contestants increases, there is too much duplication of R&D. As the value of every contestant’s R&D decreases, less R&D should be encouraged. For the second effect, from Proposition 1, when the number of contestants increases, \( \varphi \) may decrease, remain the same, or increase. For random variables with a decreasing density function, \( \varphi \) will decrease with the number of contestants. In this case, the two effects work in the same direction and the optimal prize spread increases with the number of contestants.

Will the expected value of the best R&D output always increase when the number of contestants increases? Not necessarily. There are three effects when the number of contestants increases. First, the expected value of the highest order statistics always increases. Second, for any given spillover rate, the total amount of spillovers each contestant receives increases. Third, each contestant conducts less R&D as there is more duplication. When the degree of spillovers and the change of the expected value of the highest order statistics are sufficiently small, the best R&D output will decrease with the number of contestants.

\(^{19}\) See Spence (1984) for related discussion.
number of contestants. This result is different from Loury (1979)'s result as he finds that the expected invention date always moves ahead (which is the same as the expected best output always improves) when the number of firms in the race increases.

IV. “Winner-Take-All” R&D Contests

In Section III, we have assumed that there exists a principal who chooses prizes optimally to maximize contestants’ expected payoffs. When there is such a principal to set prizes, prizes are fixed. An interesting and natural question is that what would happen when the reward is endogenously determined by R&D spending. In this section, we study “winner-take-all” competition. Now, the prize to the winner is not fixed but depends on her R&D output.

In a “winner-take-all” contest, the winner gets all surplus and each loser gets zero. For contestant $i$, if $\mu_i + \beta \sum_{j \neq i} \mu_j + \varepsilon_i$ is the highest among all contestants, her reward will be $\mu_i + \beta \sum_{j \neq i} \mu_j + \varepsilon_i$. Otherwise, she gets nothing. If contestant $i$ spends $\mu_i$, her probability of producing the best R&D output when each of her rivals spends $\mu_m$ will be $\int_{-\infty}^{\infty} F_{n-1}^{(\cdot)}[(1 - \beta)(\mu_i - \mu_m) + \varepsilon_i] f(\varepsilon_i) d\varepsilon_i$. Thus her expected payoff is

$$\Pi_i = \int_{-\infty}^{\infty} (\mu_i + \beta \sum_{j \neq i} \mu_j + \varepsilon_i) F_{n-1}^{(\cdot)}[(1 - \beta)(\mu_i - \mu_m) + \varepsilon_i] f(\varepsilon_i) d\varepsilon_i - k\mu_i^2 / 2 .$$

The first order condition for contestant $i$’s expected payoff maximization is

$$\int_{-\infty}^{\infty} F_{n-2}^{(\cdot)}[(1 - \beta)(\mu_i - \mu_m) + \varepsilon_i] f(\varepsilon_i) d\varepsilon_i$$

$$+ (1 - \beta) \int_{-\infty}^{\infty} (\mu_i + \beta \sum_{j \neq i} \mu_j + \varepsilon_i) (n-1) f[(1 - \beta)(\mu_i - \mu_m) + \varepsilon_i] F_{n-2}^{(\cdot)}[(1 - \beta)(\mu_i - \mu_m) + \varepsilon_i] f(\varepsilon_i) d\varepsilon_i$$

$$- k\mu_i = 0 .$$

20 The following is an example. Suppose the random variables are uniformly distributed on $[-a, a]$. When there is no spillover, $a < 1/2$ is a sufficient condition under which the expected highest R&D output decreases with the number of contestants.

21 In the patent race literature, the prize to the winner is usually fixed. The race determines which firm wins the prize, but not the magnitude of the prize. In our model, the prize to the winner depends on her R&D output.

22 It is assumed that the second order condition is satisfied. An example provided in the paragraph following Proposition 4 shows the second order condition is satisfied.
Equation (20) can be interpreted as follows. Suppose contestant $i$ increases her R&D spending by one unit. The benefit has two components. First, for a given probability of winning, it increases her payoff conditioning on winning by $1/n$. Second, for a given reward, it increases her expected probability of winning. Equation (20) comes from the requirement that marginal benefit should equal marginal cost.

In a symmetric equilibrium, $\mu_i = \mu_m$. Thus (20) reduces to

\[
\int_{-\infty}^{+\infty} F^{n-1}(\epsilon_i)f(\epsilon_i)d\epsilon_i + \int_{-\infty}^{+\infty} ((1 + (n - 1)\beta)\mu_m + \epsilon_i)(1 - \beta)(n - 1)f^2(\epsilon_i)F^{n-2}(\epsilon_i)d\epsilon_i - k\mu_m = 0.
\]

Remember that $\omega \equiv \int_{-\infty}^{+\infty} (n - 1)f^2F^{n-2} d\varepsilon$ is the expected marginal benefit from random factors in a symmetric equilibrium. Since $\int_{-\infty}^{+\infty} F^{n-1}(\epsilon_i)f(\epsilon_i)d\epsilon_i = 1/n$, (21) is equivalent to

\[
\frac{1}{n} + (1 - \beta)\omega = \frac{k - (1 - \beta)[1 + (n - 1)\beta]^{\frac{1}{2}}}{\mu_m}.
\]

Now we are ready to study the influence of uncertainty and spillovers on contestants’ R&D spending in the following subsections. We will also study contestants’ incentives to increase the efficiency of R&D.

**IV.1: The Influence of Uncertainty**

For a given number of contestants, in general, the expected reward to the winner when prizes have the winner-take-all nature is not equal to the optimally chosen prize spread. This raises one interesting question. For a given number of contestants, if the fixed reward is set to be the expected value of a variable reward, can we rank R&D spending in these cases? Intuitively, we may expect that the R&D spending when reward to the winner increases with R&D output to be larger than the R&D spending when reward is fixed. When prizes are fixed, increasing R&D spending only increases a contestant’s chance of winning. When the reward to the winner increases with R&D output, increasing R&D not only increases a contestant’s probability of winning, but also increases the R&D output that will be enjoyed by the winner. With this additional
incentive, a contestant may spend more on R&D to compete for a smaller variable reward than for a fixed prize. However, this intuition is incomplete. There is a third effect that $\omega$ also changes. When $\mu_i$ increases, a lower realization of $\varepsilon_i$ is needed for winning.

The formal comparison works as follows. When the market R&D spending of a contestant is $\mu_m$, the expected reward to the winner is $W_m = \mu_m [1 + (n - 1)\beta] + e_n$. From (5), the corresponding R&D spending for a fixed prize spread with the same value is

$$\mu_s \bigg| w_n = (1 - \beta) \{\mu_m [1 + (n - 1)\beta] + e_n\} \phi.$$  

Combining (22) with the above equation, we get

(23)  

$$\mu_s \bigg| w_n = \mu_m - \frac{1}{n} + (1 - \beta)\omega - (1 - \beta)e_n \phi.$$  

From (23), whether $\mu_s \bigg| w_n$ is larger or smaller than $\mu_m$ depends on the sign of $\frac{1}{n} + (1 - \beta)\omega - (1 - \beta)e_n \phi$. When there are full spillovers, $\mu_m$ is always larger than $\mu_s$. This is not surprising since the prize spread needs to increase to infinity to encourage contestants to conduct R&D for fixed prizes. When there is no spillover, the following are two examples with opposite results.

**Example 1:** Let $k=1$. Suppose random variables are uniformly distributed on $[-a, a]$. From (15), $\mu_s = 1/n$. From (22), $\mu_m = a/(2a - 1)$. From (16), $\Delta W = 2a/n$. Market reward for the winner is $W_m = \frac{a}{2a - 1} + \frac{n - 1}{n + 1} a$. When $n = 2$ and $a > 5/4$ or when $n = 3$ and $a > 7/2$, it can be easily checked that $\Delta W > W_m$ and $\mu_s < \mu_m$. So in a market equilibrium contestants are spending more for an expected smaller variable prize than for a larger fixed prize!

**Example 2:** For the power function distribution discussed in Section II, when there are two contestants and $\theta$ equals 0.6, it can be verified by checking (23) that the R&D spending when contestants compete for a variable prize is smaller than when they compete for a fixed prize.

As uncertainty is a key feature of R&D, how risk affects a contestant’s R&D spending is an interesting topic to study. Keep the mean return of a project constant and increase the variance, how will a risk neutral contestant adjust her R&D spending? There
are two effects when variance increases. First, R&D spending may decrease since now winning depends more on better luck than on higher R&D spending. Second, a contestant’s payoff is convex in the realization of random effect, though she is risk neutral. The reason is that the random effect matters only when she wins. The second effect increases a contestant’s incentive to spend on R&D. The following two examples illustrate that either effect can dominate the other.

Example 3: Consider the power function distribution in Section II. To focus on the influence of risk, suppose that there are no spillovers. Let $k=1$, $n=2$. In the first case, $\theta = 3/4$, $\gamma = 7/3$, the variance of the random variables in this project will be $16/33$. From (22), R&D spending will be $49/29$. In the second case, $\theta = 5/8$, $\gamma = 13/5$, the variance of the random variables will be $64/105$ and R&D spending will be $169/83$. In both cases, the mean equals 1. Both the variance and R&D spending in the first case is smaller than those in the second case.

Example 4: It can be checked that an increase in $a$ decreases a contestant’s R&D spending when random variables are distributed uniformly on $[-a, a]$.

IV.2: The Influence of Spillovers

A natural topic about spillovers is how a contestant's incentive to conduct R&D is affected by the degree of spillovers. We now study this relationship. Let

$$\beta_c \equiv 1 - \frac{1}{n\omega} \left( \frac{\sqrt{(n-1)\varphi[(n-1)\varphi + n^2\omega \varphi + n^2\omega^2k]}}{(n-1)\varphi} - 1 \right).$$

PROPOSITION 4: Consider a “winner-take-all” R&D competition. (i) Suppose $\omega < 0$, a sufficient condition for $d\mu_m / d\beta > 0$ is that $\beta \in [0, (n-2)/(2n-2)]$. (ii) Suppose $\omega > 0$ and $(n-2)\varphi / n + (n-1)\omega \varphi - \omega k > 0$. If $n^2\varphi \geq 4(n-1)k$, $d\mu_m / d\beta > 0$ for $\beta \in [0, (n-2)/(2n-2)]$; If $n^2\varphi < 4(n-1)k$, $d\mu_m / d\beta > 0$ for $\beta \in [0, \beta_c]$.

Proof: From (20), we get

$$\text{Sign} \left( \frac{d\mu_m}{d\beta} \right)$$

(24)
\[
= \text{Sign} \{(n-2-2(n-1)\beta)(\frac{1}{n} + (1-\beta)\omega)\varphi - \omega(k - (1-\beta)(1+(n-1)\beta)\varphi)\}.
\]

Part i is obvious from (24).

When \(\omega > 0\), for the right side of (24) to be positive, we need

\[
(n-1)\omega \varphi \beta^2 - 2(n-1)(\frac{1}{n} + \omega)\varphi \beta + (\frac{n-2}{n} \varphi + (n-1)\omega \varphi - k\omega) > 0.
\]

Let the left side of (25) equal zero and solve it as an equation of \(\beta\), we get two roots. The larger root is always larger than \((n-2)/(2n-2)\) and is discarded. When \((n-2)\varphi/n + (n-1)\omega \varphi - \omega k > 0\), the smaller root \(\beta_c\) is positive. And \(n^2\varphi = 4(n-1)k\) is the condition for \(\beta_c = (n-2)/(2n-2)\). This proves Part ii of the proposition. QED

The following is an example that R&D spending increases with the degree of spillovers. Let \(n = 3\), \(k = 1/2\), \(f(\varepsilon) = 1/6\) for \(\varepsilon \in [-3,3]\).\(^23\) In this case, \(\omega > 0\) and \(\mu_m = \frac{3-\beta}{3-(1-\beta)(1+2\beta)}\). A contestant’s expected profit is at least \(7/16\), which is positive. For \(\beta \in [0,3-\sqrt{17}/2]\), \(d\mu_m/d\beta > 0\).

Our result that a contestant’s R&D spending may increase with the degree of spillovers is striking. The intuition for Proposition 4 is the following. For a given prize spread \(\Delta W\) and a given spillover rate \(\beta\), let the corresponding optimal R&D spending be \(\mu_m(\Delta W, \beta)\). Now suppose \(\beta\) increases. From \(\frac{d\mu_m}{d\beta} = \frac{\partial \mu_m}{\partial \beta} + \frac{\partial \mu_m}{\partial \Delta W} \frac{\partial \Delta W}{\partial \beta}\), there are two effects when the spillover rate increases. The first effect comes from \(\frac{\partial \mu_m}{\partial \beta}\). An increase in the degree of spillovers always decreases a contestant’s chance of winning, thus \(\frac{\partial \mu_m}{\partial \beta} < 0\). This effect is well documented in the literature. The second effect comes from \(\frac{\partial \mu_m}{\partial \Delta W} \frac{\partial \Delta W}{\partial \beta}\). From (5), a contestant’s R&D spending always increases with the prize spread, so \(\frac{\partial \mu_m}{\partial \Delta W} > 0\). For a given level of R&D spending, a larger spillover rate causes R&D output to increase. With the “winner-take-all” assumption, a

\(^{23}\) It can also be checked that the second order condition for a contestant’s profit maximization is satisfied for those parameter values.
higher R&D output means a larger expected prize. Thus $\partial \Delta W / \partial \beta > 0$. As a result, the second effect is positive and the two effects go in opposite directions. The second effect may be termed as “a larger pie effect” and it has not been explored in the literature. Under some parameter values, the second effect dominates the first one and R&D spending increases with the degree of spillovers.

Our result that R&D spending may increase with the degree of spillovers does not depend on the assumption that return to the society is a linear function of the highest R&D output. Suppose a higher R&D output means a lower constant marginal cost. When a constant marginal cost monopolist faces linear demand, the monopolist’s return is convex in realized R&D output. If this is the case, our result will be strengthened because the reward to the winner increases at a faster rate than quality increases and the second effect will be stronger than the linear situation. If return to the society is a concave function of the highest R&D output, the interval of the spillover rates for R&D spending to increase with the degree of spillovers is shorter, but still exists.

In the literature, Spence (1984) provides a formal study of the influence of spillovers in R&D. In Spence (1984), firms conduct process R&D in the first stage and R&D spending determines production cost in the second stage. There is no uncertainty in R&D outcome and a higher R&D spending gives rise to a lower marginal cost deterministically. He shows that spillovers always reduce a firm's incentive to spend on cost reduction R&D. The intuition is the following. With a larger spillover, a firm will decrease its competitors’ cost by a larger amount and this makes its rivals tougher competitors in the production stage. Therefore, large spillovers discourage R&D spending. In our model, the winner of the contest has monopoly power to develop the product and there is no product market competition.

In the literature, there are some empirical studies showing that spillover does not necessarily discourage R&D spending. Using data from Canadian industries, Bernstein (1988) finds that there is a positive incentive effect on the R&D capital of the intra-industry spillover rate in industries with relatively large R&D propensities. Levin (1988) also reports empirical results that are consistent with our theory. Based on survey data in 130 industries, Levin (1988) finds that the four industries with the highest level of spillovers (computers, communications equipment, electronic components and aircraft)
also rank high in R&D intensity.\textsuperscript{24} His results give some support to the hypothesis that spillovers are conducive to rapid technical progress, but no support to the hypothesis that spillovers discourage R&D investment. Puzzled by the empirical finding, Levin and Reiss (1988) think that the distinction between the extent and the productivity of spillovers is a possible explanation. They argue that R&D intensity increases with the productivity of spillovers, but falls with the extent of spillovers. Cohen and Levinthal (1989) also find that spillovers encourage R&D spending. In addition, they find that a higher spillover rate is more conducive to R&D when there are more firms in an industry. This weakly supports our result as the threshold spillover rate \((n - 2)/(2n - 2)\) in Proposition 4 increases with the number of firms in an industry. To explain their empirical finding, Cohen and Levinthal assume that R&D also increases a firm’s ability to exploit industry knowledge.\textsuperscript{25} With this additional incentive, a larger spillover rate may increase a firm’s R&D spending. However, neither of the above papers gives an explicit structural model to sign the relationship between R&D spending and the degree of spillovers.\textsuperscript{26}

\textbf{IV.3: Incentives to Increase the Efficiency of R&D}

Suppose that contestants have different R&D knowledge. Contestants are still symmetric in the sense that different knowledge gives rise to the same R&D cost function. Remember that \(k\) is the efficiency of R&D spending. Suppose that \(k\) will decrease if contestants share their knowledge. That is, every contestant becomes more efficient in R&D. Will contestants have incentives to lower \(k\)? We now study contestants’ incentives to increase the efficiency of R&D.

\textbf{PROPOSITION 5:} (i) A sufficient condition for it to be profitable for contestants to increase the efficiency of R&D in a “winner-take-all” competition is that the market

\textsuperscript{24} Managers were asked to rank the degree of spillovers in a survey conducted by Levin and colleagues.

\textsuperscript{25} Following Cohen and Levinthal (1989), Kamien and Zang (2000) explore the influence of absorptive capacity in their study on research joint ventures.

\textsuperscript{26} The essence of their models is consistent with our model. In Levin and Reiss (1988), R&D increases with the productivity of spillovers. When the productivity increases, firms are in fact competing for a larger prize. In Cohen and Levinthal (1989), a firm cannot automatically benefit from R&D conducted by other firms. The learning incentive of R&D plays a similar role as the “winner-take-all” assumption in our model.
level of R&D spending is not larger than social optimum. (ii) When market R&D spending is more than social optimum, a necessary and sufficient condition for contestants to benefit from an increase in R&D efficiency is that

\[(1 - \beta)[1 + (n - 1)\beta]\varphi < (n - 1)k .

Proof: From (19), we get

\[
\Pi_m = \int_{-\infty}^{+\infty} \left[ ((1 + (n - 1)\beta)\mu_m + \varepsilon_i) F_n^{-1}(\varepsilon_i) f(\varepsilon_i) d\varepsilon_i - \frac{k\mu_m^2}{2}. \right.
\]

As a result, we get

\[
(26) \quad \Pi_m = \frac{\mu_m[1 + (n - 1)\beta]}{n} + \frac{\varepsilon_n}{n} - \frac{k\mu_m^2}{2}.
\]

Differentiate (26) with respect to \( k \), we get

\[
(27) \quad \frac{d\Pi}{dk} = \left( \frac{1 + (n - 1)\beta}{n} - k\mu_m \right) \frac{\partial \mu_m}{\partial k} = -\frac{1}{2}\mu_m^2.
\]

(i) From (22), \( \frac{\partial \mu_m}{\partial k} < 0 \). From (27), a sufficient condition for \( d\Pi/dk < 0 \) is that \( \mu_m \leq \frac{1 + (n - 1)\beta}{kn} \), which is the socially efficient level of R&D spending.

(ii) Suppose \( \mu_m > \frac{1 + (n - 1)\beta}{kn} \). For \( d\Pi/dk < 0 \), from (27), we need

\[
(28) \quad \left| \frac{\partial \mu_m}{\partial k} \right| < \frac{\mu_m^2 / 2}{k\mu_m - (1 + (n - 1)\beta) / n}.
\]

From (22) and (28), we need

\[
(29) \quad [k + (1 - \beta)(1 + (n - 1)\beta)\varphi]\mu_m < 1 + (n - 1)\beta.
\]

Since \( \mu_m > \frac{1 + (n - 1)\beta}{kn} \), (29) will be satisfied if and only if

\[(1 - \beta)[1 + (n - 1)\beta]\varphi < (n - 1)k . \]

QED

The intuition for Proposition 5 is the following. The right side of (27) consists of two terms representing the two effects from an increase in R&D efficiency. First, it increases R&D spending. Second, the cost of conducting any given level of R&D decreases. When the market R&D spending is less than social optimum, an increase in R&D spending will benefit a contestant. Thus the two effects work in the same direction.
and each contestant’s expected profit will increase. When market R&D is more than social optimum, from (28), it will be profitable for contestants to share their expertise if R&D spending will not increase too much. A larger $k$ is more likely to make this happen and this explains the condition $(1 - \beta)[1 + (n - 1)\beta]\rho < (n - 1)k$.

Market R&D level does not equal social optimal R&D spending because market prize is not equal to the socially optimal prize. From (16), the optimal prize awarded to the winner increases with the degree of spillovers. So the market prize to the winner will be more likely to be less than social optimum when the spillover rate is large. Thus, a larger spillover rate will increase contestants’ incentives to increase the efficiency of R&D. One implication is that firms in industries with higher spillover rate are more likely to engage in establishing research joint ventures.

There is some evidence that competing firms engage in activities to increase the efficiency of R&D. von Hippel (1988, Chapter 6) studies cooperation between rivals: the informal trading of technical know-how between firms. His case study involves trading of proprietary know-how between US steel minimill producers. His survey reveals that many minimills engage in routine knowledge sharing through their engineers. This kind of knowledge sharing may involve training workers of competing firms at no charge, helping competitors to set up unfamiliar equipment. Reciprocity is emphasized in this kind of trading as each firm pays great attention to what it gives to and what it can expect from its trading partners.

V. Evaluation of Market Performance
In this section, we evaluate market performance. First, we compare market R&D spending with social optimum for a given number of contestants. Then we study whether the number of contestants endogenously determined by free entry condition is socially optimal.

V.1: Fixed Number of Contestants
First, let us evaluate market performance for a given number of contestants.
PROPOSITION 6: (i) Suppose $k\omega + \phi > 0$. When there is no spillover, a contestant’s R&D spending is more than social optimum; when there is full spillover, a contestant’s R&D spending is less than social optimum. (ii) There exists a spillover rate at which the market R&D spending is socially optimal.

Proof: (i) When $\beta = 0$, from (15) and (22), $\mu_m$ is always larger than $\mu_s$.

When $\beta = 1$, from (15), $\mu_s = 1/k$. From (22), $\mu_m = 1/kn$. So $\mu_m$ is always smaller than $\mu_s$ unless $n = 1$.

(ii) We know that $\mu_s$ and $\mu_m$ are continuous in the spillover rate. Combine this with Part i of this proposition, there exists a spillover rate that the market R&D spending is socially optimal. QED

A sufficient condition for $k\omega + \phi > 0$ is that random variables are symmetrically distributed with respect to the origin. We may wonder that a contestant has no incentive to spend on R&D when there are full spillovers. However, this is not correct. Though spending on R&D does not increase a contestant’s chance of winning when there are full spillovers, it increases her reward if she does win. Even taking into account that a contestant only has $1/n$ chance of winning, it may still be profitable for her to spend on R&D. In fact, a contestant is maximizing $k\mu/n - k\mu^2/2$, and the optimal R&D spending level is $1/n$.

Another benchmark case is when there is no spillover. An increase of $\Delta \mu$ in a contestant’s R&D spending may benefit her in two ways. First, it increases her payoff by $\Delta \mu$ if she wins. Second, it increases her chance of winning. The social benefit is $(1 + (n-1)\beta)\Delta \mu$, which is larger than $\Delta \mu$ if $\beta > 0$. In general, whether $\mu_m > \mu_s$ or not is ambiguous. When there is no spillover, $\Delta \mu = (1 + (n-1)\beta)\Delta \mu$. With the existence of the second benefit, a contestant’s private incentive to spend on R&D exceeds the social incentive. As a result, when $\beta = 0$, $\mu_m$ is larger than $\mu_s$. The reason is that the society only values the highest R&D output and there is much duplication of R&D spending when there are no spillovers.
V.2: Endogenous Number of Contestants

Now we allow the number of contestants to be endogenously determined in a free entry equilibrium. We compare the market equilibrium number of contestants determined by the zero profit condition with the socially optimal number of contestants. From (26), we get

$$\Pi_m = \frac{\mu_m [1 + (n-1)\beta]}{n} + \frac{e_n}{n} - \frac{k\mu_m^2}{2},$$

where $\Pi_m$ is the profit of an individual firm in the market equilibrium.

From the above equation, we get

$$\Pi_m = k\mu_m \mu_s + \frac{e_n}{n} - \frac{k\mu_m^2}{2}. \tag{30}$$

Let $\Pi_s$ denote the profit of a contestant in a social optimum. From (8) and (12), we get $\Pi_s = k\mu_s^2 + \frac{e_n}{n} - \frac{k\mu_m^2}{2}$. As a result, we get

$$\Pi_s = \frac{k\mu_s^2}{2} + \frac{e_n}{n}. \tag{31}$$

From (30) and (31), we get

$$\Pi_s - \Pi_m = \frac{k(\mu_s - \mu_m)^2}{2}. \tag{32}$$

From (32), for a given number of contestants, the expected profit of a contestant when the prizes are optimally designed is always larger than or equal to the expected profit of a contestant when prizes have the “winner-take-all” nature.

The social planner will choose the number of contestants to maximize expected social surplus. The social benefit is the highest realized R&D output as a function of equilibrium R&D spending. The social cost is the R&D cost. Differentiate (31) with respect to $n$, we get

$$\frac{d(n\Pi_s)}{dn} = \Pi_s + (\frac{\partial e_n}{\partial n} - \frac{e_n}{n}) + n \frac{\partial (k\mu_s^2 / 2)}{\partial n}. \tag{33}$$

From the right side of (33), when an additional contestant enters the race, it has three effects on social welfare. The first term says that she contributes to social welfare by her profit. A contestant is concerned with $e_n / n$, while the social planner is concerned
with \( \partial e_n / \partial n \), this difference is the second term. A contestant does not take into account of the externalities she impose on others as there is now more duplication of R&D spending. The cost saving externality is the third term. From (32) and (33), we get

\[
(34) \quad \frac{d(n\Pi_s)}{dn} = \Pi_m + \frac{k}{2}(\mu_s - \mu_m)^2 + (\frac{\partial e_n}{\partial n} - \frac{e_n}{n}) + nk\mu_s \frac{\partial \mu_s}{\partial n}.
\]

In the right side of (34), the second term is positive, the sign of the third term is undetermined and the fourth term is negative. Examples can be found to show that the free entry equilibrium number of contestants may be less, equal to or more than the socially optimal number of contestants.

Loury (1979) shows that competitive entry induces too many firms joining the innovation race. Loury’s result depends critically on the existence of an increasing returns to scale stage.\(^{27}\)

Patent law may have two effects on R&D activities. First, it changes the rewards to innovators and imitators. Second, the spillover rate may be affected by patent policy. Without imposing additional structure on the current model, we cannot determine whether those kinds of changes will increase or decrease social welfare.

VI. Conclusion

We have established a tournament framework to study the influence of uncertainty and spillovers in R&D. This framework is also useful in studying other kinds of contests, such as rent seeking. The main results are summarized here. First, the relationship between market structure and a contestant’s R&D spending is sensitive to the form of uncertainty that characterizes the R&D process. Second, equilibrium R&D spending may

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\(^{27}\) In Loury (1979), investment at level \( x \) produces a constant and instantaneous probability \( h(x) \) of success. Loury assumes that there is an investment level \( \bar{x} \), where \( h(\bar{x})/x \) is the largest. That is, the R&D technology involves some initial increasing returns to scale, then diminishing returns are encountered. In Loury’s model, there is no fixed cost of investment. When a social planner chooses the number of firms and the level of investment of each firm jointly, he will obviously set the expenditure of a firm to be \( \bar{x} \). The social planner can then choose the number of firms so that the expected date of innovation will be optimal. When the investment is set at \( \bar{x} \), a firm will have a positive profit and this cannot be a free entry equilibrium. As a firm’s expected profit decreases with the number of firms, free entry equilibrium involves too many firms joining the race.
increase with the degree of spillovers. Third, an increase of the prize to the winner may decrease each contestant’s expected payoff.

We also use our framework to study other issues related to R&D and competition. We show that reward to the winner may not increase with the number of contestants. Also, a sufficient condition for contestants to increase the efficiency of R&D is that market R&D spending is not larger than social optimum. Finally, we evaluate market performance for exogenously given number of contestants and endogenously determined number of contestants.

The paper may be extended in various directions. The realized R&D output may need to be higher than some given standard, like the current cost level, in order to be valuable. Introducing a minimum R&D output standard will not change the analysis essentially. The realizations of random variables among contestants may be correlated rather than independent. Also, instead of assuming that contestants have equal abilities in conducting R&D, we may consider heterogeneous contestants in the future.

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