Fragility of Purely Real Macroeconomic Models

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Disclaimer

- The views expressed in this talk are my own.

- They may not be shared by others in the Federal Reserve System.
Purely Real Macroeconomic Models

- Over past thirty years, much macroeconomic research has focused on purely real models.

- Basic modelling strategy: ignore any impact of nominal rigidities on real outcomes.

- This strategy is based on two key assumptions.

- The first assumption is empirical: The price level adjusts rapidly to shocks.
  - I won’t be talking about this assumption.
Fragility: A Theoretical Problem

- The second key assumption is theoretical.

- Consider a model with nearly instantaneous price level adjustment.

- Implicitly assumed: Real outcomes in that model are close to...

- The real outcomes in an equivalent, purely real, model.

**Point of this paper: this assumption is wrong.**
Basic Intuition: Dependence on the Future

- Start with a purely real model.

- Build an equivalent model with nearly instantaneous price level adjustment.

- Assume a nominal interest rate peg (or lower bound).

- In the latter model: households expect "gaps" to disappear very quickly.
  
  - "Gaps" = difference between the two models’ outcomes.
• But suppose households expect a gap of some kind in the long run.

• Gaps are expected to disappear quickly.

• Hence, even if this long-run gap is tiny, the current gap must be large.

• In other words: in any model with nearly instantaneous price level adjust-
ment:

Current real outcomes are highly sensitive to beliefs about the long run.

• This dependence is not present in purely real models (all gaps are zero).
Fragility Problem

- All models are necessarily abstractions

- But not all models are *useful* abstractions.

- Purely real models are highly sensitive to a small perturbation.

- Purely real models are too *fragile* to be of use.
Fixing the Fragility Problem

• The fragility of purely real models comes from their incompleteness.

• To be sufficiently robust to be useful, a model needs four ingredients.

1. explicit connection between real activity and inflation (Phillips curve).

2. a central bank reaction function.

3. a fiscal policy reaction function.

4. formation of expectations about the long run.
Key Technical Element in Analysis

- I consider a wide class of real models.

- Key element: the model horizon is finite (albeit arbitrary).
  - This allows beliefs about long-run gaps to vary over dates and states.

- In an infinite horizon model, long-run gaps are necessarily zero.
References

- Literature on surprising power of forward guidance.
- Literature on impact of price rigidities at the zero lower bound.
- Commentary on "neo-Fisherian" approach to monetary policy.
Outline

1. Perturbing Purely Real Models

2. Two Results

3. Four Missing Ingredients

4. Conclusions
Perturbing Purely Real Models
A Class of Purely Real Models

• Time is discrete and indexed \( t = 1, \ldots, T \).

• Models have four components: \((X, M, \Gamma, g)\)

• \(X\) is arbitrary set ... \(x\) is a specification of economic characteristics.
  
  – Could capture evolution of capital or wealth distribution.

• \(M\) is the set of stochastic (cross-person average) marginal utility processes.
• \( \Gamma \) maps \( M \) into \( X \).

  – given marginal utility process, what economic outcomes occur?

• \( g \) maps \( X \) into \( M \).

  – given economic outcomes, what is marginal utility of consumption?
Equilibrium in a Purely Real Model

Given a model \((X, M, \Gamma, g)\), an equilibrium \((x^{REAL}, m^{REAL})\) satisfies:

\[
x^{REAL} = \Gamma(m^{REAL})
\]

\[
m^{REAL} = g(x^{REAL})
\]

- Interpretation: \(m^{REAL}\) is an (inverse) tracker of aggregate demand.

- First condition: aggregate demand \(m^{REAL}\) generates outcomes \(x^{REAL}\).

- Second condition: \(x^{REAL}\) generates aggregate demand \(m^{REAL}\).
  - Roughly: supply \(x^{REAL}\) creates its own demand \(m^{REAL}\).
Natural Real Interest Rate

- Define the natural real interest rate to be:

\[ r_{t}^{NAT} = \frac{m_{t}^{REAL}}{E_{t}m_{t+1}^{REAL}} \]

- For talk: I assume \( r_{t}^{NAT} \) is constant at \( r^{NAT} \).
Monetary Policy

• Endow agents with interest-bearing outside asset (money).

• Price level is denoted $p_t$.

• Inflation rate $\pi_t = \frac{p_t}{p_{t-1}}$.

• Central bank sets interest rate $R(\pi_t)$ in period $t$.
  
  – $R$ is strictly increasing.
Fiscal Policy

- All agents face *nominal* lump-sum tax equal in last period $T$.
  - Lump-sum tax equal to outstanding per-capita money.

- This fiscal policy is Ricardian.

- Price level is indeterminate in final period.

- Price level in earlier periods is determined by expectations of $p_T$. 
Aggregate Asset Pricing Equation

- Money is an asset, and its price satisfies Euler equation.

- Average Euler equation:

\[
\frac{m_t}{p_t} = R(\pi_t)E_t \frac{m_{t+1}}{p_{t+1}}, \quad t = 1, \ldots, (T - 1)
\]

- Note: borrowing constraints can’t bind.

- All agents need to hold sufficient financial wealth to pay period $T$ taxes.
Connecting Real Activity and Inflation: Phillips Curve

• Start with equilibrium marginal utility $m^{REAL}$ from real model.

• The inverse "Phillips curve" is:

$$\left(\frac{m_t}{m_t^{REAL}}\right) = \left(\frac{\pi_t}{\pi^*}\right)^{-\phi}$$

• Connects aggregate demand $m$ with inflation $\pi$. 
Price Level Adjustment and the Phillips Curve

- If $\phi = 0$, $m_t = m_t^{REAL}$: price level adjusts instantaneously

- If $\phi$ is near zero, then price level adjusts nearly instantaneously.

- Empirical aside: recent "disappearance" of Phillips curve implies $\phi$ is near infinity, not near zero.
Equilibrium in a Model with Price Rigidities

Start with equilibrium \((x^{REAL}, m^{REAL})\) to purely real model \((X, M, \Gamma, g)\).

The equilibrium \((x^{MON}, m^{MON}, \pi)\) in the model with price rigidities satisfies:

\[
x^{MON} = \Gamma(m^{MON})
\]

\[
m_{t}^{MON} = R(\pi_{t}^{MON})E_{t}\frac{m_{t+1}^{MON}}{\pi_{t+1}}, \quad t = 1, \ldots, T - 1
\]

\[
\frac{m_{t}^{MON}}{m_{t}^{REAL}} = (\frac{\pi_{t}}{\pi^{*}})^{-\phi}, \quad t = 1, \ldots, T
\]

In what follows, define the gap \(\hat{m}_{t} = m_{t}^{MON}/m_{t}^{REAL}\).
Two Results
Result 1

Result 1: Suppose $\phi > 0$.

For any positive $\lambda$, there exists an equilibrium such that $\ln(\hat{m}_T) \geq \lambda$.

- Intuition: Fiscal policy is Ricardian.

- The period $T$ inflation rate is indeterminate.

- Hence, period $T$ real outcomes are also indeterminate in equilibrium.
Result 2: Suppose $R(\pi_t) \geq R_{LB}$. Then, for any $\lambda > 0$, there exists an equilibrium such that:

$$\ln \hat{m}_t \geq [\lambda + \phi \ln \frac{R_{LB}}{r^{NAT} \pi^*}](1 + 1/\phi)^{T-t} - \phi \ln \frac{R_{LB}}{r^{NAT} \pi^*}$$

- Proof: By backward induction.

- Note: Raising $R_{LB}$ raises $\hat{m}_t$ and so lowers $\hat{\pi}_t$.
  - "Neo-Fisherianism" isn’t valid.
Backward Induction: Part 1

- The aggregate Euler equation implies:

\[
\hat{m}_t = \frac{R(\pi_t)}{r_{NAT} \pi^*} E_t^{NAT} \hat{m}_{t+1} \left( \frac{\pi^*}{\pi_{t+1}} \right)
\]

where \(E_t^{NAT}\) is the risk-neutral expectation in the real economy:

\[
E_t^{NAT} z_{t+1} \equiv r_{NAT} E_t \frac{m_{t+1}^{REAL}}{m_t^{REAL}} z_{t+1}
\]

- The Phillips curve implies:

\[
\hat{m}_t = \frac{R(\pi_t)}{r_{NAT} \pi^*} E_t^{NAT} \hat{m}_{t+1}^{1+1/\phi}
\]
Backward Induction: Part 2

- Lower bound + Jensen’s inequality implies:

$$\ln \hat{m}_t \geq \ln \left( \frac{R_{LB}}{r^{NAT} \pi^*} \right) + (1 + 1/\phi)E_t^{NAT} \ln(\hat{m}_{t+1})$$

- For any $\lambda$, there exists eq’m with $\ln \hat{m}_T \geq \lambda$.

- And so, for any $\lambda$, there exists an equilibrium such that:

$$\ln \hat{m}_t \geq \lambda(1 + 1/\phi)^{T-t} + \phi \ln\left( \frac{R_{LB}}{r^{NAT} \pi^*} \right)[(1 + 1/\phi)^{T-t} - 1]$$
Fragility: Infinite Sensitivity to the Long Run

- Suppose marginal utility gap $\ln(m_T)$ is (believed to be) bounded below by $\lambda$, where:

$$\left[ \lambda + \phi \ln \frac{R_{LB}}{\pi^* r N A T} \right] > 0$$

- Then, the period $t$ gap is highly sensitive to $\left[ \lambda + \phi \ln \frac{R_{LB}}{\pi^* r N A T} \right]$

- Degree of sensitivity is arbitrarily large when $\phi$ is near zero.
Four Missing Ingredients
Fragility and Incompleteness

- Purely real model’s implications are highly sensitive to tiny perturbations.

- Need to add other ingredients to robustify the model.
Ingredient 1: Nominal Rigidities

- Need to have explicit model of nominal rigidity.

- Measurement issues are very subtle.

- Outcomes may be highly sensitive to nominal rigidities exactly when they’re small.
Ingredient 2: Monetary Policy Reaction Function

• What is the lower bound $R_{LB}$?

• More generally: How does $R(\pi)$ depend on inflation?
Ingredient 3: Fiscal Policy Reaction Function

• Ricardian fiscal policy cannot pin down inflation.

• This means that inflation is a sunspot variable in period $T$.

• Inflation is determinate under other kinds of fiscal policies (price level peg).

• Key issue: How does fiscal policy depend on price level?
Ingredient 4: Model of Long-Run Belief Formation

• This paper assumes rational expectations.

• Uncertainty about the long-run is uncertainty about which period $T$ equilibrium gets played.

• More generally: how do beliefs about the long run get formed?

• How do policy choices/actions affect those beliefs?
Conclusions
Negative Conclusion

- Add a small amount of nominal rigidity to a purely real model.

- Model’s implication become highly sensitive to beliefs about the long run.

- Purely real models are too fragile to be usable.
Constructive Conclusion

Useful models need four ingredients.

- Nominal rigidities.
- Central bank reaction function.
- Fiscal policy reaction function.
- Model of beliefs about the long-run.

AFAIK, no current macro model has all four.
Importance of Using Finite Horizon Models

• In infinite horizon model, long-run gaps have to be zero.
  – or current gap would be infinite.

• This rules out a key feature of long but finite horizons.
  – potential dependence on beliefs about the long run.

• Should use finite horizon models, not infinite horizon models.