

The Great Housing Boom of China*

Kaiji Chen[†] Yi Wen[‡]

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Abstract

This paper provides a theory to explain the paradoxical features of the great housing boom in China—the persistently faster-than-GDP housing price growth, exceptionally high capital returns, and excessive vacancy rates. The expectation that high capital returns driven mainly by resource reallocation are not sustainable in the long run can induce the very productive entrepreneurs to speculate in housing during economic transition. This creates a self-fulfilling growing housing bubble, which can create severe resource misallocation. A calibrated version of the theory accounts quantitatively for both the growth dynamics of house prices and other salient features of the recent Chinese experience.

Keywords: Housing Bubble, Resource Misallocation, Chinese Economy, Development, Economic Transition.

JEL Codes: E22, E23, O11, O16, P23, P24, R31.

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[†]Economics Department, Emory University.

[‡]School of Economics and Management, Tsinghua University, and Research Division, Federal Reserve Bank of St. Louis.

1 Introduction

Housing prices in China have experienced rapid and steady accelerations coincident with its spectacular economic growth in the recent decade. Data based on 35 major Chinese cities show that average real housing prices have grown at an annual rate of around 17% for the past decade, far exceeding the 10% average growth of real GDP in the same period. Closely associated with the great housing boom is the growing number of empty or “ghost” apartments across major cities in China. In 2013 the national urban housing vacancy rate reached 22.4%, far above the level of developed countries (e.g., the homeowner vacancy rate in the United States was only about 3% during the peak of the U.S. housing bubble around 2006). Of note, the majority of the ghost apartments are sold properties, which suggests an excessively strong (speculative) demand rather than excess supply.¹

During the same period, China has also enjoyed a persistently high rate of return to capital. For example, between 1998 and 2012, China’s real rate of return to capital was consistently around 20%. This rate of return is unprecedented even compared with the best performing emerging economies. Yet housing investors in China consist of not only households but also firms, including the most productive and profitable firms.

The combination of these features—namely, (i) the decade-long faster-than-income growth of housing prices, (ii) the exceptionally high vacancy rate, and (iii) the unprecedented high rate of return to capital—is puzzling. A standard neoclassical model, either with land as a production factor or with housing services in the utility function, predicts that housing prices can grow at most as fast as aggregate income, and thus can hardly explain the phenomenal housing price growth and the alarmingly high vacancy rate. Alternatively, in the classical Samuelson-Tirole bubble economy, houses can serve as an attractive store of value even if they provide no utilities. This view can explain the massive ghost apartment phenomenon in China, but it requires the critical assumption that the rate of return to capital is excessively low for housing investors, which is at odds with the observed persistently high rate of return to capital in China.

What economic forces are at work to generate (and sustain) such a great housing boom in China? Why would productive entrepreneurs divert their rapidly rising wealth toward housing instead of productive capital? What are the economic consequences of such a great housing boom?

In this paper, we propose a growing-bubble theory to explain the great housing boom.

¹Many home buyers in China are upper-middle-income and high-income households and they often hold multiple homes for investment purpose (see Section 2). Survey data also show that the majority (62% in 2012) of home buyers purchase houses for investment purposes.

The key element of our theory is an economic transition (after economic reform) featuring massive resource (labor) reallocation from a conventional less productive sector to an emerging sector, which consists of productive but financially constrained entrepreneurial firms. During the transition, the rate of return to capital in the emerging sector remains exceptionally high because of the “surplus” labor unleashed from the traditional sector. However, high capital returns driven mainly by resource reallocation are not sustainable in the long run, and this expectation can induce even the very productive entrepreneurs in a financially backward economy to seek housing as an alternative store of value for their rapidly growing wealth, even if housing provides no rents or utilities. Such speculative arbitrage based on anticipated low future return to capital and high expected future demand for housing can create a self-fulfilling bubble that grows much faster than the national income during the transition, thus explaining China’s massive ghost apartment phenomenon and decade-long faster-than-income growth in housing prices despite high capital returns.

Our paper fits into the fast-growing literature on economic development and resource misallocation under financial frictions.² In this environment, following policy reforms that remove important sources of resource misallocation, there exists a prolonged transition in which capital and labor are reallocated gradually from the less productive to the more productive but financially constrained firms. While the bulk of the literature emphasizes the effects of resource reallocation on improving allocative efficiency and the associated saving-investment dynamics during the transition, we argue that such a transition may also be prone to asset bubbles, even when the economy enjoys fast productivity growth and high returns to capital. Moreover, the presence of such bubbles can exacerbate resource misallocation and reduce welfare.

To incorporate asset bubbles into such a transition economy, we extend the framework of Song, Storesletten, and Zilibotti (SSZ, 2011) to a setting with an intrinsically valueless asset—housing.³ The SSZ model is attractive for our purpose because it can endogenously generate and quantitatively account for some important features of China’s economic transition, such as the rapid rate of gross domestic product (GDP) growth and the persistently high rate of return to capital in the emerging sector, which we argue are key to understanding China’s great housing boom. An important property of the SSZ model is a linear endogenous-growth AK feature during the transition period, which can sustain an exceptionally high rate of return to capital in the rapidly growing entrepreneurial sector for a prolonged

²See, for example, Jeong and Townsend (2007); Restuccia and Rogerson (2008); Guner, Ventura, and Xu (2008); Song, Storesletten, and Zilibotti (2011, 2014); Buera, Kaboski, and Shin (2011); Buera and Shin (2013), Moll (2014) and Midrigan and Xu (2014).

³We also consider alternative settings where housing provides utilities in a later section.

period without diminishing returns. Once the transition ends, however, the model reaches a Lewis turning point without “surplus” labor and starts to behave like a standard neoclassical model with rapidly declining returns to capital. Our contribution and value added is to show that such a development path can sustain growing bubbles—bubbles that grow much faster than GDP despite an exceptionally high rate of return to capital.

The intuition is simple. As exceptional as it is, no rational entrepreneur would expect the high capital return to continue forever, as diminishing returns to capital will eventually take effect after the transition ends. This expectation can induce even the very productive entrepreneurs during the transition to arbitrage in the housing market when other asset markets are underdeveloped or effectively shut down, thus leading to a self-fulfilling housing bubble. Under arbitrage, the rate of return to the bubble asset (or the housing price growth rate) is dictated by the rate of return to capital facing the marginal investors, who may be the financially constrained productive entrepreneurs. The return to capital by productive entrepreneurs, moreover, remains persistently higher than the growth rate of the aggregate economy due to labor reallocation. Consequently, the economy can exhibit a persistently faster-than-GDP growth in housing prices during the transition. We show that such a mechanism can quantitatively replicate China’s house price dynamics over the past decade fairly well while being consistent with the salient features of China’s growth experience.

In addition, our model sheds light on the welfare implications of China’s housing bubble: It can significantly prolong China’s economic transition and reduce social welfare. Unlike some existing bubble models, in our model the housing bubble can exist without dynamic inefficiency, thanks to the disparity between social and private rates of return to capital.⁴ Hence, by crowding out private capital formation and other productive activities, the bubble creates a negative externality on the permanent income of all agents. Accordingly, the occurrence of the housing bubble generates a substantial degree of resource misallocation and welfare losses, prolonging economic transition and slowing aggregate economic growth. Such welfare consequences offer an additional explanation for the Chinese government’s concerns over the great housing boom and its policies to contain the bubble.

Our paper is one of the first to study growing bubbles—bubbles with values growing persistently faster than the growth rate of the economy, as opposed to static bubbles or bubbles that grow at or below the growth rate of the economy.⁵ Our paper also contributes

⁴See also Farhi and Tirole (2012) for a similar result. In both papers, agency frictions drive a wedge between the social rate of returns to capital and the equilibrium rate of returns. Accordingly, bubbles exist even in an environment with dynamic efficiency. However, the reasons bubbles may reduce welfare differs between our paper and theirs. In their paper, the presence of bubbles raises the equilibrium interest rate, which reduces the price of other external liquid assets.

⁵For the rapidly developing housing bubble literature, see Caballero and Krishnamurthy (2006); Kocherlakota (2009); Farhi and Tirole (2012); Giglio and Severo (2012); Martin and Ventura (2012); Ventura (2012);

to the emerging literature on China’s high housing price puzzle. Most works in this area focus on why the housing price level is so high in China.⁶ For example, Wei, Zhang and Liu (2012) provide a theory to link the high housing price levels in major cities in China to these areas’ high household saving rates due to an unbalanced gender ratio. In sharp contrast, our paper focuses on why housing prices in China have grown at a faster rate than the economy over a prolonged period. Models that explain only the high housing price level from the demand side are insufficient to understand China’s growing housing bubble. More importantly, by shifting the analysis from the *level* of housing prices to the *growth rate* of housing prices, our paper sheds light on why housing bubbles may create resource misallocation and prolong China’s economic transition, an issue silent in Wei, Zhang and Liu (2012).

The remaining part of the paper is organized as follows: Section 2 presents some institutional backgrounds and stylized facts about China’s housing market. Section 3 describes a simple two-period benchmark model to illustrate our essential explanations on the great housing boom, as well as the model’s qualitative implications. Section 4 extends the analysis to a multiperiod version of our model for a calibration exercise. Section 5 concludes with remarks for further research. A technical Appendix available from our Web pages contains the formal proofs and numerical algorithm.

2 Stylized Facts

2.1 Housing Price Growth

The official housing price indices published by the Chinese government do not control for housing quality, which can create serious downward bias for the measured housing prices.⁷ To correct this problem, Wu, Deng, and Liu (2014) use independently constructed housing price indices based on sales of newly built housing units in 35 major Chinese cities. These city-level series are then aggregated into a national level indicator using a weighted average formula, with the total transaction volume during 2006-2010 in each city as the weight. The resulting national-level housing price index shows a much faster growth rate than the official housing price index. For example, the national-level housing price index has increased at a

Burnside, Eichenbaum, and Rebelo (2013); Miao and Wang (2013); and Galí (2014), among many others.

⁶One exception is Garriga, Tang and Wang (2014), who explore the role of rural-urban migration in the house price dynamics in urban areas of China and especially, Beijing and Shanghai.

⁷For example, the National Bureau of Statistics of China (NBSC) provides two major housing price indices. Based on these housing price indices, the average growth rate of housing prices in China is below the average growth rate of the economy. However, Wu, Deng, and Liu (2014) argue that these measures are severely biased downward because they fail to control both the complex-level quality changes (e.g., housing suburbanization) and unit-level quality changes (e.g., developers’ pricing strategies).

rate of 17% per year between 2006:Q1 and 2010:Q4. If we ignore the negative impact of the 2008 financial crisis, the average growth rate of housing prices was more than 20% per year during this period (see Figure 1).

The increase in housing prices is also accompanied by rapidly rising land values in China. Figure 1 shows that nationwide, real constant quality land values have grown at an average rate of more than 16% per year between 2004:Q1 and 2013:Q2. Accordingly, land value has constituted an important and increasing share in housing prices. For example, according to Wu, Gyourko, and Deng (2012), in the city of Beijing the average share of land value in housing prices was 37% before 2008 and surpassed 60% after 2010.

The faster-than-income growth rates of housing prices are prevalent across almost all cities in China. Figure 2 shows that most of the 35 major cities in China have experienced a significantly faster growth in housing prices than aggregate disposable income (which takes into account population growth due to migration).⁸ For example, in large cities such as Shanghai and Beijing, the average real growth rate of housing prices during the same period is 2 to 3 times higher than the average real growth rate of disposable income. The fact that house prices grow persistently faster than aggregate disposable income at both the national and the city levels casts doubt on the conventional wisdom that China's housing price growth is driven mainly by the increased utilitarian demand for housing due to rural-urban migration.

Accompanied with the housing boom is the continuously rising and alarmingly high housing vacancy rate. According to China Household Finance Survey (2014), in 2013 the average vacancy rate in the first-, second- and third-tier cities in China was 21.2%, 21.8% and 23.2%, respectively.^{9,10} Among different groups of households, 35.1% of entrepreneurial households own vacant houses (or 29.9% of them have multiple apartments), compared with 22.6% among non-entrepreneurial households. Furthermore, the proportion of households with vacant houses increases with household income. In the top 10th percentile income group, for example, 39.7% of households have vacant (multiple) houses, which is about 22 percentage points higher than that of households in the bottom income quartile.

The faster-than-income growth of housing prices implies a rapidly rising price-to-income

⁸According to Song et al. (2014), urban population, inclusive of rural migration, grew at an annual rate of 4.1% between 2000 and 2005.

⁹The definition of the vacancy rate in China corresponds to the homeowner vacancy rate in the Housing Vacancy Survey conducted by the U.S. Census Bureau, computed as the proportion of the vacant self-owned housing units in total homeowner housing units. The definition of vacant housing units does not include those newly built, but not yet sold, housing units.

¹⁰China's first-tier cities usually refer to Beijing, Shanghai, Guangzhou, and Shenzhen which make "The Big 4." Second-tier cities include the provincial capital cities and other municipalities directly under the central government. Third-tier cities include all other cities.

ratio for average wage earners. Ge and Yang (2014) use data from China Household Income Survey and find that the growth rate of real wages has been increasing since the economic reform. Between 1998 and 2007, the average real wage growth reached 9.0% per year, almost as fast as real per capita GDP growth.¹¹ So the gap between real housing price growth and real wage growth is more than 7 percentage points. With a rising average housing price-to-income ratio, it becomes increasingly difficult for the average Chinese household (who do not yet own a house or want to purchase additional houses) to use housing as a store of value. This is not an issue for rich and upper-middle-income households since their incomes can keep pace with the housing price growth. Our model is consistent with this reality of heterogeneous income groups because in our model the home buyers are the productive (rich) households and only the marginal buyers in the housing market determine the equilibrium growth rate of housing prices. In other words, allowing the low-income households to participate in the housing market has no effect on our key results.

2.2 Returns to Capital and Resource Reallocation

It is well documented that the average real rate of return to capital in China has remained around 20% over the past decade. Following this literature, we use the marginal revenue product of capital as a proxy for the rate of return to capital, as in Bai, Hsieh, and Qian (2006).^{12,13} Panel A of Figure 3 shows that the rate of return to capital is on average 20% between 1998 and 2012.¹⁴ In particular, it increased steadily from 18% in 2001 to 26% before the financial-crisis year of 2008. Similarly, the measured average after-tax rate of return to capital excluding urban housing is about 18.2% between 1998 and 2012, approximately the same as the estimated growth rate of the aggregate housing price.¹⁵

Underlying the enduring high rate of return to capital is the massive labor reallocation in

¹¹According to the data from NBSC, the national average growth rate of real per capita disposable income between 1998 and 2012 was 9.3% per year.

¹²Specifically, we measure the capital-to-output ratio at market prices and include any expected change in the price of capital as part of its returns. Our computed series of marginal (revenue) product of capital between 1998 and 2005 are essentially the same as those of Bai, Hsieh, and Qian. (2006).

¹³Ideally, we should use the income share of reproducible capital. In China, however, the data on reproducible capital income are not available, since there was no market for land in China before the mid-1990s and the market for leaseholds is very imperfect. Bai, Hsieh, and Qian (2006) find that after 1990 the measured average rate of returns to capital is close to its counterparts in the nonagricultural and nonmining sectors.

¹⁴Until 2005, the Chinese statistical authorities classified all self-employment income as labor income. Therefore, if anything, the reported capital share understated the true capital income at least before 2005.

¹⁵The measured after-tax returns to capital excluding urban housing are computed by excluding the urban residential capital stock from the measured capital stock and by excluding its imputed rent (assumed as 3% of the original value of the residential capital stock by the National Bureau of Statistics of China) and tax on output and enterprise income from the capital income.

China. This can be seen in Panel B of Figure 3, which plots the evolution of the share of private employment in total employment. Following SSZ, we adopt two measures of the private employment share: (i) the share of domestic private enterprises (DPEs) in total employment (which equals DPEs plus SOEs employment), and (ii) $(DPE+FE)/(DPE+FE+SOE+COE)$, where FE pertains to employment of foreign enterprises, COE that of collectively-owned enterprises and SOE that of state-owned enterprises. For both measures, the private employment share increased steadily for most years during the 1998-2011 period and surpassed 60% in 2011. Such a massive labor reallocation from SOEs to the emerging private firms and the associated low wage rate, we believe, are key to sustaining the high rate of return to capital since 1998.

2.3 Crowding-Out Effects on Investment

The rapid growth in housing prices is accompanied by a spectacular boom in the real estate sector. Data from *China Statistical Yearbook (CSY) 2012* show that the share of total real estate investment in GDP increased by more than threefold, from 4.2% in 1999 to 13.2% in 2011. Booming residential investment accounts for about 70% of the real estate boom. The average nominal growth rate of residential investment is 25.5% per year, compared with an average nominal GDP growth rate of 13.9% per year. Accordingly, the share of residential investment in GDP rose from 2.4% in 1999 to 9.5% in 2011, a fourfold expansion.

On the other hand, the rapidly growing housing bubble has shown a strong crowding-out effect on China's capital formation for both SOEs and private firms. We measure the crowding-out effects by estimating the correlation coefficients between housing price growth and non-real-estate investment growth.¹⁶ To remove seasonal effects, all growth rates are on a year-to-year basis, which means the growth rate of a particular month is compared with the same month in the previous year. Table 1 presents the correlation between real housing price growth (deflated by the consumer price index) and real investment growth (deflated by the producer price index). The second and third columns show the correlations of aggregate housing price growth with growth in real estate investment and other types of investment, respectively. In addition to reporting the contemporaneous correlations, we also report lead and lag correlations.

Table 1 shows that the growth of real estate investment is significantly and positively correlated with housing price growth, while non-real estate investment is significantly nega-

¹⁶Due to data availability constraints, we are only able to decompose aggregate investment into real estate investment and the rest.

tively correlated with housing price growth. More importantly, the results show that current growth in housing prices is a strong predictor of a future drop in non-real estate investment growth, with the peak correlation between housing price growth and investment growth reached at a 5-month lead. This crowding-out effect of housing price growth on non-housing investment is consistent with the predictions of our model.¹⁷

2.4 Other Facts Concerning Model Assumptions

Our model makes the following simplifying assumptions: both the land supply and the interest rate are fixed. In addition, the focus of our model is housing price dynamics over the last decade, which corresponds to a period of massive SOE privatization in China.

SOE Reform. Under China’s planned economy, SOEs were the major employers in cities and they played the pivotal role of maintaining low unemployment and ensuring social stability. By the mid-1990s, the Chinese government realized that its gradualist reform policy could no longer manage the mounting losses of SOEs. Beginning in 1997, China moved forward with more aggressive restructuring with large SOEs, accomplished through large-scale privatization. The reallocation of labor and capital out of SOEs toward private firms has been a key source of productivity growth in the past decade.

Land Supply. In China the land available for home construction is strictly controlled by the government. During 1997-2000, new construction land was no more than 20.40 million acres; during 2001-2010, it was no more than 30.72 million acres. This restriction on the size and new release of construction land was further strengthened by the “National Land Use Plan 2006-2020,” passed by the State Council of China in August 2008. According to this regulation, the total land available for construction in urban and rural areas is limited to 506.25 million acres by 2010 and 558.6 million acres by 2020. The same document requested that the amount of cultivated land in 2010 and 2020 be maintained at 1.818 billion acres and 1.805 billion acres, the so-called “red line” lower limit for the total amount of arable land. Figure 4 shows the total amount of arable land in China. It is clear that since 2003 the amount of arable land has been fully stabilized, implying a de facto fixed supply of land for home and real estate construction.

Financial Repression and Interest Rate Control. China has made significant progress since 1978 in opening its economy to the outside world, but financial reform significantly lags its economic reform in goods-producing sectors. China’s financial repression is easily seen in Figure 5 where interest rates are essentially flat with the deposit rate lying substantially below the lending rate. Funds are channeled through state-owned banks to the

¹⁷A wide class of models (e.g., Kocherlakota, 2009 and Martin and Ventura, 2012), predict that housing bubbles, by serving as collateral, crowd in capital investment.

conventional sector occupied mainly by SOEs. There are few investment alternatives for household savings: stock markets are poorly regulated and dominated by SOEs, interest rates are set by the government, the national capital account is closed, and the exchange rate is fixed or tightly managed. Through a system of strict capital controls where the state directly manages the banking sector and financial intermediation, the government has been able to maintain or suppress interest rates at below market-clearing levels. When the interest rate is fixed at a level below the market-determined rate, SOEs would be able to survive despite productivity inefficiency.

3 The Benchmark Model

In this section, we develop a theory of China’s great housing boom consistent with the institutional background and stylized empirical facts about China and its housing market behaviors. In particular, we extend the SSZ model to a setting with an intrinsically valueless asset—housing—and prove that a housing price bubble that grows faster than GDP exists even if housing provides no rents or utilities to investors. We shut down the access of low-income households (workers) to the housing market because their participation has only a level effect on the housing prices but no growth effects. We emphasize the growing nature of the bubble because the existing bubble literature often focuses exclusively on static bubbles or bubbles that grow at most at the same rate as the economy, which is contradicted by the Chinese data. In this section we illustrate our main story in a two-period overlapping-generations (OLG) model. We extend the model to a more realistic setting with multiperiod OLGs for the calibration exercise in Section 4.

3.1 The Environment

The economy is populated by two-period lived agents with overlapping generations. Agents work when young and consume the return to their savings when old. Agents have heterogeneous skills. In each cohort, half of the population are workers without entrepreneurial skills and the other half are entrepreneurs. Entrepreneur skills are inherited from parents and we do not allow transition of social classes (for simplification without loss of generality). The total population N_t grows at an exogenous rate ν .

Before the economy starts, the government owns \bar{H} units of housing (land), which are in fixed supply. At the beginning of the first period, the government sells the housing stock to the market (if there is demand) and consumes all the proceeds.

3.1.1 Technology

There are two production sectors and thus two types of firms. Labor is perfectly mobile across the two sectors, but capital is not. The first sector is composed of conventional firms—F-firms, which (for simplicity) are owned by a representative financial intermediary (bank) and operated as standard neoclassical firms.¹⁸

The second sector is a newly emerging private sector composed of unconventional firms—E-firms. The E-firms are operated by entrepreneurs. More specifically, E-firms are owned by old entrepreneurs, who are residual claimants on profits, and they hire their own children as managers. Workers can choose to work for either type of firm.

E-firms are more productive than F-firms but are severely borrowing constrained—they cannot borrow from each other or from any other sources.¹⁹ As a result, they must self-finance capital investment through their own savings. In contrast, F-firms can rent capital from their representative financial intermediary at a fixed interest rate R . Accordingly, F-firms can still survive in the short run despite inferior technology. Over time, however, labor will be gradually reallocated from F-firms to E-firms as the capital stock of E-firms expands. Thus, the economy features a transition stage during which F-firms and E-firms coexist but with the F-sector shrinking and the E-sector expanding. When the transition ends, only E-firms exist and the economy becomes a representative-agent growth model with neoclassical features. Our focus in this paper is on the transition stage.²⁰

The technologies of the two types of firms follow constant returns to scale

$$y_t^F = (k_t^F)^\alpha (A_t n_t^F)^{1-\alpha}, \quad y_t^E = (k_t^E)^\alpha (A_t \chi n_t^E)^{1-\alpha},$$

where y^j , k^j , and n^j denote per capita output, capital stock, and labor, respectively, for a type- j firm, $j \in \{E, F\}$. The parameter $\chi > 1$ captures the assumption that E-firms are more productive than F-firms. Technological growth in both sectors is constant and exogenously given by $A_{t+1} = A_t(1+z)$. However, during the economic transition, resource reallocation can generate endogenous growth faster than the growth in A_t .

¹⁸We can assume that the F-firms have market power and our main results do not change qualitatively.

¹⁹We discuss the consequence of relaxing this assumption in a later section.

²⁰Note that the concept of transition in this paper is different from the convention in the neoclassical growth model where transition means the dynamic path from an initial point toward the steady state. This conventional transition phase shows up in our model after the F-sector disappears. To avoid confusion, we call this neoclassical transition period “post-transition.”

3.1.2 Worker's Problem

Workers can deposit their savings into the representative bank and earn a fixed interest rate R . Without loss of generality, we assume that workers do not speculate in the housing market. In an appendix available upon request, we show that allowing workers to invest in housing does not change our main results—although the housing price level would be much higher, the growth rate of housing prices is unaffected. This is because the equilibrium growth rate of housing prices in our model is determined by the rate of return to capital of the entrepreneurs, who are the marginal investors in the bubbly equilibrium.

Of note, it is the marginal investors' capital returns (i.e., the capital returns of the productive entrepreneurs in our economy) that will determine the equilibrium growth rate of the housing bubble in our model. This critical result is one of the key contributions of our paper, and it defies the conventional wisdom that attributes China's housing boom to fundamentals such as the inelastic demand of housing from the working class or the high household saving rate. Here we illustrate the possibility that such inelastic housing demand (or a high saving rate) can increase only the housing price level, but not the growth rate of the housing price.

The worker's consumption-saving problem is

$$\max_{c_{1t}^w, c_{2t+1}^w} \log c_{1t}^w + \beta \log c_{2t+1}^w$$

subject to $c_{1t}^w + s_t^w = w_t$ and $c_{2t+1}^w = s_t^w R$, where w_t is the market wage rate, c_{1t}^w, c_{2t+1}^w , and s_t^w denote, respectively, consumption when young and consumption when old, and the worker's savings.

3.1.3 The F-Firm's Problem

In each period, an F-firm maximizes profits by solving the following problem:

$$\max_{k_t^F, n_t^F} (k_t^F)^\alpha (A_t n_t^F)^{1-\alpha} - w_t n_t^F - R k_t^F,$$

where the rental rate for capital is the same as the deposit rate R . The first-order conditions imply

$$w_t = (1 - \alpha) A_t \left(\frac{\alpha}{R} \right)^{\frac{\alpha}{1-\alpha}}. \quad (1)$$

Note that during the transition, the wage rate, scaled by the level of technology, w_t/A_t , is constant, due to a constant rental rate for capital and, accordingly, a constant capital-to-labor ratio, $k_t^F/(A_t n_t^F) = (\alpha/R)^{\frac{1}{1-\alpha}}$. When the transition is completed, all F-firms disappear, so equation (1) no longer holds.

3.1.4 The E-Firm's Problem

Following SSZ, we assume that young entrepreneurs are paid a management fee m_t —that is, a fixed $\psi < 1$ fraction of the output produced, $m_t = \psi (k_t^E)^\alpha (A_t \chi n_t^E)^{1-\alpha}$.²¹ Therefore, the old entrepreneur's problem can be written as

$$\max_{n_t^E} (1 - \psi) (k_t^E)^\alpha (A_t \chi n_t^E)^{1-\alpha} - w_t n_t^E \quad (2)$$

The first-order conditions imply a linear relationship between n_t^E and k_t^E

$$n_t^E = [(1 - \psi) \chi]^\frac{1}{\alpha} \left(\frac{R}{\alpha}\right)^{\frac{1}{1-\alpha}} \frac{k_t^E}{\chi A_t}. \quad (3)$$

Such a linear relationship is obtained because of a constant wage rate, which in turn results from the constant interest rate R . Accordingly, labor is reallocated to E-firms at a speed equal to the growth of the E-firm's capital stock. Substituting (3) into (2), we obtain the profit of the E-firm,

$$\pi(k_t^E) = (1 - \psi)^\frac{1}{\alpha} \chi^\frac{1-\alpha}{\alpha} R k_t^E \equiv \rho^E k_t^E,$$

where the first equality is obtained by using (3). Whenever F-firms exist, the return to capital in E-firms, $\rho^E \equiv (1 - \psi)^\frac{1}{\alpha} \chi^\frac{1-\alpha}{\alpha} R$, is a constant because n_t^E increases linearly in k_t^E . Similar to SSZ, we impose the following assumption about an E-firm's relative productivity such that an entrepreneur's return to capital is higher than the deposit rate R :

$$\chi > \underline{\chi} \equiv \left(\frac{1}{1 - \psi}\right)^{\frac{1}{1-\alpha}}.$$

²¹SSZ provide a microfoundation for a young entrepreneur's management fee as a fixed fraction of output: There exists an agency problem between the manager and owner of the business. The manager can divert a positive share of the firm's output for her own use. Such opportunistic behavior can be deterred only by paying managers a compensation that is at least as large as the funds they could steal, which is a share ψ of output. An alternative setup is that parents would leave voluntary bequests to their children, who in turn would invest in the family firm.

3.1.5 The Young Entrepreneur's Problem

The young entrepreneur decides consumption and portfolio allocations in housing investment, bank deposits, and physical capital investment. The rate of return to capital investment is simply ρ^E . We assume that the balanced growth rate, which equals the rate of return to housing investment at steady state, is higher than the bank deposit rate, that is, $(1+z)(1+\nu) > R$. As a result, the entrepreneur will always prefer investing in housing to depositing in the bank.²² Given the housing prices, denoted as P_t^H , the young entrepreneur faces a two-stage problem.

In the first stage, a young entrepreneur's consumption-saving problem is

$$\max_{s_t^E} \log(m_t - s_t^E) + \beta \log R_{t+1}^E s_t^E$$

where $R_{t+1}^E \equiv \max\{\rho^E, P_{t+1}^H/P_t^H\}$ is the rate of returns for entrepreneurial savings and depends on the entrepreneur's portfolio choice. First-order conditions give the optimal savings of young entrepreneurs, $s_t^E = m_t / (1 + \beta^{-1})$.

In the second stage, the young entrepreneur chooses portfolio allocation given the total savings s_t^E . The fraction ϕ_t^E of savings is invested in capital such that $K_{t+1}^E = \phi_t^E s_t^E N_t$, where $K_{t+1}^E = k_{t+1}^E N_{t+1}$ is the total capital deployed by E-firms. The remaining $(1 - \phi_t^E)$ fraction of savings is invested in housing, such that $P_t^H H_t^E = (1 - \phi_t^E) s_t^E N_t$, where H_t^E denotes the total housing stock purchased by young entrepreneurs at period t . Throughout this paper, we ensure there exists an interior solution for portfolio choice, so that the following no-arbitrage condition holds:

$$\frac{P_{t+1}^H}{P_t^H} = \rho_{t+1}^E, \quad (4)$$

where $\rho_{t+1}^E = \rho^E$ during the transition. Hence, an old entrepreneur's income is simply $\rho^E s_t^E$. The above condition simply says that the rate of return to housing must equal the rate of return to entrepreneurial capital in a bubbly equilibrium.

3.1.6 The Bank's Problem

For easy of exposition, we assume that each period the bank simply absorbs deposits from young workers, rents them to F-firms at interest rate R , and invests the rest in foreign bonds

²²We will show that the housing price growth rate is constant in the transition stage and decline toward the steady-state level after the transition.

with the same rate of return R (as in SSZ). As mentioned earlier, the result would be similar if we instead allowed the bank to also invest in housing on behalf of the workers and the F-firms. This is so because the F-firm is not the marginal investor determining the growth rate of housing prices in the housing market.

3.1.7 Time Line

To summarize, in each period the economic events unfold as follows:

1. At the beginning of period t , production of E-firms and F-firms takes place. Each young worker gets paid a real wage w_t regardless of which sector they work in. Each young entrepreneur gets m_t .
2. Both the young entrepreneur and young workers make consumption/saving decisions. In addition, the young entrepreneur makes a portfolio choice ϕ_t^E .
3. The housing market opens. Old entrepreneurs sell housing stock held in the previous period, H_{t-1}^E . Young entrepreneurs make a portfolio decision ϕ_t^E .
4. F-firms repay their capital rents to the bank.
5. The currently old workers consume and die, as do the currently old entrepreneurs.

3.1.8 Laws of Motion

Since E-firm is self-financed, the law of motion for the E-firms' capital stock follows

$$K_{t+1}^E = \phi_t^E \frac{\rho_t^E \psi}{(1-\psi)\alpha} \frac{1}{1+\beta^{-1}} K_t^E, \quad (5)$$

where $\rho_t^E = \rho^E$ for all periods during the transition. As shown later, in this simple economy, the entrepreneur's portfolio share in physical capital, ϕ_t^E , is a constant, which, together with a constant ρ^E , implies that the dynamics of the model have an AK feature during the transition: the growth rate of E-firms' capital is constant. Similarly, we can obtain the implicit law of motion for housing demand as

$$P_t^H \bar{H} = (1 - \phi_t^E) \frac{\rho_t^E \psi}{(1-\psi)\alpha} \frac{1}{1+\beta^{-1}} K_t^E,$$

where we have used the housing market-clearing condition $H_t^E = \bar{H}$.

3.1.9 Post-Transition Equilibrium

We now characterize the equilibrium in the post-transition stage. Since $n_t^E = 1$, the profit of the E-firm is

$$\pi(k_t^E) = \alpha(1 - \psi)(k_t^E)^\alpha (A_t \chi)^{1-\alpha}.$$

Note that $\pi(k_t^E)$ features decreasing returns to scale at this stage. The rate of return to E-firms' capital is simply $\rho_{t+1}^E = \alpha(1 - \psi)(k_{t+1}^E)^{\alpha-1} (A_{t+1} \chi)^{1-\alpha}$.

3.1.10 The Steady State

The steady state of the economy is reached only in the post-transition stage. Since all per capita variables (except labor inputs and housing) grow at the rate A_t , we detrend them as $\hat{x}_t = x_t/A_t$.

At the steady state, the law of motion for capital (5) implies

$$\hat{k}^{E*} = \left[\frac{\psi \phi^{E*} \chi^{1-\alpha}}{(1 + \beta^{-1})(1 + z)(1 + \nu)} \right]^{\frac{1}{1-\alpha}}. \quad (6)$$

Since $\rho^{E*} = \alpha(1 - \psi) \left(\hat{k}^{E*} / \chi \right)^{\alpha-1}$, we have

$$\rho^{E*} = \alpha(1 - \psi) \frac{(1 + \beta^{-1})(1 + z)(1 + \nu)}{\psi \phi^{E*}}. \quad (7)$$

Equation (7) implies that the rate of returns to capital is negatively related to the E-firms' portfolio share in physical capital, ϕ^{E*} .

The equilibrium portfolio allocation ϕ^{E*} can be solved by the no-arbitrage condition. Since the supply of housing is fixed, the growth rate of housing prices, denoted as $\rho_{t+1}^H \equiv P_{t+1}^H / P_t^H$, equals the balanced growth rate, $(1 + z)(1 + \nu)$, in the steady state. As a result, the no-arbitrage condition implies E-firms' steady-state portfolio share in physical capital as

$$\phi^{E*} = \alpha(1 - \psi)(1 + \beta^{-1}) / \psi. \quad (8)$$

Intuitively, the larger the rate of returns to E-firms' capital, as captured by $\alpha(1 - \psi)$, the larger the share of entrepreneurial savings in physical capital. On the other hand, the larger ψ and β , which imply, respectively, a larger income share and saving propensity of young entrepreneurs, the lower the return to physical capital and thus the lower the share of entrepreneurial savings in physical capital.

3.2 Characterizing the Bubbly Equilibrium

In this subsection, we explore the equilibrium with housing bubbles. We first discuss the necessary condition for housing bubbles to exist. We then derive the growth rate of housing prices relative to that of aggregate output. Finally, we explore the normative implications of bubbles.

3.2.1 Existence of Bubbles

Note that there always exists an equilibrium without bubbles in our model—that is, all financial resources are invested in capital: $\phi_t^E = 1$ for all t . We call this equilibrium the “fundamental equilibrium.” To understand the conditions for the existence of a bubbly equilibrium, we must understand the nature of the fundamental equilibrium and under which conditions in the fundamental equilibrium a bubbly equilibrium can also emerge.

Consider the steady state first. The necessary condition for housing bubbles to exist in the steady state (i.e., $\phi^{E*} < 1$), is that the rate of returns to E-firms' capital is below the balanced growth rate in the fundamental equilibrium. Intuitively, when the returns to capital are so low in the fundamental equilibrium, it is optimal for entrepreneurs to divert savings into housing as an alternative store of value. This condition, together with (8), implies the following parameter restriction on the bubbly equilibrium:

$$\alpha(1 - \psi)(1 + \beta^{-1}) < \psi, \tag{9}$$

or $\psi > \underline{\psi} \equiv \alpha(1 + \beta^{-1}) / [1 + \alpha(1 + \beta^{-1})]$. Intuitively, a larger ψ makes the bubbles more likely to occur in two ways: First, it directly reduces the entrepreneur's rate of return to E-firms' capital. Second, by increasing the output share of the young entrepreneur, it increases the capital stock accumulated by the young, thus lowering the marginal product of E-firms' capital.

We are now able to characterize the conditions for the existence of bubbles in both the transition and the post-transition stages. The assumption (9), together with the law of

motion for capital, (5), implies that in the fundamental equilibrium, we must have

$$K_{t+1}^E > \rho_t^E K_t^E, \forall t. \quad (10)$$

This is so because the wedge between K_{t+1}^E and $\rho_t^E K_t^E$ is a constant, and this constant exceeds 1 in our benchmark model. Accordingly, given that (10) is satisfied at the steady state, it must be satisfied for all previous periods. Forwarding (10) by one period and noticing that, with full depreciation of capital, $K_{t+1}^E = I_t^E$, where I_t^E is investment in physical capital, we have

$$I_{t+1}^E > \rho_{t+1}^E I_t^E, \forall t. \quad (11)$$

The inequality (11) is analogous to the necessary condition for bubbles to exist in Abel, Mankiw, Summers and Zeckhauser (1989) (AMSZ henceforth). Intuitively, housing bubbles are possible if there exists a sequence of investment with its cost exceeding the income flow it generates in all periods.

The inequality (11) implies that in a bubbly equilibrium, the young entrepreneurs would voluntarily reduce their investment and hold housing in their portfolios, with the expectation that the revenues from selling these bubbles will exceed their forgone income from capital investment. The revenues are nothing but the reduced investment by the next generation of young entrepreneurs. To see this point, note that equation (5) implies that in the fundamental equilibrium,

$$I_{t+1}^E = (1 + \varepsilon) \rho_{t+1}^E I_t^E, \quad (12)$$

where $\varepsilon \equiv \psi / [(1 - \psi) \alpha (1 + \beta^{-1})] - 1 > 0$. Take the total derivative with respect to (12), and let $dI_t^E = -(P_t^H \bar{H} - 0)$; that is, the resources generated from a reduction in capital investment (the left-hand side) are invested in housing (the right-hand side). Then, we have $P_{t+1}^H / P_t^H > \rho_{t+1}^E$; that is, the return to housing investment would be greater than the return to capital if all entrepreneurial savings are invested in physical capital. As a consequence, expecting that inequality (11) holds for all future periods and thus a positive future demand for housing, the young entrepreneurs in period t would opt to divert savings into housing, which would raise the housing price P_t^H until the no-arbitrage condition $P_{t+1}^H / P_t^H = \rho_{t+1}^E$ holds.

Hence, the existence of housing bubbles is based on the rational expectation of a young entrepreneur in any given period about strong housing demand by future young entrepre-

neurs: If the future capital return ρ_t^E in the post-transition stage becomes sufficiently low, purchasing housing today can yield a high capital gain tomorrow. For cohorts born during the transition, such an expectation can be self-fulfilled if inequality (11) holds. In contrast, if, for some future period t_0 and from that period on, the condition (11) does not hold in the fundamental equilibrium, the future young entrepreneurs at and beyond that period would have no incentive to hold bubbles. By backward induction, this will discourage any previous generations from purchasing bubbles.

Our results are in contrast to those in the existing bubble literature in at least two aspects. First, the existing bubble literature emphasizes a condition similar to (10) for bubbles to exist. However, in our model, due to the presence of agency frictions (i.e., $\psi > 0$), there is a wedge between the private and the social rates of return to capital. The social rate of return to E-firms' capital is simply the marginal product of E-firms' capital, denoted as MPK^E .²³ By the definition of ρ^E , we then have $\rho^E < MPK^E$. Hence, in contrast to the standard bubble theory, housing bubbles in our economy may exist even under dynamic efficiency. This has dramatically different and important welfare implications from those of the existing bubble literature.²⁴

Second, the definition of dynamic inefficiency (e.g., Tirole, 1985) requires (10) to hold only at the steady state, which is equivalent to $(1+z)(1+\nu) > \rho^{E*} |_{\phi^E=1}$. In our economy, although ρ^{E*} at the steady state is very low (in fundamental equilibrium, it is lower than $(1+z)(1+\nu)$), during the transition, the equilibrium ρ_{t+1}^E is sufficiently high and much higher than $(1+z)(1+\nu)$. However, condition (10) still holds during the transition, which encourages young entrepreneurs to purchase housing despite their very high (absolute) returns to capital. This expectation is fueled in every period by the high expected future demand for housing, as entrepreneurs in each period expect condition (10) to hold in each successive future period.

We now explore further the existence of bubbles under dynamic efficiency. Dynamic efficiency implies that the steady-state MPK^E in the fundamental equilibrium is larger

²³Implicitly, the planner solves a constrained optimization problem without agency frictions, but with financial market imperfectness as in the benchmark model.

²⁴The original AMSZ test uses the economy-wide rate of returns for capital, instead of the marginal buyer's, as our paper does. As argued by Giglio and Severo (2012), looking at the average rate of return in the economy is not sufficient to judge the dynamic efficiency of market allocations, because there might be over accumulation among those who cannot park funds in productive assets (e.g. Martin and Ventura, 2012). Our argument is stronger, that is, the satisfaction of the AMSZ criteria for the marginal bubble buyer's returns to capital, as equation (10) does, is not sufficient to judge dynamic inefficiency due to the wedges between private and social rates of returns to capital.

than the balanced growth rate

$$MPK^{E*} |_{\phi_E=1} > (1+z)(1+\nu). \quad (13)$$

With (7), the condition (13) requires the following parameter restriction:

$$\psi < \alpha(1 + \beta^{-1}). \quad (14)$$

Intuitively, the smaller ψ is, the smaller is the steady-state capital and the higher its marginal product. Also, similar to standard OLG models, a higher α or a lower β makes the economy less likely to be dynamically inefficient. A combination of (9) and (14) gives further parameter restrictions for bubbles to exist when the economy is dynamic efficient:

$$\alpha(1 - \psi)(1 + \beta^{-1}) < \psi < \alpha(1 + \beta^{-1}). \quad (15)$$

3.2.2 The Growth Rate of Housing Prices Relative to Output

Lemma 1 *The growth rate of housing prices is equal to the growth rate of the total output of E-firms in both transition and post-transition stages.*

$$\frac{P_{t+1}^H}{P_t^H} = \frac{Y_{t+1}^E}{Y_t^E}, \forall t. \quad (16)$$

The intuition for Lemma 1 is as follows. In both transition and post-transition stages, the optimal portfolio choices by entrepreneurs will equalize the returns to capital investment and the returns to bubbles through arbitrage. In this simple economy, the equilibrium portfolio ϕ_t^E is a constant, as the wedge between K_{t+1}^E and $\rho_t^E K_t^E$ in the fundamental equilibrium is constant.²⁵ This gives $K_{t+1}^E = \rho_t^E K_t^E$ in the bubbly equilibrium. Accordingly, the growth rate of the total output of E-firm equals the rate of returns to capital for entrepreneurs,

²⁵More formally, the Appendix shows that the portfolio share in physical capital follows

$$\phi_t^E = \frac{\alpha(1 + \beta^{-1})(1 - \psi)}{\psi}, \forall t.$$

which in turns equals to the growth rate of housing prices.²⁶

With Lemma 1, the following proposition captures the growth rate of housing prices relative to that of aggregate output.

Proposition 1 *The growth rate of housing prices is higher than that of aggregate output during the transition, and it converges to that of aggregate output when the transition ends. Specifically,*

$$\Delta \log P_{t+1}^H - \Delta \log Y_{t+1} = \Delta \log \frac{Y_t^E}{Y_t^E + Y_t^F}. \quad (17)$$

Equation (17) implies that the gap between the growth rate of housing prices and that of aggregate output equals the growth rate of the E-firms' output share in aggregate output. During the transition, the aggregate output growth is a weighted average of the output growth of the E-firms and F-firms. Since F-firms keep downsizing because of labor reallocation, this sector's output growth rate is less than that of E-firms, rendering a lower growth of the aggregate economy than that of E-firms.²⁷ Therefore, housing prices will grow faster than the growth rate of the economy at this stage. In the post-transition stage, the economy becomes essentially neoclassical: The growth rate of aggregate output equals the growth rate of E-firms' total output. As a result, the housing price grows at the same rate of aggregate output even before reaching the steady state.

Furthermore, standard algebra shows that E-firms' output share in total output is given by

$$\log \frac{Y_t^E}{Y_t^E + Y_t^F} = \log \frac{N_t^E/N_t}{1 - \psi + \psi N_t^E/N_t}. \quad (18)$$

Therefore, together with Proposition 1, equation (18) implies that the growth rate of housing prices relative to that of aggregate output depends positively on the growth rate of E-firms' employment share, N_t^E/N_t . Note that during the transition, the growth rate of E-firms' employment share is constant due to the constant growth rate of the E-sector's capital stock

²⁶More formally, the growth rate of E-firms output is

$$\frac{Y_{t+1}^E}{Y_t^E} = \frac{Y_{t+1}^E}{K_{t+1}^E} \frac{K_{t+1}^E}{K_t^E} \frac{K_t^E}{Y_t^E} = \frac{\rho_{t+1}^E}{(1-\psi)\alpha} \rho_t^E \frac{(1-\psi)\alpha}{\rho_t^E} = \rho_{t+1}^E.$$

²⁷More formally, the growth rate of F-firms' total output follows

$$\frac{Y_{t+1}^F}{Y_t^F} = \frac{RK_{t+1}^F/\alpha}{RK_t^F/\alpha} = \frac{K_{t+1}^F}{K_t^F}.$$

as implied by the AK feature.²⁸ This implies a persistently higher growth rate of housing prices than that of aggregate output. For a similar composition effect, the aggregate rate of return to capital increases during the transition, despite the constant capital returns in both E-firms and F-firms.²⁹

The key to delivering a persistently faster-than-GDP housing price growth rate in our economy is the presence of a transition stage featured by labor reallocation from unproductive to productive, (but financially constrained) firms. During this transition, the entrepreneurs' high rate of return to capital is sustained by the existence of “surplus” labor in the F-sector, and it is expected to fall only after the transition, when the diminishing returns to capital start to take effect. It is precisely this trajectory of return to capital that renders the marginal buyers (investors) in the housing market the productive agents (entrepreneurs), rather than the unproductive agents (workers or F-firms).³⁰ This is in contrast to the results of Martin and Ventura (2012). In their paper, the marginal buyers of bubbles are the unproductive agents who face an extremely low rate of return to capital (as in the dynamic inefficiency literature). Consequently, their model predicts a slower-than-GDP growth in the housing bubble, which is inconsistent with the Chinese data. Moreover, in their paper, bubbles improve allocative efficiency by crowding in (instead of crowding out) investment of the productive firms—which is again inconsistent with the Chinese data shown in Section 2. By contrast, as the next section shows, bubbles in our model can worsen the allocative efficiency by crowding out investment of even the productive firms—a serious concern of the Chinese government for many years.

3.2.3 Economic Consequences of Growing Bubbles

An interesting issue is the normative implication of bubbles in our model. Because bubbles can exist in our model without dynamic inefficiency, they may reduce, rather than increase,

²⁸More formally, the growth rate of E-sector's employment share is

$$\frac{N_{Et+1}/N_{t+1}}{N_{Et}/N_t} = \frac{K_{Et+1}/N_{t+1}}{K_{Et}(1+z)/N_t} = \frac{\rho_t^E}{(1+z)(1+\nu)}.$$

²⁹More formally, the aggregate rate of returns to capital is computed as

$$\rho_t = \frac{\rho^E K_t^E + \rho^F K_t^F}{K_t^E + K_t^F} = \frac{R}{1 - \frac{N_t^E}{N_t} \left[1 - \chi ((1 - \psi) \chi)^{-\frac{1}{\alpha}} \right]}.$$

³⁰We show in an appendix (available upon request) that our results for housing price dynamics hold in a corresponding partial equilibrium two-sector economy, in which the young entrepreneur faces an exogenously given sequence of the rate of returns to capital taken from the current general equilibrium setup.

social welfare. We now explore the welfare implications of growing bubbles in detail.

We first study the implications of bubbles for aggregate consumption in both the transition and post-transition stages. With condition (14), the law of motion for capital (5) implies that in the fundamental equilibrium, at each period t ,

$$K_{t+1}^E < MPK_t^E \cdot K_t^E. \quad (19)$$

In other words, investment in bubbles is not optimal for the social planner, despite the incentive of entrepreneurs to hold housing as stores of value. The reason is simple: Given the sufficiently high marginal product of E-firms' capital, housing bubbles reduce the resource available for aggregate consumption by crowding out productive investment.³¹

The criterion of dynamic efficiency applies to the effect of bubbles on aggregate consumption. To allow bubbles to reduce total entrepreneurial consumption, we further assume

$$\psi < \alpha(1 + \beta^{-1}) [\psi + \alpha(1 - \psi)]. \quad (20)$$

Note that $\psi + \alpha(1 - \psi)$ is the share of E-firms' output accrued to young and old entrepreneurs. Since $\psi + \alpha(1 - \psi) < 1$, the inequality (20) is sufficient for the condition of dynamic efficiency, (14), to hold.³² We now derive the effects of bubbles on each type of agent. Defining the period- t aggregate consumption of agent-type $j \in \{w, E\}$ as $\tilde{c}_t^j \equiv \tilde{c}_{1,t}^j + \tilde{c}_{2,t}^j(1 + \nu)^{-1}$, we have the following proposition:

Proposition 2 *Given that (9) and (20) are satisfied, a housing bubble reduces aggregate consumption and welfare of both entrepreneurs and workers.*

The intuition is as follows. In addition to the forgone capital returns, entrepreneurial housing investment reduces the lifetime income of future entrepreneurs and, thus, generates a negative externality on consumption.³³ It is easy to show that during the transition, since the rate of return to capital is constant, a reduction in lifetime income reduces the entrepreneur's lifetime utility. In the Appendix, we also show that bubbles reduce the entrepreneurial

³¹A similar wedge between social and private rates of return for capital occurs in the endogenous growth models of Grossman and Yanagawa (1993) and King and Ferguson (1993), in which the labor productivity of individual firms depends positively on the aggregate stock of capital.

³²A combination of (9) and (20) implies $(1 - \psi)(1 - \alpha) < \psi$, which is guaranteed by $m_t \geq w_t$, the necessary condition for the young entrepreneur to work as a manager rather than a worker.

³³Given the initial capital stock and constant returns to capital, the permanent incomes of the old and young entrepreneurs alive in the first period are unchanged when a housing bubble is introduced.

lifetime utility at the steady state. For entrepreneurs born during the post-transition stage, a sufficient condition for welfare loss is $\alpha(1 + \beta^{-1}) > 1 - \alpha^2$.

Regarding the impact of housing bubbles on workers' consumption, note that the wage rate, a constant along the transition, is unaffected by the presence of a bubble during the transition. Hence, the welfare of workers during the transition is unaffected by the bubble. However, when the transition ends, their lifetime utility decreases as a result of the housing bubble. This is because the workers' wage income starts to depend positively on E-firms' capital stock, while the rate of return to saving (the deposit rate) is still fixed.

The next question is whether it is desirable to crash a bubble once it exists, given that bubbles crowd out productive investment. The answer is no. In our economy, the housing bubble serves as a store of value that enables the young entrepreneurs to finance retirement consumption when old. Eliminating the bubble will therefore erode the retirement wealth of the old entrepreneurs who happen to hold housing at the time the bubble bursts. To ensure that no household is left worse off after the housing bubble bursts, the policymaker needs to compensate these old entrepreneurs for their losses—say, by issuing government bonds to the current-period young entrepreneurs. But the resources needed to compensate old entrepreneurs are the same resources that would have been released by the young entrepreneurs for capital accumulation. This implies that the policymaker is simply substituting another form of bubble for housing without crowding in productive investment. Such a policy dilemma may explain why the Chinese government has been so reluctant to levy aggressive property taxes to burst the housing bubble, despite its rapid growth and sheer size, as well as its apparent crowding-out effects on capital investment.

3.2.4 Discussion of Assumptions

We briefly discuss several simplifying assumptions that may have shaped our results before moving to the numerical exercises. First, we abstract from housing as a durable consumption good in our model. This simplifying assumption allows us to focus on the role of housing as an investment good. Even if housing can yield utilities, speculative housing investment would still exist in our model in the sense that housing investors would overpurchase (or overinvest in) the housing stock beyond what is justified by the marginal utility of housing relative to the marginal cost. The equilibrium growth rate of housing prices can still exceed that of aggregate income, provided that the rate of return to capital is sufficiently high relative to the marginal utility of housing.³⁴ However, the arbitrage condition needs to be

³⁴For example, this can be achieved by the assumption that the marginal utility from housing services approaches zero when housing services are beyond some threshold level.

modified such that the rate of return to housing should now include both the capital gains and the utility gains of housing. This would induce a slower growth rate of housing prices, but the growth rate can still exceed that of aggregate income.

Alternatively, we could allow the presence of both housing investors (holding housing for resale purposes only) and homeowners, who are subject to a tight borrowing constraint.³⁵ For example, housing services are valued by the workers, not the entrepreneurs. Another possibility is to allow housing services to be valued by some entrepreneurs (e.g., the middle-aged) but not the others (e.g., the old). Given the tight collateral constraint of homeowners and the speculative housing demand by investors, the marginal buyers who determine the growth rate of housing prices would still be the entrepreneurs who hold housing for resale purposes. Hence, such a modification will affect only the level, not the growth rate, of the equilibrium housing prices.

In addition, entrepreneurs in our model cannot borrow. In an appendix available upon request, we relax this assumption by assuming that young entrepreneurs can borrow against housing as collateral. This would modify the no-arbitrage condition by adding the shadow value of housing as collateral into the overall marginal benefit of housing. However, our main prediction about the faster-than-GDP growth in housing prices remains qualitatively unchanged. More interestingly, we show analytically that relaxing the collateral constraint would further crowd out productive capital investment in both transition and post-transition stages.

Also, bubbles in our economy are assumed to be deterministic. We can introduce stochastic bubbles by assuming some exogenous probability of bubbles bursting. This would change only the no-arbitrage condition by replacing the actual growth rate of housing prices with the expected growth rate of housing prices. And this change would make the realized housing price growth rate in the bubbly equilibrium even higher.

3.3 A Numerical Example

Figure 6 provides the key predictions of the benchmark model. In Panel A, labor demand by E-firms increases steadily until period T , when the transition ends. Panel B shows that the rate of returns to E-firms' capital, which equals the growth rate of housing prices, is persistently high during the transition. In the post-transition stage, however, both series start to decline as entrepreneurs continue to accumulate capital. Panel B also plots the dynamics of the growth rate of aggregate output, which is much lower than the returns to

³⁵See, for example, Arce and Lopez-Salido (2011) and Zhao (2013) for a similar assumption.

E-firms' capital during the transition.³⁶ In the post-transition stage, however, these two series overlap. Accordingly, the ratio of housing price to aggregate output keeps increasing during transition but becomes a constant in the post-transition stage (Panel D).

Panel E of Figure 6 illustrates the portfolio share of E-firms' saving in housing. In this simple economy, the equilibrium ϕ_t^E is constant. This is because the wedge between K_{t+1}^E and $\rho_t^E K_t^E$ in the fundamental equilibrium is constant, due to full capital depreciation and log utility and thus a constant growth rate of entrepreneurial wealth. Finally, Panel F shows that during the transition, aggregate total factor productivity (TFP) increases monotonically due to the increase in E-firms' employment share.³⁷ When the transition is completed, however, the (detrended) TFP level becomes constant.

To summarize, in our economy, the (productive) entrepreneurs demand housing as a store of value for their growing wealth, anticipating an eventual fall in the rate of returns to capital. Accordingly, the growth rate of housing prices, which equals the growth rate of E-firms' total output, is always higher than that of aggregate output during the transition. Moreover, the presence of housing bubbles, by crowding out productive investment, reduces aggregate consumption and welfare for all types of agents.

4 The Quantitative Multiperiod Model

Can a calibrated version of the model account for the rapid growth of housing price in China, both in absolute terms and relative to GDP, during the 2004-2012 period when data are available? To answer this question, we extend our two-period benchmark model to a J -period model to facilitate a quantitative calibration exercise.

4.1 Environment

Agents live for J periods, are born with zero wealth, and cannot die with negative wealth. Workers supply one unit of labor each period. They retire after J^R years of work. Young entrepreneurs work for the old entrepreneurs in the first J^E periods of life. For simplicity, we assume that an age- j ($j < J^E - 1$) young entrepreneur can only make deposits in the bank with a fixed interest rate R . From age $J^E - 1$ on, she can have a portfolio choice by

³⁶The aggregate output growth rate at period t is the growth rate of aggregate output between period t and $t - 1$. Thus, aggregate output growth rate at the first period is not well defined.

³⁷Aggregate TFP is computed as a standard Solow residual of a one-sector aggregate production function using aggregate capital and labor as inputs.

purchasing housing or investing in her own business. In this economy, we assume the capital depreciation rate $\delta < 1$.

4.1.1 The F-Firm's Problem

The F-firm's problem is similar to that in the benchmark model:

$$\max_{k_t^F, n_t^F} (k_t^F)^\alpha (A_t n_t^F)^{1-\alpha} - w_t n_t^F - R^l k_t^F + (1 - \delta) k_t^F.$$

For calibration purposes, we assume that lending to an F-firm is subject to a constant iceberg cost ξ , which represents the intermediation cost. In equilibrium, the lending rate for F-firms is $R^l = R / (1 - \xi)$.

4.1.2 The E-Firm's Problem

An age- j old entrepreneur in time t solves the following problem

$$\pi(k_{j,t}^E) = \max_{n_{j,t}^E} (1 - \tau_t^y) (1 - \psi) (k_{j,t}^E)^\alpha (A_t \chi n_{j,t}^E)^{1-\alpha} - w_t n_{j,t}^E + (1 - \delta) k_{j,t}^E,$$

where $k_{j,t}^E$ and $n_{j,t}^E$ denote the capital and labor deployed by an age- j old entrepreneur at period t . We can derive the rate of returns for E-firms' capital as

$$\rho_t^E \equiv \pi(k_{j,t}^E) / k_{j,t}^E = \alpha (1 - \psi) (1 - \tau_t^y)^{\frac{1}{\alpha}} [(1 - \alpha) (1 - \psi) A_t \chi / w_t]^{\frac{1-\alpha}{\alpha}} + 1 - \delta.$$

Note that, despite the heterogeneity in capital stock, the rate of return to capital is the same for all entrepreneurs alive in period t under the Cobb-Douglas production function.

Again, for calibration purposes, we assume that the production of E-firms is subject to a time-varying output wedge τ_t^y . The purpose of introducing this wedge is to target the time path of the private employment share in China. Such an output wedge may capture, in reality, the preferential or distortionary policy toward private firms. For example, in the early stage of privatization, the Chinese government provided various supports (e.g., credits, tax deductions) to private firms, which encouraged their fast growth.³⁸ This would show up

³⁸For example, on June 29, 2002, the Ninth National People's Congress Standing Committee passed the Small and Medium-sized Enterprise Promotion Law, to be implemented on January 1, 2003. In 2005, the State Council issued "Several Opinions of the State Council on Encouraging, Supporting and Guiding the Development of Individual and Private Economy and Other Non-Public Sectors of the Economy," or the so-called "36 items for the non-public economy," to support the development of private enterprises via

as an implicit output subsidy to E-firms ($\tau_t^y < 0$). Over time, however, such preferential policies have started to be replaced by various government policies that restrict the growth of private firms (e.g. entry barriers for private firms into “strategic” industries and a heavy tax burden), which had contributed to the so-called “the state advances, and the private sector retreats,” or “Guo Jin Min Tui.”³⁹ This would show up in our model as an increase in the value of τ_t^y . Since, in reality, government policies affect the overall profitability of private firms, we assume that such an output wedge also applies to young entrepreneurs’ managerial compensation.

4.1.3 The Worker’s Problem

For a worker of age i in period q , his problem for the remaining of his life is

$$\max \sum_{j=i}^J \beta^{j-i} \log c_{j,t}^w$$

subject to

$$\begin{aligned} c_{j,t}^w + s_{j,t}^w &= w_t + R s_{j-1,t-1}^w, \quad \text{for } j < J^R \\ c_{j,t}^w + s_{j,t}^w &= R s_{j-1,t-1}^w, \quad \text{for } j \geq J^R \\ s_{J,t}^w &= 0, \quad s_{0,t-1}^w = 0, \end{aligned}$$

where the subscript $t \equiv q + j - i$ is the calendar time for the age- i agent to become age j .

4.1.4 The Entrepreneur’s Problem

An entrepreneur of age i in period q has the following consumption-saving problem

$$\max \sum_{j=i}^J \beta^{j-i} \log c_{j,t}^E$$

preferential credit and tax policies.

³⁹For example, in 2007, the state government issued an document (the 39th Decree), which requests a transition from preferential corporate income tax rates to legal tax rates. According to this document, those who enjoyed a 15% corporate income tax rate before 2008 would have tax rates of 18%, 20%, 22%, 24%, and 25% for each year between 2008 and 2012, respectively.

subject to

$$\begin{aligned}
c_{j,t}^E + s_{j,t}^E &= m_t + R s_{j-1,t-1}^E, \quad \text{for } j < J^E - 1 \\
c_{j,t}^E + s_{j,t}^E &= \rho_t^E s_{j-1,t-1}^E, \quad \text{for } j \geq J^E - 1 \\
s_{j,t}^E &\geq 0 \text{ for } j \geq J^E - 1 \\
s_{J,t}^E &= 0, \quad s_{0,t-1}^E = 0.
\end{aligned}$$

Here again, for the no-arbitrage condition to hold, we assume that the inner solution of entrepreneurial portfolio choice exists. Given the savings, $s_{j,t}^E$, the age- j entrepreneur at period t then makes the portfolio choice $\phi_{j,t}^E$.

4.1.5 Aggregation

Proposition 3 *There exists an equilibrium in which all cohorts alive at period t will invest the same share of wealth in housing; that is, $\phi_{j,t}^E = \phi_t^E$, for $j \in [J^E - 1, J - 1]$.*

Proposition 3 allows us to derive the following equations for aggregate capital and housing in equilibrium.

$$\begin{aligned}
K_{t+1}^E &= \phi_t^E \sum_{j=J^E-1}^{J-1} N_{j,t}^E s_{j,t}^E, \\
P_t^H H_t^E &= (1 - \phi_t^E) \sum_{j=J^E-1}^{J-1} N_{j,t}^E s_{j,t}^E,
\end{aligned}$$

where $N_{j,t}^E$ denotes the number of entrepreneurs of age j at period t . Therefore, we can solve the aggregate labor demand and portfolio choices by aggregation. The appendix describes the numerical algorithm to solve for the calibrated economy.

4.2 Calibration

We use data from the NBSC to calibrate the model. The model economy starts in 1998, when China started to privatize its SOEs. Each period in our model corresponds to one calendar year.

Consider the first set of parameters, whose values are set exogenously. Agents in our model enter the economy at age 22 and live for 50 years. This is consistent with an average life expectancy between males and females of 71.4 years according to the 2000 Chinese Population Census. Workers retire after 30 years. The population growth rate is set to $\nu = 0.03$, consistent with the average urban population growth rate in China during 1998-2012.

In terms of technology parameters, the capital income share is set to $\alpha = 0.5$, consistent with Bai, Hsieh and Qian (2006).⁴⁰ The capital depreciation rate is set to $\delta = 0.1$, which is the average depreciation rate between 1998 and 2012, computed using the method of Bai, Hsieh, and Qian (2006). The land supply is normalized to $\bar{H} = 1$, and the bank deposit rate is set to $R = 1.0175$, following Song, Storesletten, and Zilibotti (2011). Finally, the initial assets of the workers and retirees are set to be the same as the corresponding wealth in an initial steady state where there are only F-firms.

Now we turn to the second set of parameters, whose values are set endogenously to target some data moments. We calibrate $\beta = 0.998$ to match the average aggregate saving rate between 1998 and 2012 of 40%. We then set $\psi = 0.53$ to match the aggregate rate of returns to capital at 1998, which is 20%. Following SSZ, we calibrate the productivity parameter of E-firms to be $\chi = 5.64$ to match the following moment: The capital-to-output ratio of Chinese SOEs is 2.65 times higher than domestic private firms. The iceberg cost ξ is set to 0.0693 to match the marginal product of capital (MPK) for SOEs to be 0.093. The rate of labor-augmented technological growth is set to $z = 3.8\%$ to match the average growth rate of GDP of 10% during 1998-2012.

The initial entrepreneurial wealth is set to match an initial employment share of private firms at 1998. According to NBSC data, the employment share of private firms (DPE + FE) in total employment is 17% in 1998. Accordingly, the initial life-cycle distribution of wealth for managers and entrepreneurs is a scaled-down version of the life-cycle distribution of wealth for workers in the initial steady state. For the time path of output wedges, we assume a linear pattern between τ_{1998}^y and τ_{2012}^y , such that $\tau_t^y = \tau_{2012}^y - (2012 - t)\kappa$ for $1998 \leq t \leq 2012$. For $t \geq 2013$, we assume $\tau_t^y = 0$. We then calibrate τ_{2012}^y and κ to best fit the trajectory of private employment share between 1998 and 2011. This gives $\kappa = 0.029$, $\tau_{2012}^y = 0.01$.

⁴⁰Since in our model housing does not provide services and is in fixed supply, our measured capital stock corresponds to the concept of total reproducible capital stock, which we constructed following the method of Bai, Hsieh, and Qian (2006).

4.3 Results

To assess the model’s performance, we compare the simulated housing prices with both the annualized constant-quality housing price indices between 2006 and 2010 and the annual constant-quality national-level land price indices by Wu, Gyourko, and Deng (2012).⁴¹ The land price data cover the period of 2004-2012 and were updated on Gyourko’s webpage.⁴² Since the land price tracks the housing price closely (see Figure 1), we view the land price as a good proxy for the housing price for those years where the housing price data are missing. To illustrate the growth rate of the housing price relative to that of GDP, we also construct the ratio of housing prices to GDP in both the model and the data. We then normalize the simulated and actual housing prices and housing price-to-GDP ratio in 2004 to 1.

Figure 7 shows the main predictions of the model, together with their data counterparts. In Panel A, we see that the simulated housing prices replicate the actual data fairly well until 2011, with an average growth rate of 19% between 2004 and 2011. The overprediction of housing prices in 2012 may be due to the fact that since 2010, the Chinese government adopted several policies to control housing prices, from which our model is abstracted. Panel B shows that the simulated housing price-to-GDP ratio increases from 1 in 2004 to 1.77 in 2012, or an average of 7.43% per year. Hence, our model can replicate the magnitude of the increase in housing prices *relative to* GDP reasonably well. We view this result as further support of our mechanism for the housing price dynamics in China, since labor reallocation from F-firms to E-firms is key to delivering a sustained high return to E-firms’ capital, not only in absolute terms, but also relative to the growth rate of aggregate output.

Panel C of Figure 7 shows that our model can match closely the dynamics of the private employment share. Note that as time approaches the year 2012, the increase of the private employment share starts to slowdown. This is because entrepreneurs slow their accumulation of physical capital in anticipation of an increase in implicit output distortion.

In Panel D of Figure 7, the aggregate rate of returns to capital is sustainably high between 1998 and 2012. It starts to decline around 2008, which coincides with the turning point of the data. Two opposite effects in the calibrated economy drive this dynamics. On the one hand, the increase of the private employment share increases the average returns to capital, thanks to a higher E-firm rate of returns to capital. On the other, the increase in implicit output distortion on E-firms during this period tends to reduce their net returns to capital.

Panel E of Figure 7 shows that the aggregate output growth in our model replicates China’s GDP growth reasonably well and is sustainable around 10% between 1998 and 2012.

⁴¹The housing price indices provided by NBSC, although dated back to 1998, do not control for quality.

⁴²See the weblink <http://real.wharton.upenn.edu/~gyourko/chineselandpriceindex.html>.

Again, the observed hump shape of output growth rates is due to the above-mentioned two opposite effects.

Panel F of Figure 7 plots the dynamics of aggregate TFP. Between 1998 and 2006, the average TFP growth rate is 6.66 percent. This is in line with the estimation of Brandt, Van Biesebroeck, and Zhang (2009), who report an estimate for manufacturing sector TFP growth of 6.1% to 7.7% during 1998-2006. Resource reallocation contributes 4.76% to annual TFP growth. Therefore, 71.5% of the TFP growth between 1998 and 2006 in our model is due to reallocation, which is broadly consistent with the findings of Brandt, Van Biesebroeck and Zhang (2009). A falling TFP growth rate in the second half of our sample period is due to an increase in implicit output distortion toward E-firms, which, in turns, slows the pace of labor reallocation.

4.4 Counterfactual Experiments

We now conduct several counterfactual experiments to shed additional light on the structure of our model.

Firm Heterogeneity. A standard representative-agent neoclassical growth model also features a “transitional” period of capital accumulation before reaching the steady state, along which the MPK declines gradually. Would this declining MPK along the neoclassical transition generate a faster-than-GDP growth in housing prices, everything else equal? The answer is no. To illustrate this, we shut down F-firms in our model so the counterfactual economy is essentially neoclassical with only E-firms and without the transition stage featuring “surplus” labor and resource reallocation—the focus of our analysis in the previous sections. In this counterfactual economy, the parameters χ and \widehat{k}_t^E measure the aggregate, rather than E-firm-specific, productivity and capital stock, respectively. Therefore, we set \widehat{k}_1^E to be the same as the initial aggregate capital stock in the benchmark economy. We then recalibrate χ to target an initial aggregate rate of return to capital of 0.20. We keep all other parameters the same as in our benchmark economy. Figure 8 reports the simulated results in this counterfactual economy, together with their counterparts in the original model. Without firm heterogeneity, the private employment share always equals to 1 (Panel C). Panel A shows that in this counterfactual economy, housing prices grow significantly slower than the benchmark counterparts. This is due to two reasons: First, in this counterfactual economy, the growth rate of housing prices equals the aggregate rate of return to capital. Second, the aggregate rate of return to capital falls over time along with capital accumulation (Panel D). In contrast, in our benchmark economy, the housing price growth rate equals the rate of

return to E-firm’s capital, and the AK feature of E-firms helps to sustain the high return to capital during the transition. Accordingly, in this counterfactual economy, the housing price relative to GDP is essentially flat (Panel B). This is despite a declining output growth rate (Panel E). Intuitively, without firm heterogeneity, the dynamics of aggregate output growth track closely the dynamics of the aggregate return to capital, which implies that housing prices grows at a rate similar to that of aggregate output.⁴³ Another noticeable result is that the TFP growth rate in this counterfactual economy is exogenous and equal to the trend growth rate. As Panel F shows, without resource reallocation, the TFP growth rate is significantly lower than its counterpart in the benchmark economy, though in the long run, their magnitudes are the same.

Output Wedge. We shut down E-firm-specific output wedges to isolate their roles in the calibrated economy. We then recalibrate the initial entrepreneurial wealth so that the private employment share in 1998 equals that in our benchmark economy. Panel A of Figure 9 shows that without output wedges, house prices grow somewhat slower than their benchmark counterparts, at an average 14% per year between 1998 and 2012. Accordingly, the growth of housing prices relative to GDP also becomes slower than the benchmark economy (Panel B). Intuitively, the initial implicit output subsidy to E-firms pushes up the rate of returns for E-firms’ capital. Panel C shows that the private employment share increases significantly more slowly than its benchmark counterpart, reaching a level of only 40% in 2011, in contrast to a level of 62 percent in the benchmark economy. Accordingly, the aggregate return to capital and the aggregate output growth rates are also lower than their counterparts in the benchmark economy (Panels D and E). A slower labor reallocation from F-firms to E-firms also reduces the aggregate TFP growth rate for most years between 1998 and 2012.

The Crowding-out Effects of Housing Bubbles. We now explore the normative implications of housing bubbles. To this end, we shut down entrepreneurs’ access to the housing bubble by setting $\phi_t^E = 1$ for all t . Accordingly, without demand the housing prices will always equal zero. All parameters remain the same as in the original calibration. Figure 10 plots the transition path for both the counterfactual and the original economies. Panel A shows that without housing bubbles, the private employment share rises much faster. For example, by 2011, the private employment share is around 74%, 14% higher than the benchmark counterpart. This implies that the presence of a housing bubble prolongs the economic transition.⁴⁴ Panel B shows that without housing bubbles, aggregate output grows faster

⁴³When physical capital is not fully depreciated, the growth rate of E-firms’ total output will be somehow less than the aggregate or entrepreneurs’ returns to capital, because the growth rate of entrepreneurs’ wealth is decreasing over time before the economy reaches the steady state.

⁴⁴During the transition, the aggregate rate of return for capital is lower in the benchmark economy than

between 1998 and 2011, with an average growth rate of around 11%. Accordingly, the steady-state aggregate output is 9.51% higher than its counterpart in the bubbly equilibrium. Panel C shows a similar pattern for TFP growth rate. Between 1998 and 2011, TFP grows at an average rate of 5.9% per year, in contrast to the rate of 5.1% in our benchmark economy. Such a disparity suggests that housing bubbles exacerbate resource misallocation by crowding out productive investment, slowing labor reallocation from unproductive firms (F-firms) to productive firms (E-firms), and thus resulting in a lower aggregate productive and efficiency. Panel D suggests a significant welfare loss due to the presence of a housing bubble. Between 1998 and 2012, aggregate consumption would be 9.63% higher without the housing bubble. Even at steady state, aggregate consumption is 6.59% higher in the fundamental equilibrium. Intuitively, housing bubbles, by crowding out productive capital investment, reduce permanent incomes of future cohorts via both managers' compensation and wage rate.

4.5 Discussion of Results

While the focus of our paper is China, our model can shed light on the occurrence of housing bubbles in other emerging economies during their rapid-growth transition periods. The highlight of our theory is that economic transitions driven by massive resource reallocation must eventually come to an end and that the associated exceptionally high capital returns are thus unsustainable in the long run. Such rational expectations can induce even the productive agents in the economy to seek alternative stores of values (such as housing and land) for their rapidly growing wealth. Indeed, a similar transition pattern has also occurred in other East Asian economies during their earlier stages of industrialization, such as in South Korea and Taiwan. For both economies, intersectoral labor reallocations from agriculture to manufacturing (or the service sector) contributed importantly to their fast economic growth.⁴⁵ Therefore, a brief analysis of the two economies' housing booms serves as a useful examination of the relevance of our theory for other emerging countries.

South Korea experienced an acceleration of output growth in the second half of the 1980s. According to Young (1995), during 1985-1990, the average growth rates of aggregate output and TFP were 10.7% and 2.6%, respectively, both significantly higher than in the early 1980s and the 1970s. The capital productivity (or return to capital), as measured by the ratio of

the fundamental economy due to a lower private employment share.

⁴⁵Song, Storesletten and Zilibotti (2011) emphasize the resource reallocation to financially constrained private firms within the manufacturing sector as a key reason to explain the coexistence of acceleration of productivity growth and a foreign surplus in Korea and Taiwan in the 1980s. We view the intersectoral labor reallocation as essential for the housing booms experienced in these two countries during the 1980s.

gross value added to total asset, was persistently high during the 1980s, especially for small and medium-sized enterprises (Nugent and Yhee 2002, Table 6).⁴⁶ Meanwhile, land prices almost tripled during 1985-1991, compared with only a 60% increase during 1980-1985.⁴⁷ Interestingly, this housing boom coincided with important structural changes in the Korean economy. In the 1980s, Korea experienced a massive reallocation of labor from agricultural to other sectors. The share of agricultural employment dropped by 20% between 1980 and 1992, compared with a mere 6% drop between 1992 and 2007.⁴⁸ Since the mid-1980s, labor costs had been rising significantly, reflecting a pending shortage of labor (see Smith, 2000, Figure 3.3). While the rate of increase slowed by the early 1990s, the real average monthly earnings in manufacturing still grew by an average of 7.8 per year between 1992 and 1996, while productivity growth lagged. This increase in labor costs increasingly undermined Korean firms' competitiveness, which led to a rapid increase in relative export prices during this period (Smith, 2000, Figure 3.2).

Similar to the Korean and Chinese experiences, the housing boom in Taiwan happened during a time of massive labor reallocation and fast economic growth. According to Koo and Park (1994), Taiwan witnessed a sharp increase in land prices in the second half of the 1980s.⁴⁹ Accordingly, average housing prices in the Taipei area more than quadrupled from late 1986 to early 1990. During the same period, real gross national product (GNP) grew at an annual rate of 9% and capital productivity peaked in 1987.⁵⁰ At the micro level, Taiwan experienced a fast reallocation of labor from agriculture to manufacturing and service sectors since the 1970s. The share of agricultural employment dropped from 25% in 1978 to 13% in 1989. A labor shortage had gradually become apparent in the manufacturing sector since the late 1970s and more so after the mid-1980s. Labor costs rose as a result of this labor shortage. The average monthly real wage, which increased by 6.5% per year during 1981-1986, increased by 11.4% per year between 1986 and 1990. Accordingly, Taiwan also experienced a sharp increase in the relative export prices in the second half of the 1980s (Smith 2000, Figure 3.2).

In conclusion, despite important cultural and institutional differences, South Korea and Taiwan shared some common features in their development paths and, especially in their

⁴⁶Specifically, the ratio of gross value added to total assets is more than 35 percent for small and medium enterprises and more than 24 percent for large enterprises during 1985-1991. This is in contrast to a declining pattern of capital productivity for both types of firms between 1992 and 1997.

⁴⁷After 1992, both land and housing prices leveled off until the 1997-98 financial crisis.

⁴⁸See Lee (2010) for details.

⁴⁹Accompanying the soaring land price is the sharp increase in stock prices.

⁵⁰See Chen (2001) and Hsiao and Park (2002).

housing booms, with the recent Chinese experience. All three economies experienced rapid and faster-than-GDP growth in housing prices and simultaneous high capital returns during their respective economic transitions—driven largely by cheap labor and resource reallocation. The completion of the transition process eventually led to rising labor costs and the cooling down of housing bubbles. These features are consistent with the predictions of our theory.

5 Conclusion

This paper provides a framework to explain the paradoxical features of the great housing boom in China—the persistently faster-than-GDP housing price growth, exceptionally high capital returns, and excessive vacancy rates, or the ghost-town phenomenon. Our theory suggests that the decade-long housing boom contains a rational bubble naturally arising from China’s unprecedented economic transition, which is featured by resource reallocation from low-productivity to high-productivity firms. The model’s predictions are consistent (qualitatively and quantitatively) not only with China’s broad growth pattern, but also with the three paradoxical features of the great housing boom. We also show that such a growing housing bubble can crowd out productive capital investment, thus prolonging the economic transition and reducing social welfare.

A number of simplifications have made our model tractable. For example, housing in our model does not provide utility services. A richer model with both homeowners and speculators in housing markets would enrich the welfare implications of growing housing bubbles. For example, a growing housing bubble distorts homeowners’ life-cycle consumption patterns under borrowing constraints by forcing people to save excessively when young to enter the housing market. Furthermore, the rapidly rising housing prices tend to worsen wealth inequalities across income classes, as the housing price growth is driven largely by the upper-middle-income class that enjoyed the most rapid income growth during the economic development. By contrast, more and more low-income households are excluded from the housing market—because their income growth falls behind that of housing price growth. Moreover, our model abstracts from several institutional details of China’s housing and land markets (e.g., local government’s heavily reliance of revenue from land sales), which, we believe, might also be a contributing factor to the size of China’s gigantic housing bubble. These are all important issues for our future research.

Despite its simplicity, a calibrated version of our model quantitatively matches the growth dynamics of housing prices and other salient features of the recent Chinese experience reasonably well. We therefore view our model as a useful starting point to study the macroeconomic

implications of growing housing bubbles in China. For example, a growing housing bubble reduces the private sector's incentive to innovate. Because of the relatively low risk, low entry costs, low technology, and high profits in housing investment, the housing bubble has enticed many productive and high-tech firms in China to reallocate resources from R&D to the real estate market. In an economy in transition from a labor-intensive economy to a capital-intensive economy, such resource misallocation can be very costly: It may substantially prolong China's economic transition and reduce China's TFP growth, especially when its population is aging fast and labor costs are rapidly rising. We plan to empirically validate and quantify such resource misallocation within our framework in future works.

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Tables and Figures

Table 1. Correlation: Housing Price Growth and Fixed Investment Growth

Nationwide		
	Real Estate Investment	Other Investment
Current	0.5255*	-0.3212*
$t - 1$	0.4765*	-0.4046*
$t - 2$	0.4115*	-0.4499*
$t - 3$	0.3320*	-0.5025*
$t - 4$	0.2710*	-0.5467*
$t - 5$	0.2025	-0.5438*
$t - 6$	0.1288	-0.5171*

Source: The aggregate monthly house price data from January 2006 to December 2011 are from Wu, Deng, and Liu (2014). The corresponding monthly investment data are from CSY (various issues). To remove seasonality, the growth rates for housing prices and investment are computed as the growth rate over the same month of the previous year. *Significant at the 5% level.

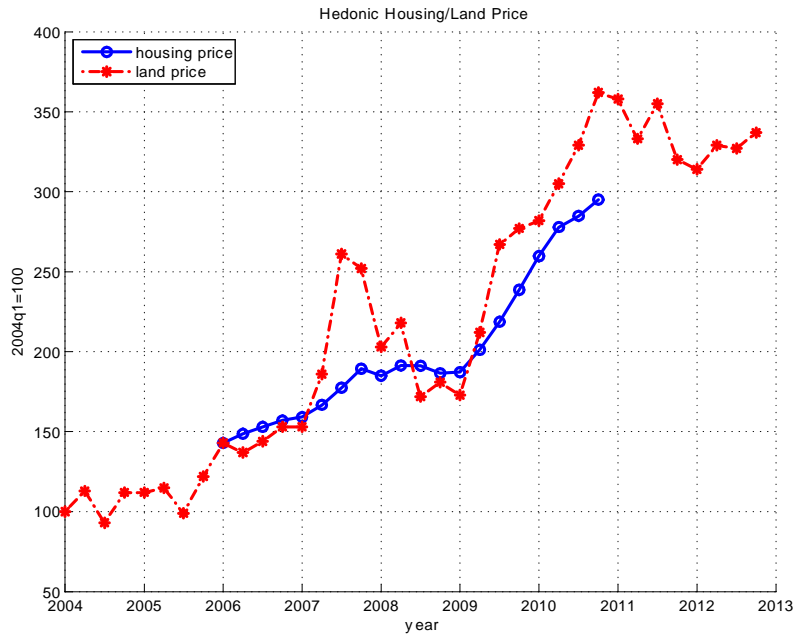


Figure 1. Housing/Land Prices in China

Source: The hedonic house price data are from Wu, Deng and Liu (2014), and the hedonic land price data are from Wu, Gyourko, and Deng (2012).

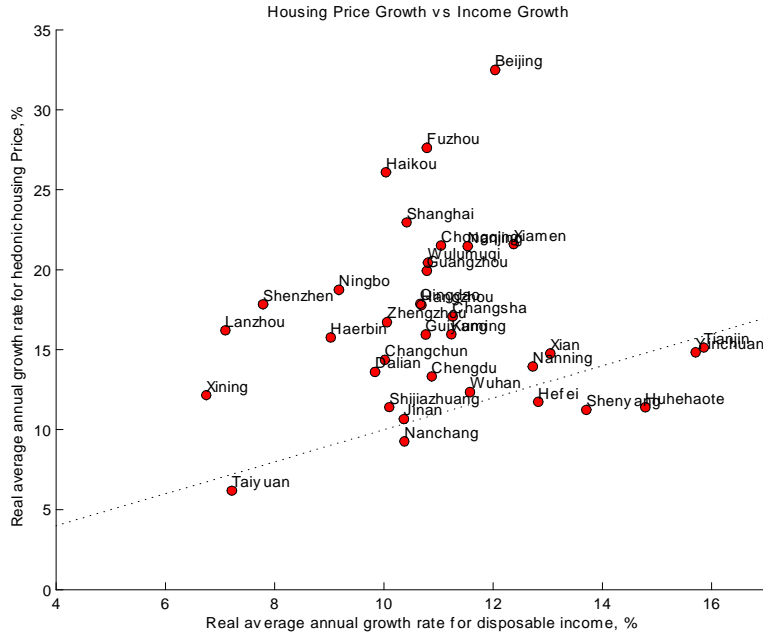


Figure 2. Growth Rate of Housing Prices and Aggregate Income across Major Cities in China

Source: The hedonic house price data for 35 major cities are from Wu, Deng and Liu (2014). The growth of disposable income is computed by the authors based on the growth rate of real disposable income and the growth rate of the urban-residing population from various issues of the statistics communiqués for different cities. The dashed line in Panel B is the 45-degree line.

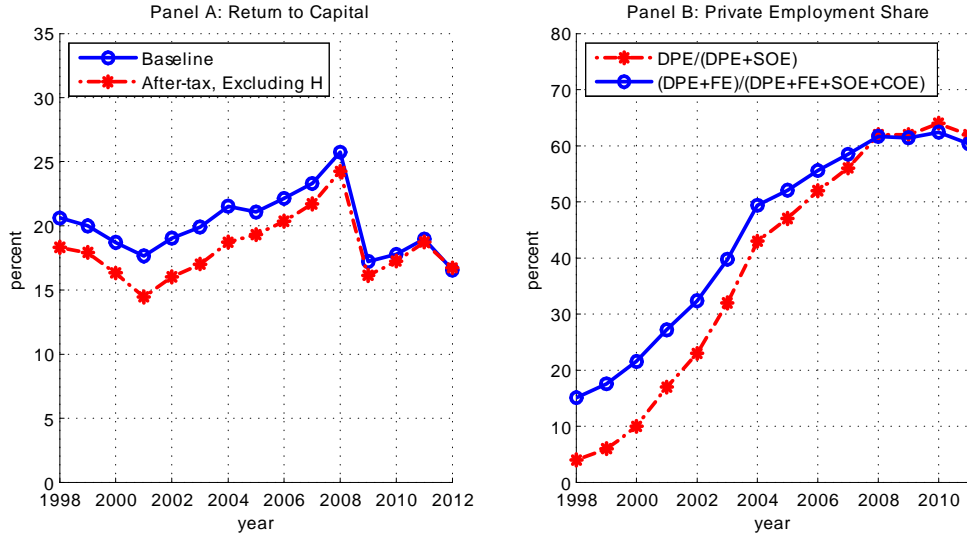


Figure 3. Returns to Capital and Labor Reallocation.

Note: DPE, domestic private enterprises; FE, foreign enterprises; SOE; state-owned enterprises; COE, collectively owned enterprises.

Source: In Panel A, the rate of return to capital for various years is computed by the authors using the approach of Bai, Hsieh, and Qian (2006) and data from CSY (various issues). The circle line refers to the gross rate of return to capital, and the dashdot line refers to the after-tax return to capital excluding urban residential housing. In Panel B, the private employment shares are computed by the authors using data from CSY (various issues).

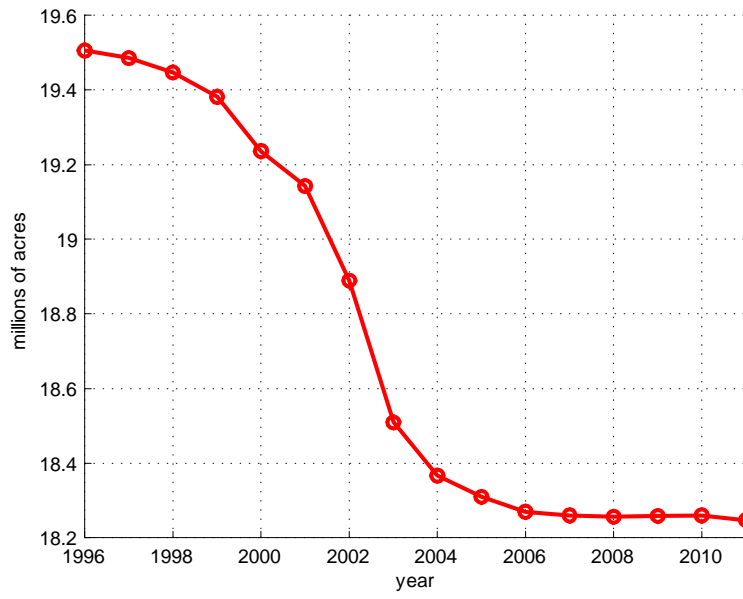


Figure 4. Total Amount of Arable Land

Source: CSY (various issues).

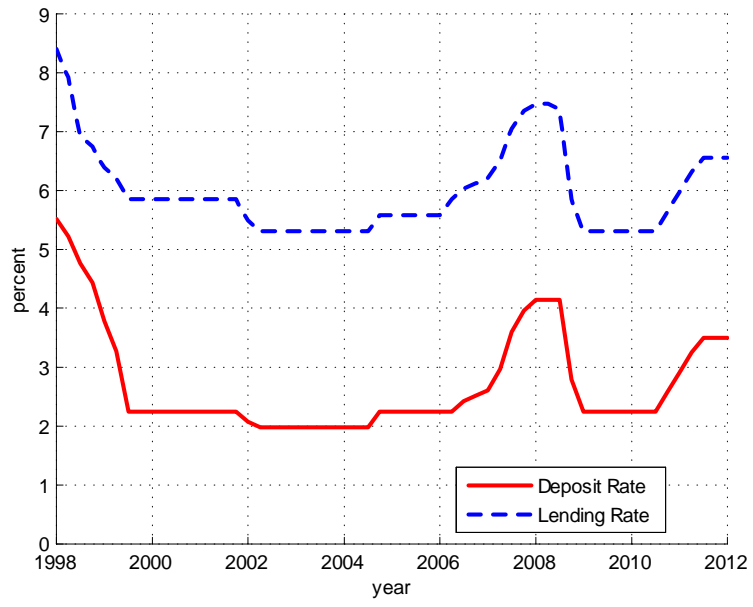


Figure 5. China's One-Year Benchmark Nominal Interest Rates

Source: CEIC Database.



Figure 6. Transition in the Two-Period Economy.

Notes: The figure shows the evolution of key variables in the analytical economy. In Panel B, the solid line refers to ρ_t^E and the dashdot line refers to the growth rate of aggregate output. Time T denotes the end of transition, when all workers are employed by E-firms.

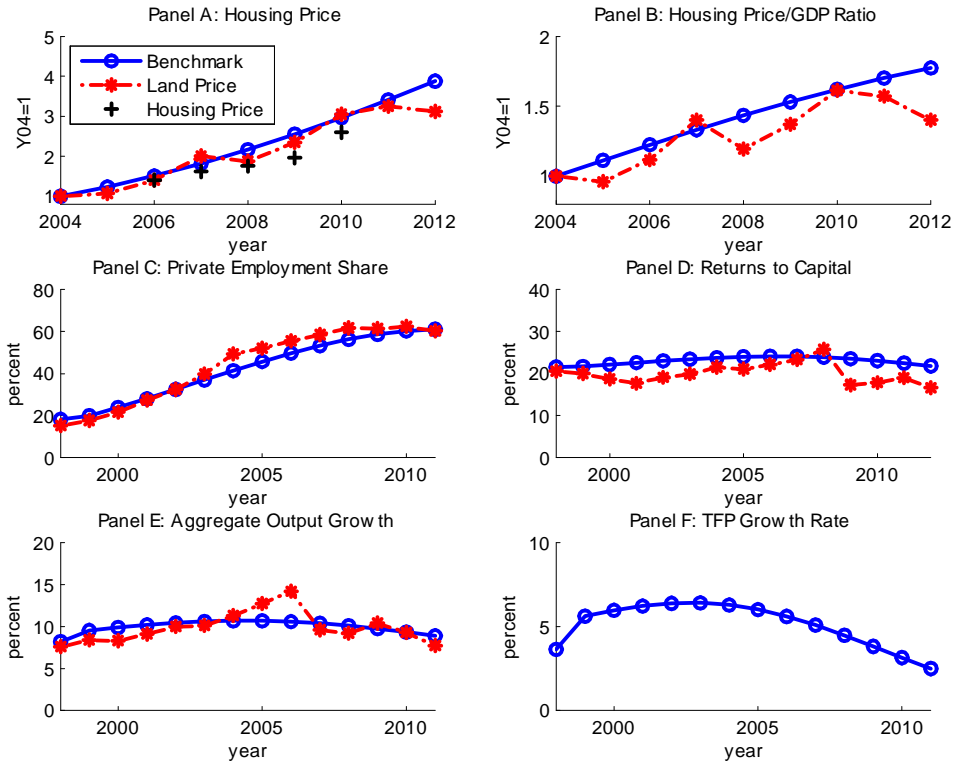


Figure 7. Transition in the Multiperiod Calibrated Economy

Notes: This figure shows the evolution of key variables during and after transition of the calibrated economy. The solid and dashdot lines refer to the simulated results from the model and the data, respectively. In Panel A, the plus refers to annualized housing price data from Wu, Deng, and Lin (2014), whereas the dashdot line refers to annual land price data from Wu, Gyorko, and Deng (2012). Both the simulated housing price data and actual land price data are normalized to 1 in 2004. The housing price data in 2006 are normalized to equal the normalized actual land price data in 2006.

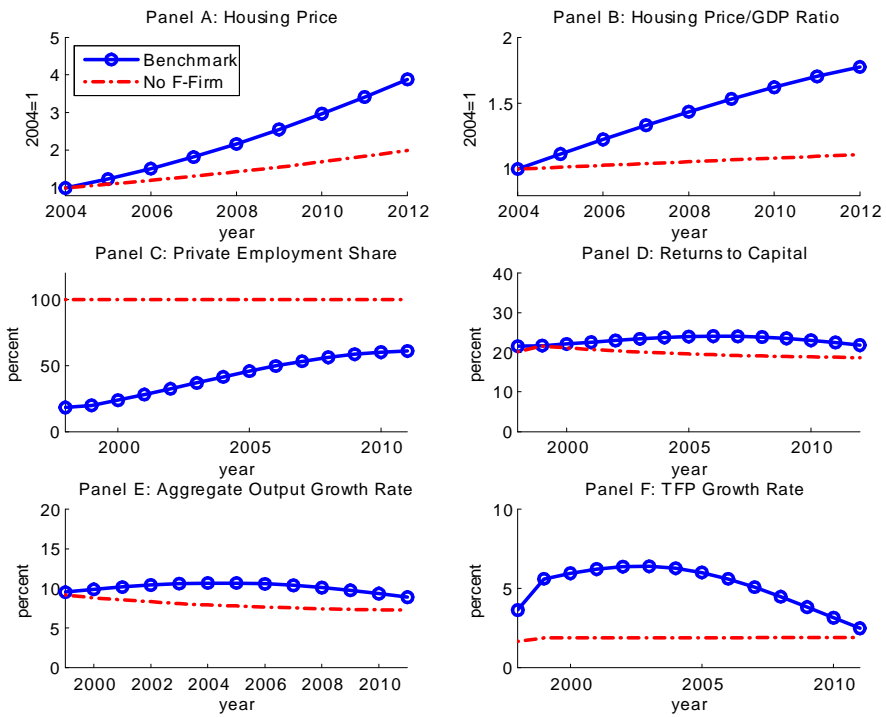


Figure 8. The Role of Firm Heterogeneity in the Calibrated Economy

Notes: The figure shows the evolution of key variables in the calibrated economy (solid line) and the counterfactual economy in which there are no F-firms (dashdot line).

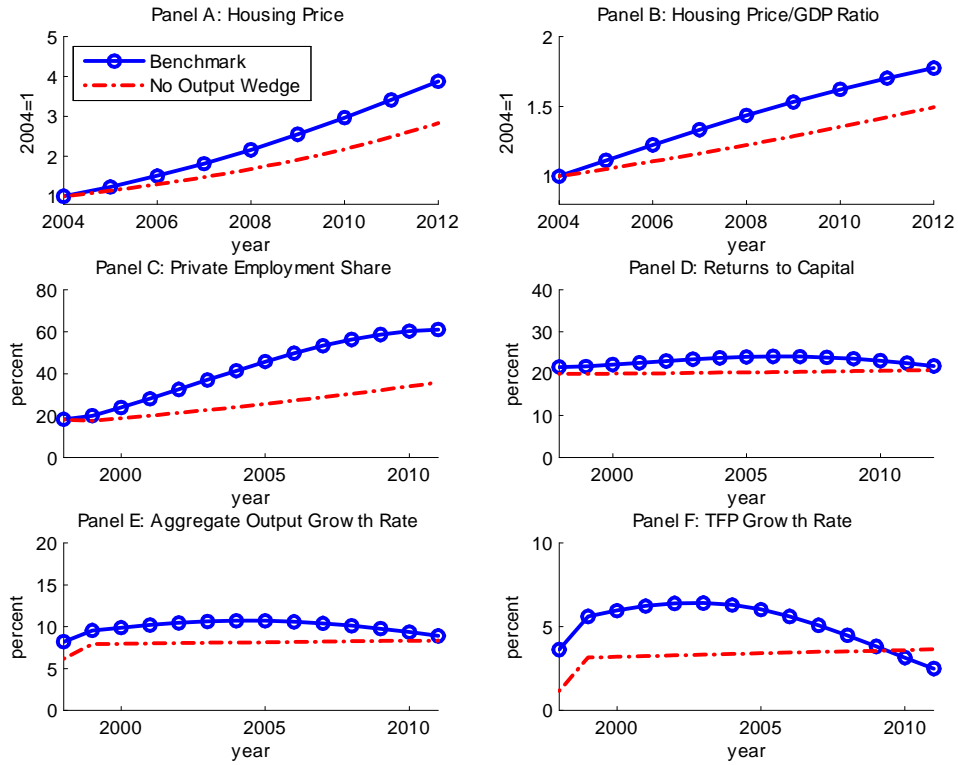


Figure 9. The Role of the Output Wedge in the Calibrated Economy

Notes: The figure shows the evolution of key variables in the calibrated economy (solid line) and the counterfactual economy in which there are no output wedges (dashdot line).

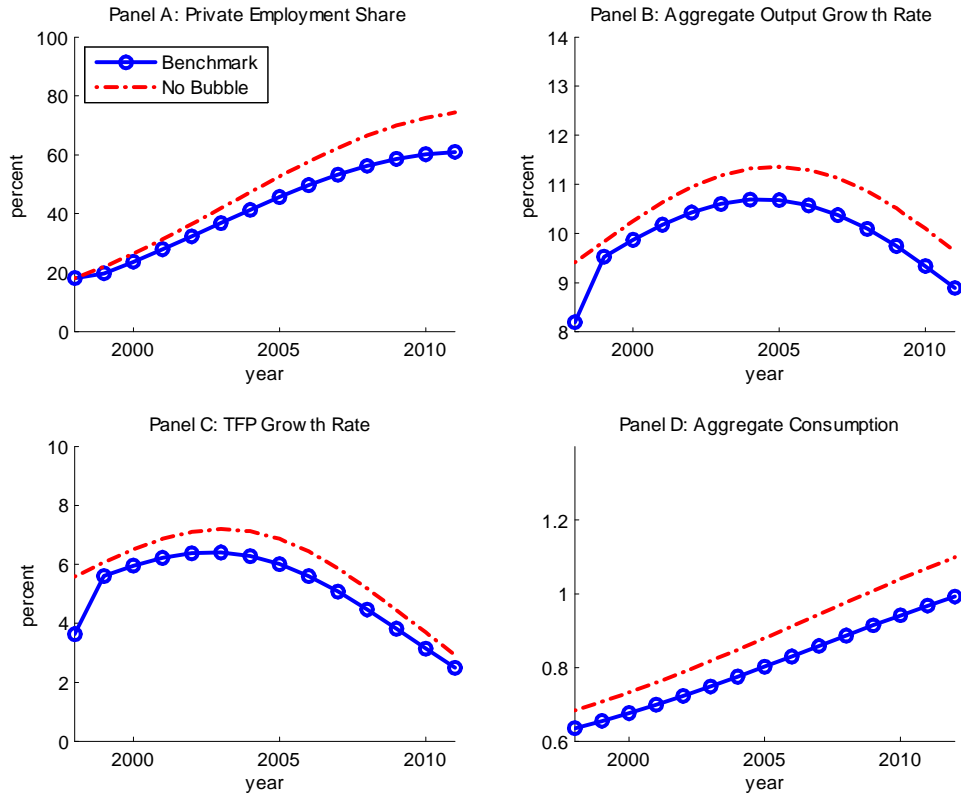


Figure 10. The Welfare Effects of Housing Bubbles in the Calibrated Economy

Notes: The figure shows the evolution of key variables in the calibrated economy (solid line) and the counterfactual economy in which there are no housing bubbles (dashdot line).

APPENDIX (not intended for publication)

A. Proof of Propositions and Lemmas

In this section, we prove the various lemmas and propositions.

Proof of Lemma 1. The growth rate of E-firms output is

$$\frac{Y_{t+1}^E}{Y_t^E} = \frac{Y_{t+1}^E}{K_{t+1}^E} \frac{K_{t+1}^E}{K_t^E} \frac{K_t^E}{Y_t^E} = \frac{\rho_{t+1}^E}{(1-\psi)\alpha} \frac{K_{t+1}^E}{K_t^E} \frac{(1-\psi)\alpha}{\rho_t^E}, \quad (21)$$

where K_{t+1}^E/K_t^E depends on the equilibrium portfolio share of entrepreneurs in physical capital, ϕ_t^E . We now solve for the equilibrium portfolio share of entrepreneurial savings in housing. Using the housing market-clearing condition, $\bar{H} = H_t^E$, we have

$$(1 - \phi_t^E) \frac{1}{1 + \beta^{-1}} \psi (K_t^E)^\alpha (A_t \chi n_t^E N_t)^{1-\alpha} = P_t^H \bar{H}. \quad (22)$$

Forwarding (22) by one period, and with $(K_{t+1}^E)^\alpha (A_{t+1} \chi n_{t+1}^E N_{t+1})^{1-\alpha} = K_{t+1}^E \rho_{t+1}^E / [\alpha(1-\psi)]$, equation (22) can be rewritten as

$$(1 - \phi_{t+1}^E) \frac{1}{1 + \beta^{-1}} \frac{\psi \rho_{t+1}^E K_{t+1}^E}{\alpha(1-\psi)} = P_{t+1}^H \bar{H}. \quad (23)$$

With the law of motion for capital (5), (23) can be rewritten as

$$(1 - \phi_{t+1}^E) \frac{1}{1 + \beta^{-1}} \frac{\psi \rho_{t+1}^E}{\alpha(1-\psi)} \frac{\phi_t^E}{1 + \beta^{-1}} \psi (K_t^E)^\alpha (A_t \chi n_t^E N_t)^{1-\alpha} = P_{t+1}^H \bar{H}. \quad (24)$$

Dividing (24) by (22) for all t , we have

$$\frac{1 - \phi_{t+1}^E}{1 - \phi_t^E} \frac{\phi_t^E}{1 + \beta^{-1}} \frac{\psi \rho_{t+1}^E}{\alpha(1-\psi)} = \frac{P_{t+1}^H}{P_t^H} = \rho_{t+1}^E,$$

or simply

$$\frac{1 - \phi_{t+1}^E}{1 - \phi_t^E} \frac{\phi_t^E}{1 + \beta^{-1}} \frac{\psi}{\alpha(1-\psi)} = 1. \quad (25)$$

Equation (25) is a first-difference equation capturing the dynamics of ϕ_t^E . One solution to equation (25) is that

$$\phi_t^E = \frac{\alpha(1 + \beta^{-1})(1 - \psi)}{\psi}, \forall t. \quad (26)$$

To solve for K_{t+1}^E/K_t^E , we substitute (26) into (5) and obtain

$$K_{t+1}^E = \rho_t^E K_t^E. \quad (27)$$

Equation (27) is a variant of the no-arbitrage condition. Comparing (10) in the fundamental equilibrium and (27) in the bubbly equilibrium, we see that in the bubbly equilibrium the optimal portfolio choice by an entrepreneur equalizes the return to capital investment and the return to bubbles by crowding out E-firms' capital investment.

Finally, substituting (27) into (21), we obtain $Y_{t+1}^E/Y_t^E = \rho_{t+1}^E = \rho_{t+1}^H$. ■

Proof of Proposition 1: We first decompose the ratio of housing value to aggregate output as

$$\frac{P_t^H \bar{H}}{Y_t} = \frac{P_t^H \bar{H}}{Y_t^E} \frac{Y_t^E}{Y_t^E + Y_t^F}. \quad (28)$$

The first argument on the right side of (28), $P_t^H \bar{H}/Y_t^E$, can be further expressed as

$$\frac{P_t^H \bar{H}}{Y_t^E} = \frac{P_t^H \bar{H}}{K_{t+1}^E} \frac{K_{t+1}^E}{Y_t^E} = \frac{1 - \phi_t^E}{\phi_t^E} \frac{K_{t+1}^E}{Y_t^E}. \quad (29)$$

Equation (27) implies

$$K_{t+1}^E = (1 - \psi) \alpha Y_t^E. \quad (30)$$

With both (26) and (30), it is straightforward that $P_t^H \bar{H}/Y_t^E$ is a constant. Therefore, log-differencing (28), we obtain (17).

Finally, we derive $Y_t^E/(Y_t^E + Y_t^F)$. Using (3), Y_t^E can be expressed as

$$Y_t^E = \frac{N_t^E}{N_t(1 - \psi)} \kappa_F^\alpha A_t N_t, \quad (31)$$

where $\kappa_F \equiv k_{Ft}/(n_{Ft}A_t) = (\alpha/R)^{\frac{1}{1-\alpha}}$. Similarly, it is easy to show that

$$Y_t^E + Y_t^F = \left(1 + \frac{\psi}{1-\psi} \frac{N_t^E}{N_t}\right) \kappa_F^\alpha A_t N_t. \quad (32)$$

■

Proof of Proposition 2. To prove this proposition, consider the fundamental equilibrium—that is, $\phi_t^E = 1$ for all t . According to (6), introducing housing reduces the steady-state physical capital. Hence, we only need to show under which condition a marginal reduction in physical capital reduces total entrepreneurial consumption. Aggregating the budget constraints of the young and old entrepreneurs at period t , and using the capital market-clearing condition, we obtain

$$[N_t c_{1,t}^E + N_{t-1} c_{1,t-1}^E + N_{t+1} k_{t+1}^E] / 2 = N_t [m_t + \rho_t^E k_t^E] / 2. \quad (33)$$

With the definition of c_t^E and $m_t + \rho_t^E k_t^E = [\psi + (1-\psi)\alpha] y_t^E$, a detrended version of (33) is

$$\widehat{c}_t^E = [\psi + (1-\psi)\alpha] \widehat{y}_t^E - \widehat{k}_{t+1}^E (1+z)(1+\nu). \quad (34)$$

Taking the derivative of the right side of (34) with respect to \widehat{k}^E at the steady state, we can obtain the following sufficient condition for introducing bubbles to reduce aggregate consumption for entrepreneurs:

$$[\psi + (1-\psi)\alpha] MPK^{E*} |_{\phi^E=1} > (1+z)(1+\nu). \quad (35)$$

With (7) and the definition of MPK^E , the inequality (35) can be rewritten as

$$[\psi + (1-\psi)\alpha] \alpha (1 + \beta^{-1}) (1+z)(1+\nu) / \psi > (1+z)(1+\nu). \quad (36)$$

Reordering (36), we obtain (20).

The proof of welfare implication for entrepreneurs in both transition and post-transition stages is straightforward. Substituting the detrended version of (5) into (34), we obtain

$$\widehat{c}_t^E = \left[\psi + (1-\psi)\alpha - \frac{\psi}{\alpha(1+\beta^{-1})} \right] \widehat{y}_t^E.$$

Since E-firms' capital increases monotonically, we need to prove only

$$\frac{\partial \widehat{c}_t^E}{\partial \widehat{k}_t^E} = \left[\psi + (1 - \psi) \alpha - \frac{\psi}{\alpha (1 + \beta^{-1})} \right] MPK_t^E |_{\phi^E=1} > 0. \quad (37)$$

By assumption (20), $\partial \widehat{c}_t^E / \partial \widehat{k}_t^E > 0$ for all period t . Hence, housing bubbles, by crowding out physical capital, reduce total entrepreneurial consumption.

For entrepreneurs born during the transition, (38) becomes

$$\begin{aligned} & \log (m_t - s_t^E) + \beta \log \rho^E s_t^E \\ &= (1 + \beta) \log k_t^E + \widehat{C}, \end{aligned}$$

where \widehat{C} is a function of parameters. Therefore, it is easy to see that a reduction in capital stock would reduce the welfare of entrepreneurs born during the transition.

For the entrepreneur born in the post-transition stage, but before reaching the steady state, the lifetime utility can be expressed as

$$\begin{aligned} & \log (m_t - s_t^E) + \beta \log \rho_{t+1}^E s_t^E \\ &= \log \left(\frac{\psi \rho_t^E k_t^E}{(1 + \beta) \alpha (1 - \psi)} \right) + \beta \log \frac{\rho_{t+1}^E k_{t+1}^E (1 + \nu)}{\phi_t^E}. \end{aligned} \quad (38)$$

At steady state, equation (38), after being detrended, becomes

$$\begin{aligned} & (1 + \beta) \log \rho^{E*} \widehat{k}^{E*} - \beta \log \phi^{E*} + C \\ &= (1 + \beta) \log \alpha \frac{1 - \psi (1 + \beta^{-1})}{\psi} \frac{(1 + z) (1 + \nu)}{\phi^{E*}} \left[\frac{\psi \phi^{E*} \chi^{1-\alpha}}{(1 + \beta^{-1}) (1 + z) (1 + \nu)} \right]^{\frac{1}{1-\alpha}} \\ & \quad - \beta \log \phi^{E*} + C \\ &= \left[\frac{\alpha (1 + \beta)}{1 - \alpha} - \beta \right] \log \phi^{E*} + \widetilde{C}, \end{aligned}$$

where both C and \widetilde{C} are functions of parameters. Hence, introducing housing (i.e., a reduction in ϕ^{E*}) reduces the steady-state welfare if $\frac{\alpha(1+\beta)}{1-\alpha} > \beta$, or $\alpha(1 + \beta^{-1}) > 1 - \alpha$. Note that the joint participation and incentive constraints of the young entrepreneur im-

plies $m = \psi y^E > w = (1 - \alpha)(1 - \psi)y^E$, which gives the following parameter restriction: $\psi > (1 - \alpha)(1 - \psi)$, or equivalently, $\frac{\psi}{\psi + \alpha(1 - \psi)} > 1 - \alpha$. Therefore, with assumption (20), introducing housing reduces the entrepreneurial lifetime utility at the steady state.

For entrepreneurs born during the post-transition stage, it is easy to show that equation (38) becomes

$$\begin{aligned} & \log(m_t - s_t^E) + \beta \log \rho_{t+1}^E s_t^E \\ &= \alpha(1 + \alpha\beta) \log k_t^E - (1 - \alpha)\beta \log \phi_t^E + \bar{C}, \end{aligned} \quad (39)$$

where \bar{C} is a function of parameters. We would like to compare (39) under the fundamental and the bubbly equilibrium. In the bubbly equilibrium, since k_t^E is smaller due to previous cohorts' investment in housing, a sufficient condition for welfare loss with a reduction in ϕ_t^E is

$$\alpha(1 + \alpha\beta) > (1 - \alpha)\beta$$

or $\alpha(1 + \beta^{-1}) > 1 - \alpha^2$. ■

Proof of Proposition 3: We consider the portfolio choice of an entrepreneur of age j at period t . Suppose that all other entrepreneurs alive at period t hold the same share of savings in housing, $\phi_{k,t}^E = \phi_t^E$ for $k \neq j$, so that the no-arbitrage condition holds, $\frac{P_{t+1}^H}{P_t^H} = \rho_{t+1}^E$. Accordingly, the age- j entrepreneur is indifferent between housing and physical capital. So $\phi_{j,t}^E = \phi_t^E$ is an equilibrium solution. The same logic applies for the portfolio choice of other entrepreneurs alive at period t . Hence, there exists a solution that all entrepreneurs hold the same share of housings in their net worth. ■

B. Numerical Algorithm

Again, we detrend all per capita variables (except labor inputs and housing) as $\hat{x}_t = x_t/A_t$. For total labor inputs on both the supply and demand sides, we detrend them by dividing them by the size of population, N_t . Denote $n_t^E \equiv \sum_{j=1}^{J^E-1} n_{j,t}^E$ as the total detrended labor demand by E-firms. Since the aggregation holds, the following equation determines total labor allocated to E-firms:

$$n_t^E = [(1 - \psi)(1 - \tau_t^y)\chi]^\frac{1}{\alpha} \left(\frac{R^l - 1 + \delta}{\alpha} \right)^\frac{1}{1-\alpha} \hat{k}_t^E / \chi.$$

Similarly, denote $n_t^w \equiv \sum_{j=1}^{J^R-1} n_{j,t}^w$ as the total detrended labor supply by workers. If $n_t^E > n_t^w$, we have

$$\begin{aligned}\widehat{w}_t &= (1 - \psi)(1 - \tau_t^y)(1 - \alpha) \left(\widehat{k}_t^E / n_t^E \right)^\alpha \chi^{1-\alpha} \\ n_t^E &= n_t^w, \quad n_t^F = \widehat{k}_t^F = 0.\end{aligned}$$

Otherwise,

$$\begin{aligned}\widehat{w}_t &= (1 - \alpha) \left(\frac{\alpha}{R^l - 1 + \delta} \right)^{\frac{\alpha}{1-\alpha}} \\ n_t^F &= n_t^w - n_t^E \\ \widehat{k}_t^F &= (\alpha / (R^l - 1 + \delta))^{\frac{1}{1-\alpha}} n_t^F.\end{aligned}$$

Also, we have the following equations for both the transition and post-transition stages:

$$\begin{aligned}\rho_t^E &= \alpha(1 - \psi) [(1 - \alpha)(1 - \psi)(1 - \tau_t^y)\chi / \widehat{w}_t]^{\frac{1-\alpha}{\alpha}} + 1 - \delta \\ \widehat{m}_t &= \psi(1 - \tau_t^y) \left(\widehat{k}_t^E \right)^\alpha (\chi n_t^E)^{1-\alpha} / \sum_{j=1}^{J^E-1} n_{j,t}^E \\ H_t^E &= \overline{H} \\ \widehat{p}_t^H &= \widehat{p}_{t+1}^H (1 + z)(1 + \nu) / \rho_{t+1}^E \\ \widehat{p}_t^H \overline{H} &= (1 - \phi_t^E) \sum_{j=J^E-1}^{J-1} n_{j,t}^E \widehat{s}_{j,t}^E \\ \widehat{k}_{t+1}^E &= \phi_t^E \sum_{j=J^E-1}^{J-1} n_{j,t}^E \widehat{s}_{j,t}^E.\end{aligned}$$

We assume transition takes T periods. At period T , the economy enters the steady state. The algorithm to solve for the transition takes the following steps:

1. Guess the sequence of $\left\{ \phi_t^E, \widehat{k}_{t+1}^E, \widehat{p}_t^H \right\}_{t=1}^{T-1}$.

2. Given \widehat{k}_1^E , compute $\left\{ n_t^E, \widehat{w}_t, n_t^F, \widehat{k}_{t+1}^F, \rho_t^E, \widehat{m}_t, \widehat{s}_{j,t}^E, \widehat{s}_{j,t}^w, H_t^E \right\}_{t=1}^{T-1}$.

3. Check the following conditions for each period $t = 1, 2, \dots, T - 1$:

$$\begin{aligned}\phi_t^E &= 1 - \frac{\widehat{p}_t^H \overline{H}}{\sum_{j=J^E-1}^{J-1} n_{j,t}^E \widehat{s}_{j,t}^E} \\ \widehat{p}_t^H &= \widehat{p}_{t+1}^H (1+z)(1+\nu) / \rho_{t+1}^E \\ \widehat{k}_{t+1}^E &= \phi_t^E \sum_{j=J^E-1}^{J-1} n_{j,t}^E \widehat{s}_{j,t}^E / [(1+z)(1+\nu)],\end{aligned}$$

and (since ρ_{T+1}^E not known)

$$\widehat{k}_{T+1}^E = \widehat{k}^{E*} = \left[\frac{\phi^{E*} \psi (1 - \tau^y) \chi^{1-\alpha}}{(1 + \beta^{-1}) (1 + z) (1 + \nu)} \right]^{\frac{1}{1-\alpha}}.$$