Collateralized Debt and Sovereign Default *

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Abstract

A fundamental problem of the sovereign debt is the limited legal enforcement compared to the corporate debt. There have been a set of initiatives to establish a global standard on sovereign lending and borrowing. The United Nations Conference on Trade and Development (UNCTAD) aims to set "commonly accepted principles and practices"; however, over the past century, a number of other initiatives have pursued the same goal with no success. This paper exemplifies on the idea that equity-based contracts can provide a solution to non-enforceable sovereign debt contracts. In particular, had Argentina utilized equity-based contracts prior to its 2001 debt default, it would have gained from the following advantages: higher debt-to-GDP ratios, larger welfare gains, lower probability of default and thus lower spreads.

Keywords: sovereign default, sovereign debt, equity based contracts, Islamic finance
JEL Codes: F30, F34, K33

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1 Introduction

Argentina's president announced that the country would not abide by the U.S. Supreme court rulings ordering a payment that is due from Argentina's 2001 default. The court ruled that Argentina breached its contractual obligation of treating all-bondholders equally. The court rules that unless Argentina settles its debt dispute from 2001 default, it is barred from making its payment to the main bond-holders and yet Argentina claims that the decision is not fair. The government did not make a progress on reaching a deal to resolve the case and Argentina defaulted on its debt for the second time in 13 years.

A fundamental problem of the sovereign debt is the limited legal enforcement compared to the corporate debt. With the recent European debt crises along with Mexican, Russian and Argentinean sovereign debt defaults, the initiatives to establish a global standard on sovereign lending and borrowing have resurfaced and attracted increased attention. There have been a set of initiatives established by the United Nations Conference on Trade and Development (UNCTAD) which aims to provide a global standard on sovereign lending and borrowing. The UNCTAD aims to set "commonly accepted principles and practices" for sovereign debt contracts through Principles of Responsible Sovereign Lending and Borrowing. Yet this ambitious goal raises issues with sovereignty and enforceability. Over the past century, there have been other initiatives pursuing similar goals by the UNCTAD and by the League of Nations in the 1920s and 1930s which have all been ineffective (Weidemaier (2013)).

To address enforceability of sovereign debt contracts, this paper proposes equity based contracts to be used. Equity based contracts are common in the commercial sector but, have recently begun to grow in public sector. Borrowing can be collateralized both on existing assets such as land or on future receivables such as future oil revenues. So in the event of a default, the creditor has the right to collect the assets. Mexico for instance, collateralized 17 percent of its debt on future oil revenues according to report of the IMF (2003).

Even though equity based sovereign contracts are practiced all over the world, it is mostly exercised in Islamic finance and there is more available data. With the recent global financial crisis, the proper functioning of conventional banking is now being questioned and has prompted increased attention to Islamic Banking. Hasan and Dridi (2010) and Beck et al.

(2013) have pointed out Islamic banking's superiority during the financial crisis. To provide some intuition about the difference between Islamic compliant finance and conventional finance, principles of Islamic finance are discussed shortly. These principles are: the prohibition on *riba* (generally defined as interest or excessive interest), the prohibition on *gharar* (principle of not selling what you do not possess, thus, short-selling and derivative products are not allowed because the buyer gets into a contract for an intangible asset), and the prohibition on the production of goods and services that are against Islamic values (such as drug, alcohol and pork) (Singh and Sheng (2012)).

Although *sukuk* and bonds are fundamentally different, sukuk is commonly viewed as Islamic equivalent of conventional bonds. Accounting and Auditing Organization for Islamic Financial Institutions (AAOIFI), which is an Islamic autonomous non-profit corporate that was established to monitor if the financial products and the contracts comply with *Sharia* (Islamic law) standards, provides the difference between *sukuk* and conventional bonds. The guidelines indicate that sukuk is not a debt owed to the certificate holder; rather, it is a contract where the owners share the returns and carry out the risk of losses. Thus, the underlying regulations both in conventional bonds and sukuk are not only distinctly different, but sukuk must also be Shariah compliant.

Islamic finance has been growing at a very fast rate. As it is shown in Figure 1, the global sukuk issuances have skyrocketed since 2001 even though it plunged in 2008 due to the global financial crises. Although Islamic finance is still a niche market and its share in the World finance is small, it has nevertheless poised an alternative financial tool as the economies proceed both in the Muslim and non-Muslim world. The United Kingdom has issued £200 million worth of sukuk on June 2014[1] and other non-Islamist states such as Hong Kong, South Africa, Singapore, Brazil and Philippines have announced their plan to issue sukuk in 2014 (Bloomberg (2013)). Private large western banks such as Goldman Sachs and HSBC have already set their part in the market and issued sukuk in 2012. Governments issuing sukuk are: Gambia, Indonesia, Malaysia, Kazakhstan, Kuwait, Pakistan, Qatar, Saudi Arabia, Turkey and United Arab Emirates.

[1] "The bond will be underpinned by rental income from three central government office properties" (www.gov.uk)

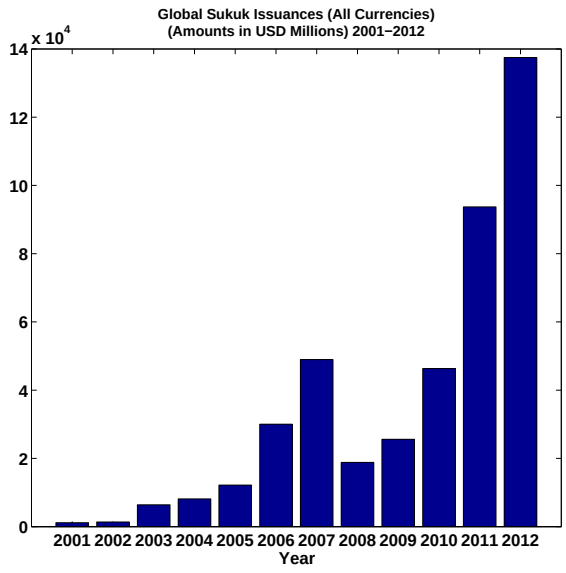

Figure 1: Global sukuk issuance, source: IIFM (2013)

The literature on sovereign default has had an explosive comeback in recent years. After booming in 1980s, the trend began to fade away towards the end of 1990s. Following the domestic debt default by Russia in 1998 and Argentina's international debt default in 2001, the sovereign default literature has gained momentum and peaked with the 2008 European debt crisis. A recurring theme in literature has evolved in which the default mechanism is explained in theoretical framework; the question of how default can be explained in the absence of a legal framework. The recent trend is to provide theoretical explanations to the debt restructuring and consequently there has been an explosion in the empirical literature (see Panizza et al. (2009)).

By far, the most prominent sovereign default litigation case is Argentina's 2001 default. By the end of 2004, there were more than 140 law suits that had been filed in the US, Germany and Italy and, at present (2014), there are still many lawsuits which have not settled and are accumulating excessive legal expenses.[2] Nevertheless, attempts of creditors to block debt restructuring through litigation to keep Argentina out of the financial markets have not been successful. Sturzenegger and Zettelmeyer (2007) provide a summary on the

[2] "plaintiffs sought to attach the representation office of the province of Buenos Aires in New York, diplomatic facilities, U.S. accounts of Correo Argentino S.A. (the renationalized postal service), and most significantly $105 million in reserves held by the Central Bank of Argentina in New York. All these requests have been denied (the latter, on appeal by the U.S. Supreme Court in October 2007)" (Panizza et al. (2009)).

advancement of sovereign debt law and litigation and point out that full repayments are very rare and in most of the cases the creditors receive almost nothing.

This paper is closely related with Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), who study long-term debt models where the sovereign bond contracts cannot be legally enforced. They extend the baseline model of one-period debt and use Argentina as a test case. In addition, Chatterjee and Eyigungor (2012) shows the existence of an equilibrium price function for a long-term debt.

The contribution of this paper is to propose a framework such that it provides a solution for the limited enforcement problem of sovereign debt. This paper focuses primarily on contracts that are equity based. Equity based contracts are widely utilized in Islamic finance in the form of sovereign sukuk issuances. The use of such contracts has shown to increase the aggregate welfare of an economy and assist sovereign states in borrowing higher amounts with lower spreads. In particular, if Argentina's contracts had been designed as equity debt contracts, the probability of default would be significantly lower and Argentina would be able to raise at least twice more in funds.

2 The Model

2.1 Environment

Sukuk-contracts avoid the concept of interest and are based on the notion of a profit-risk sharing idea. The most widely exercised sukuk-contracts can be listed as follows; *Mudaraba*, *Musharaka*, *Murabaha*, *Ijarah* and *Hybrid*. In this paper will I be using sukuk ijarah contract to set up my model as it is the most common Islamic banking product between the government and shareholders (Figure 2). Under an ijarah contract, a government issues sukuk which represents undivided proportionate ownership in the underlying asset and sells the product to the shareholders. The government continues to use the underlying asset (i.e. land, building) and pays rent for the occupation of the land. At maturity, the government may repurchase the asset back from shareholders depending on the nature of the underlying asset. For instance,

the government may not need to repurchase if the market value of an asset at the maturity date is zero (Rohmatunnisa (2008)) [3].

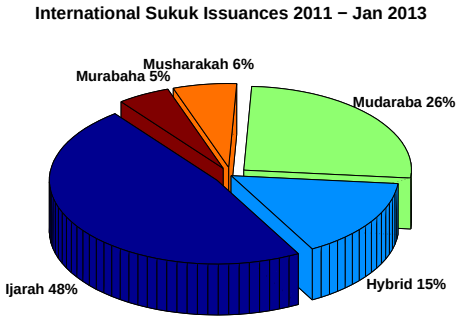

Figure 2: Structural Breakdown of International Sukuk Issuance (% by Volume), Source: IIFM (2013).

In a *Mudaraba* contract, the government acts as an entrepreneur while the Mudaraba holder acts as a capital provider. The government has full discretionary power in any kind of operating decisions and both parties agree on the share of profit at the time of signing the contract, although the losses are borne solely by the sukuk holders. A *Murabaha* contract refers to an agreement where the government purchases the goods on behalf of the investors, puts a profit margin and sells it back to the investors, very similar to leasing contracts. The *Musharaka* contract, on the other hand, requires the government and the sukuk holders to jointly contribute to the project in which profits and losses are being shared among the investors. *Hybrid* is a portfolio of investments including ijarah, Murabaha, shares and/or sukuk certificates.[4]

It is assumed in this paper that the government can only issue ijarah based sukuk contracts[5] and it does not commit to honoring contracts. It is natural to think that different types of sukuk inherently may carry out different risks. However, to make the model more tractable

[3] Central Bank of Malaysia (2009) provides guidelines for an ijarah contract.

[4] Asutay et al. (2013) provide a short history about when modern Islamic finance started to evolve along with different types of Sukuk issuances.

[5] Almost 50% of all sukuk issuances are contracted as an ijarah contract.

and solvable only one type of asset, ijarah sukuk, is used. If the government defaults, it does not pay any current or future rent obligations contracted in the past. Since sukuk-ijarah holders ('shareholders') are the beneficial owners of the underlying assets, shareholders can use up the asset. One major contractual difference of sovereign sukuk ijarah from the sovereign bond therefore is that the agents possess a tangible asset under an ijarah contract. Thus, ijarah sukuk holders will be able to rectify some portion of their loss in the event of defult. For instance, if the government stops paying rent for occupying the land or the building, ijarah holders can sell their portion of the asset and reap the proceeds. However, it may not be possible for an ijarah holder to completely recoup any loss because the value of the land may have diminished at the time of a default state or the shareholder may not be able to sell the asset immediately and may have to incur fees for the maintenance. In the analysis below, it is assumed that shareholders would be able to recoup 10% or 50% worth of the underlying asset in the event of a default.

Why Islamic finance contracts are enforceable, or in other words, if a sovereign does not pay its rent obligations for occupying the land, can the owner of the land (ijarah holder) receives his asset back? More specifically, if Argentina had an ijara contract and defaulted on it, would Argentina let the ijarah holders to exercise their rights on the underlying asset? By contract, shareholders hold the usufruct of the asset and rent their asset to government for a mutually agreed period. It is always possible for a government to take the private property for a purpose of public interest with eminent domain; however, this is a risk of "expropriation", not a risk of default. Unlike sovereign default laws, there are strict national and international laws if a government exercises expropriation. The government cannot take the property of an alien without a fair compensation. The United Nations General Assembly Resolution 1803 requires giving appropriate compensation in cases of lawful state taking. Modeling the risk of expropriation is not the scope of this paper; even so, additional cost terms should be introduced if a sovereign exercises eminent domain on top of defaulting. Thus, introducing expropriation risk would make defaulting costlier, and boost up the results.

In sovereign debt literature, there is a common assumption of exclusion from the international capital market as a form of penalty in the case of a default. However, within the confines of an ijarah contract, the parties have agreed to carry out the risk of loss. As a

consequence, a government is technically able to borrow again after a default. Nevertheless, in practice a government will not be able to find new shareholders due the loss in confidence. Since the first sovereign sukuk issuance in 2000, there has not been a default on a sovereign sukuk default. However, there have been a series of corporate sukuk defaults during the financial crisis in the Gulf Corporation Countries (GCC) such as Dubai Worlds Nakheel Sukuk in 2009 which prevented them from issuing new sukuk for a period of time[6]. Thus, it is assumed that governments will be out of the international capital markets for a certain period of time.

Since ijarah contracts promise a series of rent payments, this paper is modelled as a long-term contract and Macaulay (1938) definition of duration is used to compute the duration of sukuk ijarah contracts. Macaulay Duration D_t is defined as the weighted average maturity of future cash flows. An ijarah issued at period t, paying periodic rents a at dates $n+1$, $n+2$,..., $n+j$ and the final price of M has duration D_t:

$$D_t(a) = \frac{1}{q_t}\left(\sum_{j=1}^{J} n_j \frac{a}{(1+i_t)^{n_j}} + n_J \frac{M}{(1+i_t^m)^{n_J}}\right)$$

where i_t^m is the periodic yield delivered by sukuk ijarah and it satisfies

$$q = \sum_{j=1}^{\infty} a_j (1+i_t^m)^{-j}.$$

So,

$$i^m = \frac{1}{q} - \delta.$$

Since I assumed a rent structure such that sukuk ijarah promises to pay infinite sequence of rents that decrease at a rate δ with a final price of zero $(1, (1-\delta), (1-\delta)^2, ..., 0)$, D_t

[6]Majid et al. (2012) provides how corporates issuing sukuk are being rated and provides statistics about corporate defaults and Siswantoro (2013) analyzes the corporate ijarah sukuk default in Indonesia.

becomes[7]:

$$D = \frac{1}{q(1-\delta)} \lim_{J\to\infty} \sum_{j=1}^{J} j \left(\frac{1-\delta}{1+i^m}\right)^j \quad (1)$$

$$= \frac{1+i^m}{i^m+\delta}. \quad (2)$$

One period model is a particular model of my framework. Observe that equation (2) is 1 for $\delta = 1$. The rent structure assumed in this paper is closely related to ijarah fixed rate (IFR). It is permissible for the rent to be fixed or floating provided that both parties mutually agreed on it.

It is worth noting that optimal maturity of an investment is fixed in this model and cannot be chosen by the government. Studying how the sovereigns decide on the maturity structure of an investment is an interesting question but it is beyond the scope of this paper. Buera and Nicolini (2004) provides a theoretical analysis for optimal maturity structure of government debt.

Parameter $\theta \in [0,1]$ denotes the portion of the asset that shareholders enjoy in case of a default.For θ being 0 corresponds to the benchmark economy (conventional finance) and for θ bigger than 0 represents the economies using equity based contracts. First, the model is solved for $\theta = 0$ and the target statistics are matched for Argentina's economy. Then to answer the question of: what would have happened in 2001 if the debt contracts had been written as an ijarah contract, I re-run the model with two different collateral limits, 10% and 50% using the same parameter values matching the benchmark economy. The results are presented and discussed in section 4.

2.2 Time Line

The timing of events can be summarized as follows:

1. Period t starts and the government first learns its endowment y which is public information.

[7]To obtain equation (2), following formula is used: $\sum_{k=0}^{\infty} kx^k = \frac{x}{(1-x)^2}$ for $x \in [0,1)$.

2. The government then chooses whether to default or not:

   - If it chooses to default, ijarah holders will be able to receive θ portion of their sukuk holdings. The country will stay in financial autarky, the output of which equals $y - \phi(y)$ while in default and will come back to the markets with an exogenous probability $\psi \in [0, 1]$ next period.
   - If the government repays, it may choose to issue more ijarah or purchase back some of its outstanding ijarah holdings.

3. Period $t + 1$ starts.

2.3 Government's Problem

I will use the noble framework of Eaton and Gersovitz (1981) in modelling the sovereign default. As in previous studies, I will analyze a small open economy where the households are identical and have preferences given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{3}$$

where E denotes the expectation operator, $0 < \beta < 1$ is the discount factor, c_t denotes aggregate consumption at time t, and utility function $u(.) : [0, \infty) \to \mathcal{R}$ is increasing, strictly concave, continuous, and bounded above by the quantity U. It is assumed to be an endowment economy and the agents receive an income of the consumption good $y \in \mathcal{Y} \subset \mathbb{R}_{++}$ which follows a Markov process. The government has no access to any commitment technology that is, the government cannot commit to future repayment and borrowing decisions. My paper will focus on Markov Perfect Equilibrium (MPE). Hence, equilibrium default and borrowing decisions are payoff-relevant today.

Following Arellano (2008) and Chatterjee and Eyigungor (2012), the cost of a default increases more than proportionally with income. As in Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009), I assume the following quadratic loss function for income during a default episode: $\phi(y) = d_0 y + d_1 y^2$. This assumption is consistent with evidence provided by Chuhan and Sturzenegger (2003) and Arteta and Hale (2008) and also the

property of endogenous default costs presented in Mendoza and Yue (2012). This assumption of default cost allows the equilibrium default models to match the debt-to-GDP ratios and sovereign bond yield.

It is assumed that the government can choose ijarah i from a finite set $\mathcal{I} = \{i_0, i_1, ...i_{I-1}, i_I\}$ where $i_I > i_{I-1} > ... > i_1 > i_0$. For simplicity, I assume that government is a net issuer of the ijarah asset; I do not allow the sovereign to accumulate rent. Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2012) model long-term debt as an infinite stream of coupons decreasing at a constant rate δ. Thus, the model is tractable. Similarly, a sukuk ijarah issued in period t promises to pay $(1-\delta)^{n-1}$ units of goods as a rent payment in period $t+n$, for all $n \geq 1$. Let i be the outstanding rent payments at the beginning of period t, and i' be the outstanding rent payments at the end of period $t+1$. Let b denote time t period issuance level, then the debt dynamics can be represented as follows:

$$i_{t+1} = (1-\delta)i_t + b_t,$$

where i_t is the total number of rents due at the beginning of period t, and b_t is the total number of ijarah issued at period t. Throughout the paper, x and x' denote any variable x at period t and $t+1$, respectively.

Let V denote the value function of a government that is not currently in default. For any sukuk ijarah price function q, the function V satisfies the following functional equation:

$$V(i,y) = \max\left\{V_R(i,y), V_D(i,y)\right\}, \tag{4}$$

where the government's value of repaying is given by

$$\begin{aligned} V_R(i,y) &= \max_{i' \in \mathcal{I}} \left\{u(c) + \beta \mathbb{E}_{y'|y} V(i',y')\right\}, \\ &\text{subject to} \\ c &= y - i + q(i',y)\left[i' - (1-\delta)i\right], \end{aligned} \tag{5}$$

the value of defaulting is given by:

$$V_D(i,y) = u(c) + \beta \mathbb{E}_{y'|y}\left[(1-\psi)V_D(0,y') + \psi V(0,y')\right], \tag{6}$$

subject to

$$c = y - \phi(y) - \theta i.$$

When the government does not pay its rent, it faces a default cost of $\phi(y) = d_0 y + d_1 y^2$, the shareholders receive $\theta \in [0,1]$ share of their assets and the government stays in autarky. With an exogenous probability of $\psi \in [0,1]$, the government returns back to the international markets with no debt next period. In Appendix A, I present a model in which governments have to pay some portion of its debt in order to come back to the international markets.[8]

2.4 Shareholders' Problem

The market for sukuk ijarah contracts is competitive and shareholders are assumed to be risk-neutral so that shareholders take the price schedule $q(i',y)$ as given. The opportunity cost of funds is given by the exogenous risk free interest rate $r^f > 0$.

The solution to the government's problem implies a default decision rule $\hat{d}(i,y) \in \{0,1\}$, 1 if the government defaults, 0 otherwise; and sukuk ijarah (debt) holdings decision rule $\hat{i}(i,y)$ when value of repayment is higher. In equilibrium, defined in section 2.5, shareholders use these decision rules to price contracts. With the zero-profit assumption, the sukuk ijarah (debt)-price function solves the following functional equation:

$$q(i',y)(1+r^f) = \mathbb{E}_{y'|y}\left\{[1-\hat{d}(i',y')][1+(1-\delta)q(i'',y')]\right\}. \tag{7}$$

where

$$b'' = \hat{b}(i',y'). \tag{8}$$

[8] Yue (2010) and Benjamin and Wright (2008) present models of endogenous renegotiation stage after a debt default.

The term on the right hand side of the equation (7) indicates that expected value of investing in an ijarah contract has to be equal to the left hand side of the equation (7); that is, value of selling ijarah at the current price $q(i', y)$ and investing in a risk-free asset that has a return of r^f. If the shareholders decide to hold the ijarah contract and the government does not default, then shareholders receive one unit of rent and a discounted value of all future rent receivables. $(1 - \delta)$ is the remaining amount of rent obligations for the next period and $q(i'', y')$ is the price of an ijarah contract issued next period.

When the government defaults, it is assumed that is also defaults on its future rent obligations. This is a standard assumption in the literature and consistent with defaulting governments' behavior. For instance, when Argentina defaulted, it defaulted on all of its existing debt. A sovereign debt contract often includes clauses such that in a default event, all the future debt obligations become current.

2.5 Definition of Equilibrium

This paper focuses on Markov Perfect Equilibrium (MPE) wherein the government's equilibrium default and borrowing decisions depend only on payoff relevant state variables.

Definition 1 *A Markov Perfect Equilibrium is characterized by*

1. *a collection of value functions V, V_R and V_D;*
2. *rules for default $\hat{d}$ and next-period ijarah (debt) holdings; and*
3. *an ijarah (debt) price function q;*

such that:

i. *given a price function q; $\left\{V, V_R, V_D, \hat{d}, \hat{i}\right\}$ solve the Bellman equations (4), (5), and (6).*

ii. *given policy rules $\left\{\hat{d}, \hat{i}\right\}$, the price function q satisfies condition (7).*

Proposition 1 *If default is optimal under an ijarah contract, then it is also optimal to default under a conventional sovereign contract as well.*

Proof. *Intuition* - This result comes from the strict monotonicity of u and value of the asset under ijarah contract when the government defaults is positive. Please see Appendix B for the details. ■

3 Parameterization

One period in the model is a quarter. The utility function is assumed to be CRRA form:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

where σ is the constant coefficient of relative risk aversion. The endowment is modelled as a first order autoregressive process and is denoted by $y \in \mathcal{Y} \subset \mathbb{R}_{++}$ and follows a Markov process:

$$\log(y_t) = (1-\rho)\,\mu + \rho \log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$, and $\varepsilon_t \sim N(0, \sigma_\epsilon^2)$.

The model is solved using cubic spline interpolation (the IMSL subroutine DCSDEC) over ijarah levels and linear interpolation over endowment levels. The algorithm for the numerical solution relies on the solution method by Hatchondo et al. (2010). As shown by Krusell and Smith (2003), there may exist multiple MPE in infinite-horizon economies. One way to address this issue is to solve the model as a finite-horizon economy with sufficiently large periods such that the value function and ijarah-price function for the first and second periods of the economy satisfy the convergence criteria of 10^{-6}. Then, I utilize the equilibrium value function and the price function for the first-period as infinite horizon equilibrium functions. Chatterjee et al. (2007) and Chatterjee and Eyigungor (2012) show the existence of an equilibrium for one period bonds and long-duration bonds, respectively.

Table 1 presents the model parameters that was used in the paper. The parameterization is done for Argentina since Argentina's default in December 2001 is the largest ever recorded and corresponds to 39% of its 2001 GDP. The stochastic process for endowment is estimated from the series of Argentina's GDP for the period between 1993Q4-2001Q3. The longer

horizon estimates for autocorrelation coefficient and the standard deviation of innovations yield very similar results. (see Aguiar and Gopinath (2006).)

Default costs are very similar to Hatchondo and Martinez (2009), Aguiar and Gopinath (2006) and D'Erasmo (2008). Risk aversion rate σ, discount factor β and the probability of reentry ψ are standard in the sovereign debt literature. (see Hatchondo and Martinez (2009) and Arellano (2008)).

Parameter		Value
Risk aversion rate	σ	2
Risk free interest rate	r^f	0.015
Discount factor	β	0.96
Probability of reentry	ψ	0.282
Income autocorrelation coefficient	ρ	0.9
Standard deviation of innovations	σ_ϵ	2.7%
Mean log income	μ	$(-1/2)\sigma_\epsilon^2$
Duration of debt (ijarah)	δ	0.0375
Income cost of defaulting	d_0	-0.69
Income cost of defaulting	d_1	1.017
Collateral	θ	{0.1, 0.5}

Table 1: Model Parameters

In order to compute the yield on sukuk ijarah in the simulations, it is defined as the discounted value of ijarah rents equal to its price. That is, given price q, the yield i^m satisfies

$$q_t = \sum_{n=1}^{\infty} \frac{(1-\delta)^{n-1}}{(1+i^m)^n}.$$

So,

$$i^m = \frac{1}{q_t} - \delta.$$

The sovereign spread is defined as the difference between the yield i^m on sukuk ijarah and the default free rate r^f:

$$r_t^s = \left(\frac{1+i^m}{1+r^f}\right)^4 - 1$$

where r_t^s is the annualized spread reported in the tables. Ijarah (debt) levels reported in the simulations are calculated as the present value of future rent obligations discounted at the default free rate r^f, in particular it is calculated as $\frac{b'}{\delta+r^f}$.

The data in Table 2 are quarterly real series seasonally adjusted and are computed using detrended series where trends are calculated using the Hodrick-Prescott (HP) filter and a default smoothing parameter of 1,600 to separate the data into trend and cyclical components. The business cycle statistics are taken from Hatchondo and Martinez (2009).

4 Results

Last column of Table 2 reports the moments in the data and the fourth column of the table reports the benchmark moments. In terms of the calibrated parameters, the benchmark model matches the mean debt holdings, spreads and approximately matches the volatility of income, consumption and the trade balance. After matching the targeted moments in the benchmark economy, I rerun the economy with an ijarah contract with $\theta = 0.1$ and $\theta = 0.5$. The simulation results show that the funds generated by ijarah contracts are higher, spreads and the default probabilities are lower. As collateral limit θ increases, default probabilities and the spreads fall and the amount of financing increases significantly.

Output is negatively correlated with interest rate spreads. Trade balance is countercyclical and positively correlated with spreads and consumption is also more volatile than output.

Figure 3 shows the equilibrium pricing functions for ijarah holdings and income for different values of collateral constraints. Figure 4 plots the pricing function for the mean income under different collateral constraints and $\theta = 0$ corresponds to the benchmark. As the value of θ increases, the cost of defaulting increases; thus the probability of defaulting plunges. With lower default probabilities, the cost of borrowing drops meaning that the price of ijarah sukuk increases (remember that the price is inversely related with the spreads).

Figure 5 plots a reduction in ijarah holdings as income falls. This can be explained by the interest rates implied by the price of ijarah holdings. As income drops, price of ijarah holdings decreases non-monotically as showed in Figure 3. So it is not optimal for the government to issue more ijarah as issuing more causes interest rates to increaese further.

Table 2: Simulation results for ijarah contracts and conventional contracts

	Ijarah θ=0.5	Ijarah θ=0.1	Benchmark Model	Data
Mean debt (Ijarah)-to-GDP (%)	97.6	47.8	39	39
Mean r_s	1.98	5.04	7.47	7.44
$\sigma(r_s)$	0.72	1.99	2.83	2.51
Defaults per 100 years	1.79	3.9	4.1	3.3
$\sigma(y)$	3.13	3.02	3.01	3.17
$\sigma(c)$	4.76	4.07	3.83	2.98
$\sigma(tb/y)$	1.69	1.01	0.76	1.35
$\rho(c, y)$	0.97	0.99	0.99	0.97
$\rho(r_s, y)$	-0.76	-0.76	-0.76	-0.65
$\rho(r_s, tb/y)$	0.94	0.91	0.89	0.56
Duration (years)	4.8	4.3	4.0	4.2

Note: The standard deviation of x and the correlation of (x, z) is denoted by $\sigma(x)$ and $\rho(x, z)$, respectively. Moments correspond to the mean value of each moment in 500 simulation samples where each sample includes 120 periods corresponding to 30 years without any default, and the samples start at least five years after a default.

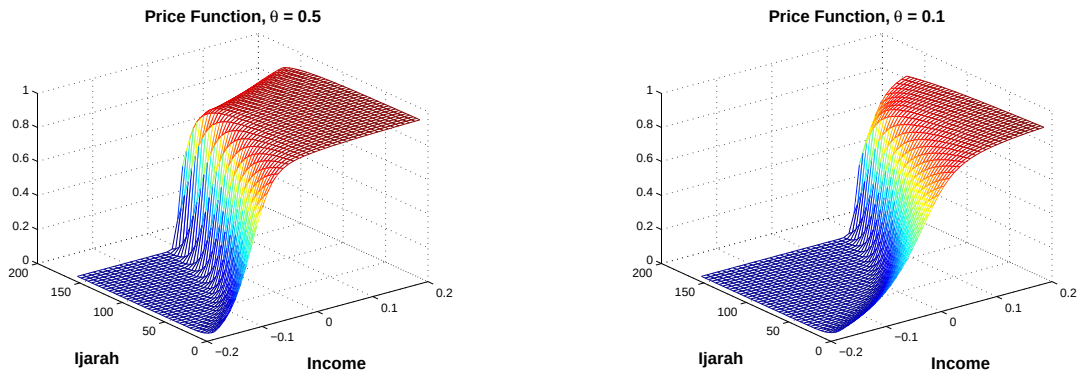

Figure 3: Pricing of Sukuk ijarah under different collateral limits

4.1 Welfare Gains

Welfare gains are defined as the proportional change in consumption that would make agents indifferent between living in an economy under sukuk ijarah contracts and living in an economy with conventional finance contracts. It is measured by

$$\left(\frac{V_I(i,y)}{V_C(i,y)}\right)^{\frac{1}{1-\sigma}} - 1,$$

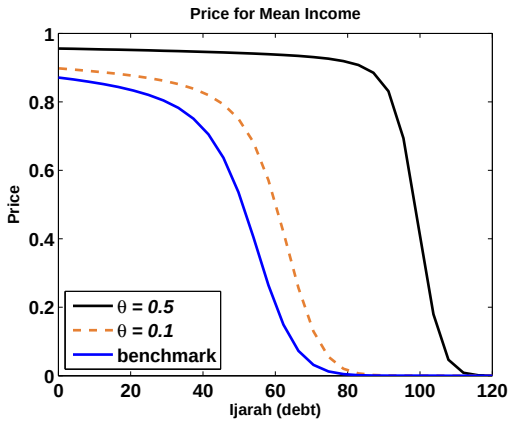

Figure 4: Government's equilibrium pricing function for mean income in the economy when the value of sukuk ijarah is lower (i.e., when $\theta = 0.1$), when the value of sukuk ijarah is higher (i.e., when $\theta = 0.5$), and when it is not an equity based contract (i.e., when $\theta = 0$ corresponds to benchmark economy).

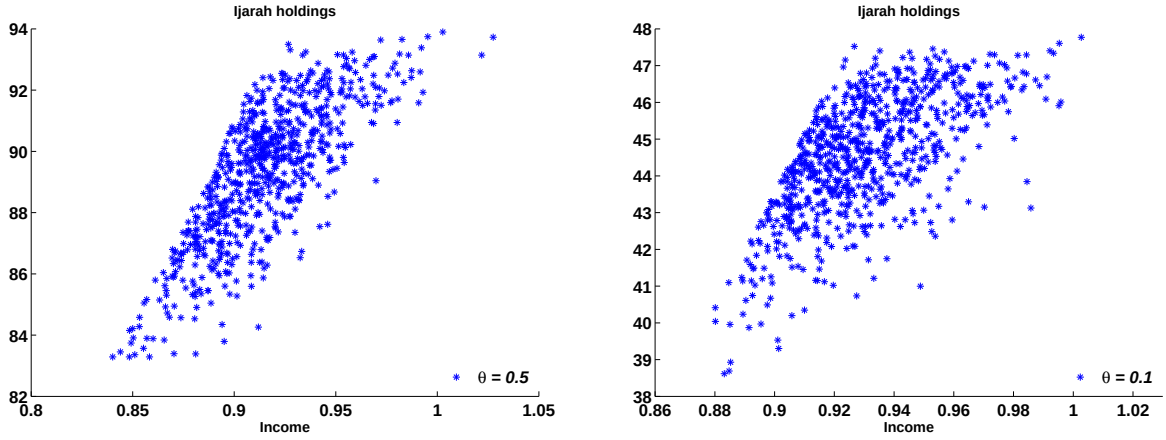

Figure 5: Income realizations and ijarah holdings for economies with a collateral value of $\theta = 0.5$ and $\theta = 0.1$. The figure shows 250 samples of 120 periods (30 years) before a default episode and starts at least five years after a default.

where V_I and V_C denote the value functions with ijarah contracts and conventional finance contracts, respectively. Welfare gains are presented in Figure 6 and Table 3. Positive welfare gain means that agents would prefer to live in an economy with ijarah contracts. In an economy with ijarah contracts, agents benefit from cheaper financing since default risks are mitigated with sukuk ijarah. Furthermore, as showed in Table 3 and Figure 6, while the risk of the initial state increases, sukuk ijarah contracts became more attractive and as the market value of collateral decreases, welfare gains fall.

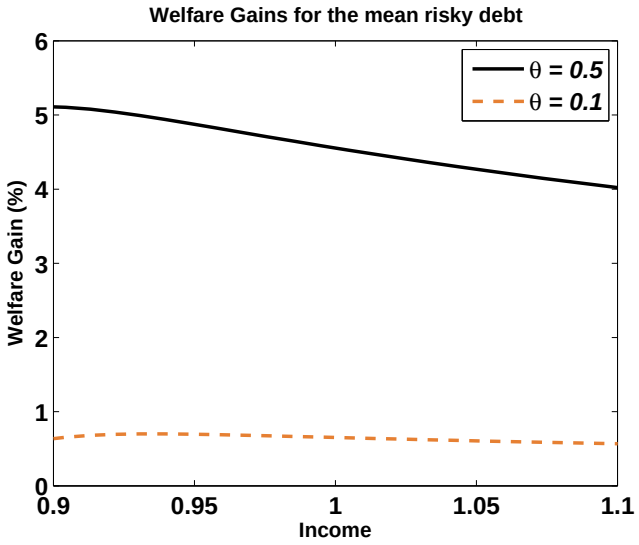

Figure 6: Necessary consumption change in order to leave a consumer indifferent between living in a conventional finance contract economy and living in the equity based contract economy different collateral limits.

Table 3: Effects of the introduction of sukuk ijarah

	Highest risk	Med. risk	Lowest risk
Spread in conventional economy	15%	7.5%	5.3%
Spread in ijarah economy, $\theta = 0.1$	11.9%	5.8%	4.2%
Spread in ijarah economy, $\theta = 0.5$	5.4%	2.2%	1.6%
Wel. gain from the intro. of ijarah, $\theta = 0.1$	0.62%	0.62%	0.59%
Wel. gain from the intro. of ijarah, $\theta = 0.5$	4.9%	4.61%	4.3%

Figure 7 shows that the default regions shrink with an economy with ijarah contracts. As the value of θ increases, default regions decrease even more.

5 Conclusion

Sovereign debt crisis have drastic consequences in the sense that it leads to output declines, collapses in foreign credit to the private sector, consequently financial disruptions and financial exclusion while the debt is being restructured. These facts mainly generated by the lack of legal enforcement which motivated a set of proposals beginning in the 1920s, and particularly following the 1995 Mexican, 1998 Russian, 2001 Argentinean and most recently the European debt crisis. The proposals have aimed to set global standards on sovereign borrowing and

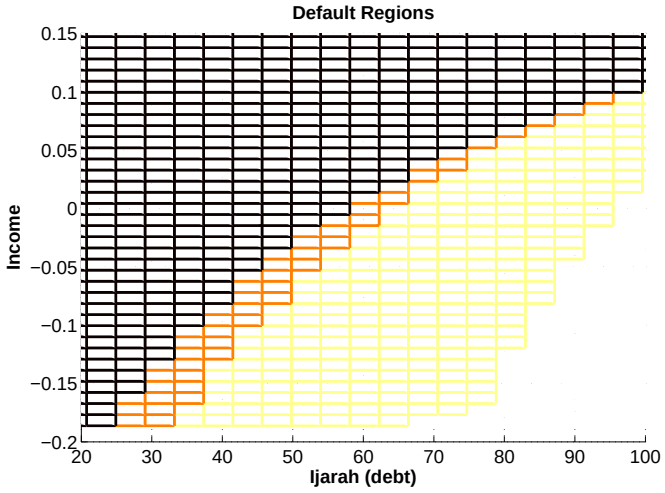

Figure 7: Government's equilibrium default decision in the benchmark economy and in the economy with sukuk ijarah with different collateral limits. The figure presents combinations of income and ijarah (debt) levels for which: (i) government would not default in the benchmark economy (black area), (ii) the government would not default if it is an ijarah economy with collateral of 10% i.e., when $\theta = 0.1$ (black and orange area), and (iii) the government would not default only if it is an ijarah economy with collateral of 50% i.e., when $\theta = 0.5$ (yellow area). (There is no set of income and ijarah (debt) combination in which the government would default if it is an ijarah economy but not in the benchmark economy).

lending. All the initiatives to mitigate the costs of defaults have failed. This paper has proposed equity based contracts, which is currently being widely practiced in Islamic finance, as a resolution to deal with the litigation of sovereign debt defaults. Ijarah sukuk are the most common type of sukuk in Islamic finance and the major difference from the conventional bonds is that the holders of ijarah sukuk share the returns and carry out the risk of any losses. In particular, in case of a default event, sukuk holders are entitled to receive the underlying asset. This paper shows that if the sovereign debt contracts had been designed as an equity based contract, in particular sukuk ijarah, sovereigns would have lower default probabilities, higher welfare gains and larger funds to finance their projects.

Most of the governments issuing sukuk, issue conventional bonds as well. Therefore, introducing conventional sovereign bonds along with sukuk borrowings can be the focus of future research[9].

[9] Hatchondo et al. (2013) study a framework solving a model with two assets: defaultable and non-defaultable (risk-free) debt.

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A Appendix A

When a government defaults, it usually pays some of its defaulted debt and regains access to the international markets. With the analysis in here, two economies are going to be compared: one with the conventional borrowing and the other with equity-based borrowing. Argentinean government commits itself to repay 22.5 % of its trend income (comes out as 35 % of its total mean debt) in case of a default to regain access to the international markets. This economy is going to be compared with equity-based borrowing where the underlying asset of the contract (say land or building) loses 65 % of its value with a default.

Different than the model presented in the text, in this section the government has to pay some amount of its defaulted debt to the lenders to regain access to the international markets and the amount it pays is denoted by i_R.

To simplify the analysis, the amount i_R is taken to be independent from the amount of total debt default as in Hatchondo et al. (2013). Nevertheless, this assumption gives the government the incentive to issue infinite amount of debt before defaulting which would distort the analysis because of a large increase in consumption. To avoid this issue, the governments are not allowed to sell bonds with a price lower than $\underline{q}$. Even though $\underline{q}$ rarely

binds in the simulations, the assumption is needed to prevent the consumption hikes before the default.

Let V denote the value function of a government that is not currently in default. For any sukuk ijarah price function q, the function V satisfies the following functional equation:

$$V(i,y) = \max\left\{V_R(i,y), V_D(i,y)\right\}, \tag{9}$$

where the government's value of repaying is given by

$$\begin{aligned} &V_R(i,y) = \max_{i' \in \mathcal{I}} \left\{u\left(c\right) + \beta \mathbb{E}_{y'|y} V(i',y')\right\}, \\ &\text{subject to} \\ &c = y - i + q(i',y)\left[i' - (1-\delta)i\right], \end{aligned} \tag{10}$$

the value of defaulting is given by:

$$\begin{aligned} &V_D(i,y) = u\left(c\right) + \beta \mathbb{E}_{y'|y}\left[(1-\psi)V_D(0,y') + \psi V(i_R,y')\right], \\ &\text{subject to} \\ &c = y - \phi(y) - \theta i. \end{aligned} \tag{11}$$

The solution to the government's problem yields the following decision rules: default rule $\hat{d}(i,y) \in \{0,1\}$, 1 if the government defaults, 0 otherwise; and next-period sukuk ijarah holdings $\hat{i}(i,y)$. In equilibrium, shareholders use these decision rules to price the contracts. Due to the zero-profit assumption, the sukuk ijarah-price function solves the following functional equation:

$$q(i',y)(1+r^f) \;=\; \mathbb{E}_{y'|y}\left\{[1-\hat{d}(i',y')][1+(1-\delta)q(i'',y')]\right\}. \tag{12}$$

where

$$i'' \;=\; \hat{i}(i',y'). \tag{13}$$

$$q(b',y)(1+r^f) = \mathbb{E}_{y'|y}\left\{[1-\hat{d}(i',y')][1+(1-\delta)q(i'',y')] + \hat{d}(i',y')q_D(i',y')\right\} \tag{14}$$

where

$$\begin{aligned} q_D(i,y)(1+r^f) &= \mathbb{E}_{y'|y}\left\{\left[(1-\psi)+\psi\hat{d}(i_R,y')\right]q_D(i,y') \right. \\ &\quad \left. +\psi[1-\hat{d}(i_R,y')]\frac{i_R}{i}[1+(1-\delta)q(i''_R,y')]\right\} \end{aligned}$$

denotes the price of a defaulted bond, and

$$i'' = \hat{i}(i',y').$$

The difference between the two economies are the constant θ and i_R. In a conventional borrowing economy, θ is 0 and i_R corresponds to 22.5% of the mean trend income, whereas in an equity-based borrowing economy θ is $\in$ (0,1] and i_R is zero.

Definition 2 *A Markov Perfect Equilibrium is characterized by*

1. *a collection of value functions V, V_R and V_D;*
2. *rules for default $\hat{d}$ and next-period ijarah (debt) holdings; and*
3. *an ijarah (debt) price function q;*

such that:

i. *given a price function q; $\left\{V, V_R, V_D, \hat{d}, \hat{i}\right\}$ solve the Bellman equations (9), (10), and (11).*

ii. *given policy rules $\left\{\hat{d}, \hat{i}\right\}$, the price function q satisfies condition (14).*

Results using the same parameter values listed in Table 1 are provided in Table 4. By setting $i_R = 22.5\%$, the average debt haircut is 66%. To compare this economy with the

equity-based economy, θ was set to be 0.34 and the results are presented in Table 4. In both economies, debt-to-trend income ratios are very similar; however, the interest rate spread and the mean default rates are higher in conventional economy. Results show that the economy enjoys welfare gains if it rather utilized equity based contracts.

Table 4: Simulation results for ijarah contracts and conventional contracts

	Ijarah θ=0.34	Haircut 66%
Mean debt (Ijarah)-to-GDP (%)	65.5	63.7
Mean r_s	2.78	4.25
$\sigma\left(r_s\right)$	1.16	1.5
Defaults per 100 years	2.53	5.1
$\sigma(y)$	3.31	3.2
$\sigma(c)$	4.61	3.7
$\sigma(tb/y)$	1.51	0.62
$\rho(c, y)$	0.97	0.99
$\rho(r_s, y)$	-0.74	-0.76
$\rho(r_s, tb/y)$	0.93	0.7
Welfare gains (%)	0.33	

Note: The standard deviation of x and the correlation of $(x,\ z)$ is denoted by $\sigma\left(x\right)$ and $\rho(x, z)$, respectively. Moments correspond to the mean value of each moment in 500 simulation samples where each sample includes 120 periods corresponding to 30 years without any default, and the samples start at least five years after a default.

B Appendix B

Proof of proposition 1. *If default is optimal under an ijarah contract, then it is also optimal to default under a conventional sovereign contract as well.*

Proof - Following the continuity properties of Chatterjee and Eyigungor (2012), fix i and y. By definition, θ =0 corresponds to the benchmark economy and $\theta > 0$ corresponds to the ijarah economy.

$u(y - \phi(y) - \theta i) + \beta \mathbb{E}_{y'|y}\left[(1-\psi)V_D(0, y') + \psi V(0, y')\right] > V_R(i, y)$. From the strict concavity of u, it follows that $u(y - \phi(y)) > u(y - \phi(y) - \theta i)$ and thus $u(y - \phi(y)) + \beta \mathbb{E}_{y'|y}\left[(1-\psi)V_D(0, y') + \psi V(0, y')\right] > V_R(i, y)$.

■