Testing for structural breaks with local smoothers: A simulation study

Serda Selin Öztürk
Istanbul Bilgi University

Thanasis Stengos
University of Guelph

June 24, 2014

Abstract

By means of an extensive Monte Carlo simulation study based on the design of Chen and Hong (2012) we compare the performance of the tests they proposed for parameter stability with the linearity test of Li, Huang, Li and Fu (2002) and the functional form test of Li and Wang (1998). We find that the test of Li et al. (2002) test adapted to testing for parameter stability performs favorably well in terms of size and equally well in terms of power compared with the others, whereas the test by Li and Wang has no power.

1 Introduction

Testing for parameter stability has been an active area of research in econometrics, see for example Andrews (1993) and Bai and Perron (1998). Recently Chen and Hong (2012) (CH hereafter) developed within the context of a smooth coefficient semiparametric model a consistent test for smooth structural changes as well as abrupt structural breaks with known or unknown change points. Their test is based on a contrast of estimators approach in the contest of a smooth time-varying coefficient model with the contrasting estimators being the estimated regression function (fitted values) of a restricted constant parameter model and the unrestricted time-varying parameter model. By means of a Monte Carlo simulation comparison, the tests proposed by CH perform quite well when compared with other tests in the literature such as the Andrews (1993) and the Bai and Perron (1998) tests mentioned above. The CH time-varying parameter framework was originally proposed by Robinson (1989) and extended by Li, Huang, Li and Fu (2002) (hereafter LHLF) who also developed a test statistic to test the null hypothesis of a linear model against a smooth coefficient semiparametric alternative. In this paper we adapt the linearity test proposed by LHLF to test for smooth structural changes.

Within the framework of the smooth coefficient model we conduct an extensive Monte Carlo simulation study based on the original design of CH. We provide evidence on the performance of the CH test with other local-smoother based consistent tests in the literature notably the test based on the LHLF formulation and the functional form test of Li and Wang (1998), (hereafter LW). The LHLF test adapted to the case of parameter stability performs favorably well in terms of size and quite well in terms of power, when compared to the proposed CH tests. On the other hand the LW test does not do well in terms of power since it is designed to test against general nonlinear alternatives and not specifically against unknown structural breaks. This is an interesting finding by itself as it suggests that testing against a general nonlinear alternative would not allow one to detect structural breaks. The Monte Carlo comparison includes both the asymptotic and bootstrap versions of the different test statistics examined. In the next section we present the main framework of analysis based on the smooth coefficient model. We then proceed to present the results and concluding remarks.

2 Testing for structural breaks using a smooth coefficient model

A semiparametric varying coefficient model imposes no assumption on the functional form of the coefficients, and the coefficients are allowed to vary as smooth functions of other variables. Following Robinson (1989, 1991) , varying coefficient models are linear in the regressors but their coefficients are allowed to change smoothly with the value of other variables. One way of estimating the coefficient functions is by using a local least squares method with a kernel weight function. A general semiparametric smooth coefficient model is given by:

$$y_i = X_i^T \delta(z_i) + \varepsilon_i \tag{1}$$

where y_i denotes the dependent variable, X_i denotes a $p \times 1$ vector of explanatory variables of interest, z_i denotes a $q \times 1$ vector of other exogenous variables and $\delta(z_i)$ is a vector of unspecified smooth functions of z_i. One can estimate $\delta(z)$ using a local least squares approach, where

$$\begin{aligned}\widehat{\delta}(z) &= [(nh^q)^{-1} \sum_{j=1}^{n} X_j X_j^T K(\frac{z_j - z}{h})]^{-1} \{(nh^q)^{-1} \sum_{j=1}^{n} X_j y_j K(\frac{z_j - z}{h})\} \\ &= [B_n(z)]^{-1} A_n(z)\end{aligned}$$

where $B_n(z) = (nh^q)^{-1} \sum_{j=1}^{n} X_j X_j^T K(\frac{z_j - z}{h})$, $A_n(z) = (nh^q)^{-1} \sum_{j=1}^{n} X_j y_j K(\frac{z_j - z}{h})$, $K(.)$ is a kernel function and $h = h_n$ is the smoothing parameter for sample size n. LHLF provide a thorough discussion of the properties of the smooth coefficient estimator $\widehat{\delta}(z)$.

In a time varying coefficient context the variable z_t is expressed as t/T, with the sample size n being denoted by T. In other words, $\delta(z_t) = \delta(t/T)$ and the model becomes

$$y_t = X_t^T \delta(t/T) + \varepsilon_t$$

The properties of the varying coefficient estimator of $\delta(t/T)$ have been analyzed by Robinson (1989, 1991) and more recently by Cai (2007).

2.1 Testing

Following the analytical framework of CH with the necessary assumptions to establish the properties of their proposed test statistics, the testing framework centers on the null hypothesis of no structural breaks: $\delta(t/T) = \delta_0$, where δ_0 is some constant value of δ under the null

$$H_0 : y_t = X_t^T \delta_0 + \varepsilon_t \tag{2}$$

against the alternative which is given by the functional coefficient model where $\delta(t/T) \neq \delta_0$.

$$H_1 : y_t = X_t^T \delta(t/T) + \varepsilon_t \tag{3}$$

We can test the adequacy of (2), the H_0, against the semiparametric alternative (3) using the following test statistic based on a Hausman type of contrast estimators of the regression function under the null and alternative as

$$\widehat{Q}_T = \frac{1}{T} \sum_{t=1} (X_t^T \widehat{\delta}(t/T) - X_t^T \widehat{\delta}_0)^2 \tag{4}$$

The generalized (suitably standardized) Hausman statistic proposed by CH is used a test for structural change with an asymptotic standard normal distribution. CH also propose another Chow type test based on the standardized difference of the Sum of Squared Residuals under the null and the alternative, $SSR_0 = \sum_{t=1} (y_t - X_t^T \widehat{\delta}_0)^2$ and $SSR_1 = \sum_{t=1} (y_t - X_t^T \widehat{\delta}(t/T))^2$ respectively. The asymptotic distributions of the standardized versions of these statistics are derived by CH under conditional homoskedasticity as well as under conditional heteroskedasticity. They also define bootstrap versions of the above statistics[1].

In a previous study LHLF used the same framework to propose a test statistic for linearity against a smooth coefficient alternative given by the model in equation (1). By adapting their test statistic for linearity to serve as a test for structural break we can have the following statistic as a plausible test for structural change in addition to the ones proposed by CH. The test statistic in that case would take the form

$$\begin{aligned}\widehat{I}_T &= \frac{1}{T^2h^q}\sum_t\sum_{t\neq s}X_t^T(y_t - X_t^T\widehat{\delta}_0)X_s(y_s - X_s^T\widehat{\delta}_0)K(\frac{s/T - t/T}{h}) \qquad (5)\\ &= \frac{1}{T^2h^q}\sum_t\sum_{t\neq s}X_t^T X_s\widehat{\varepsilon}_s\widehat{\varepsilon}_s K(\frac{s/T - t/T}{h})\end{aligned}$$

where $\widehat{\varepsilon}_i$ denotes the residual from parametric estimation (under H_0). Under H_0, $J_T = Th^{q/2}\widehat{I}_T/\widehat{\sigma}_0 \longrightarrow N(0,1)$, where $\widehat{\sigma}_0^2$ is a consistent estimator of the variance of $Th^{q/2}\widehat{I}_T$, see LHLF for details[2]. As the statistics proposed by CH one can use a bootstrap version of the above test statistic, since bootstrapping improves the size performance of kernel based tests for functional form, see Li and Wang (1998). As a final test for structural break we also consider the latter to test the null of linearity (no structural breaks) against a nonlinear alternative. In that case the null hypothesis would be given by equation (2) and the alternative would be a generic nonlinear model given by $H_1 : y_t = m(X_t)+\varepsilon_t$. Of course such a test is only used as a benchmark as it is not expected to have

[1] CH provide a thorough and exhaustive discussion of all the necessary assumptions for the derivation of the properties of the test statistics they propose, such as mixing conditions for data dependence as well as kernel and bandwidth conditions that need to be observed for the asymptotics to carry through.

[2] The linearity test proposed by LHLF as well as the properties of the smooth coefficient estimator $\widehat{\delta}(z_t)$ were derived within an *i.i.d.* analytical framewrok. However, following CH, the analysis can be recast in the CH general framework allowing for data dependence and conditional heteroskedasticity.

much power against smooth alternatives given by equation (3), which constitutes the basis of the analytical framework under consideration.

3 Monte Carlo Simulations

We follow exactly the same simulation framework and design of CH in order to be able to replicate their findings and also be able to put the comparisons with the other kernel based tests we consider on an equal footing. The finite sample performance of the structural change tests we consider are based on the following design. In terms of the size of the test statistic, we consider the following DGP:

DGP 1 - No structural change:

$Y_t = 1 + 0.5X_t + \varepsilon_t,$

$X_t = 0.5X_{t-1} + \nu_t \quad \nu_t \sim i.i.d.N(0,1)$

We also consider all three cases CH proposed for $\{\varepsilon_t\}$ to examine the robustness of the test: (i) $\varepsilon_t \sim i.i.d.N(0,1)$; (ii) $\varepsilon_t = \sqrt{h_t}u_t$, $h_t = 0.2 + 0.5\varepsilon_{t-1}^2$, $u_t \sim i.i.d.N(0,1)$; (iii) $\varepsilon_t = \sqrt{h_t}u_t$, $h_t = 0.2 + 0.5X_t^2$, $u_t \sim i.i.d.N(0,1)$. 5000 data sets of the random sample $\{X_t, Y_t\}_{t=1}^T$ for each $T = 100, 250$ and 500 are generated.

The size of all the tests using asymptotic theory is different than the nominal level in small samples and may be sensitive to bandwidth selection. Therefore we use the following bootstrap procedure for all statistics in our comparison, illustrated in the case of LHLF as follows:

Step (i). Use the generated random sample $\{Y_t, X_t'\}_{t=1}^T$ for the model estimation via OLS and compute the Kernel test statistic $\widehat{LHLF}$ and the the OLS residual $\widehat{\varepsilon}_t = Y_t - \widehat{\alpha}X_t$.

Step (ii). Center the OLS residual $\overline{\varepsilon_t} = \widehat{\varepsilon}_t - T^{-1}\sum_{t=1}^T \widehat{\varepsilon}_t$ and then obtain a wild bootstrap residual by $\widehat{\varepsilon}_t^* = a\overline{\varepsilon_t}$ with probability $1-a/\sqrt{5}$ and $\widehat{\varepsilon}_t^* = (1-a)\overline{\varepsilon_t}$

with probability $a/\sqrt{5}$, where $a = (1+\sqrt{5})/2$.Construct the bootstrap sample $\{Y_t^*, X_t'\}_{t=1}^T$where $Y_t^* = X_t'\widehat{\alpha} + \widehat{\varepsilon}_t^*$.

Step (iii). Compute the bootstrap Kernel test statistic $\widehat{LHLF}^*$ in the same way we compute the Kernel test statistic $\widehat{LHLF}$ by replacing the original sample by the wild bootstrap sample.

Step (iv). Repeat steps (ii) and (iii) B times to obtain a series of bootstrap test statistics $\left\{\widehat{LHLF}^*\right\}_{t=1}^T$.

Step (v). Compute the bootstrap p-value $p^* \equiv B^{-1}\sum_{t=1}^T \mathbf{1}(\widehat{LHLF}^* > \widehat{LHLF})$, where $\mathbf{1}(\cdot)$ is the indicator function.

We use $B = 499$ bootstrap iterations for each of the generated 5000 data sets of random sample $\{Y_t, X_t'\}_{t=1}^T$. In terms of the power of the test statistics, we consider the following five alternative DGPs:

DGP 2- Single Structural Break:

$$Y_t = \begin{cases} 1 + 0.5X_t + \varepsilon_t, & \text{if } t \leq 0.3T, \\ 1.2 + X_t + \varepsilon_t, & \text{otherwise.} \end{cases}$$

DGP 3- Multiple Structural Breaks:

$$Y_t = \begin{cases} 0.6 + 0.3X_t + \varepsilon_t, & \text{if } 0.1T \leq t \leq 0.2T \quad \text{or} \quad 0.7T \leq t \leq 0.8T, \\ 1.5 + X_t + \varepsilon_t, & \text{if } 0.4T \leq t \leq 0.5T, \\ 1 + 0.5X_t + \varepsilon_t, & \text{otherwise.} \end{cases}$$

DGP 4- Nonpersistent Temporal Structural Breaks:

$$Y_t = \begin{cases} 1 + 0.5X_t + \varepsilon_t, & \text{if } t \leq 0.4T \text{ and } t \geq 0.6T, \\ 1.5 + X_t + \varepsilon_t, & \text{otherwise.} \end{cases}$$

DGP 5- Smooth Structural Changes:

$$Y_t = F(\tau)(1 + 0.5X_t) + \varepsilon_t,$$

where $\tau = \frac{1}{T}$ and $F(\tau) = 1.5 - 1.5\exp\left[-3(\tau - 0.5)^2\right]$.

DGP 6- Unit Root in Parameters:

$Y_t = \rho_{1t} + \rho_{2t} X_t + \varepsilon_t$

where $\rho_{jt} = \rho_{jt-1} + u_{jt}$, $u_{jt} \sim i.i.d.N(0, 1/5)$, and $j = 1, 2$.

For all five DGP's in the power comparison, we generate 1000 data sets of the random sample $\{X_t, Y_t\}_{t=1}^{T}$ for each $T = 100, 250$ and 500.

Tables 1 and 2 present the size and (size adjusted) power results respectively. The test statistics that are included in the comparison are the two tests proposed by CH, the first one given by equation 4 denoted by H, and the second one is the Chow-type test, denoted by C. The statistics $H-$robust and $C-$robust denote the robust versions of these statistics to conditional heteroskedasticity. The test statistics $LHLF$ and LW are the statistics proposed by LHLF and LW respectively. From Table 1 one can see that overall, the H and C tests tend to over-reject, whereas the $LHLF$ test tends to slightly under-reject. However, the latter under-rejection disappears when we consider the bootstrap version of the test even for small sample sizes, whereas for the bootstrap versions of the H and C tests the distortions persist even in larger samples. Our results confirm the findings of CH for the H and C tests as we have used the same design as them. Interestingly enough, the LW test for functional form appears to have good size characteristics, especially its bootstrap form.

Table 2 presents the power results. Again, the results here confirm the findings of CH when it comes to the performance of the H and C tests. At the same time it is clear that the $LHLF$ test performs equally well. It is interesting that with respect to power, LW has none. This is a useful finding on its own right, as the presence of structural breaks is not mistaken at all as functional form misspecification, something that one might have expected. In such a case, the test statistics that are appropriately designed to capture such deviations from the null of parameter constancy perform quite well and cannot be replaced by generic tests for functional form misspecification. In that context we note that the $LHLF$ test, originally proposed as a test for linearity performs remarkably well as a test for structural change and it seems to outperform its competitors in terms of size characteristics.

4 Acknowledgments

We would like to thank participants of the Istanbul International Econometrics Workshop, June 2014 for helpful comments, while the second author would like to acknowledge financial support from The Scientific and Technological Research Council of Turkey (TUBİTAK).

5 References

Andrews, D. W. K. (1993): “Tests for Parameter Instability and Structural Change With Unknown Change Point,” *Econometrica*, vol. 61, 821–856.

Bai, J., and P. Perron (1998): “Estimating and Testing Linear Models With Multiple Structural Changes,” *Econometrica*, vol. 66, 47–78.

Cai, Z. (2007), "Trending Time-Varying Coefficient Time Series Models With Serially Correlated Errors," *Journal of Econometrics,* vol.136, 1, 163-188.

Chen, B. and Y. Hong (2012), "Testing for Smooth Structural Changes in Time Series Models Via Nonparametric Regression," *Econometrics,* vol. 80, 3, 1157-1183.

Li, Q., C. Huang, D. Li. and T. Fu (2002), "Semiparametric Smooth Coefficient Models," *Journal of Business and Economics Statistics,* vol. 20, 3., 412-422.

Li, Q. and S. Wang (1998), "A Simple Consistent Bootstrap Test For a Parametric Regression Function," *Journal of Econometrics*, vol. 87, 1, 145-146.

Robinson, P. M. (1989), "Nonparametric Estimation of Time-Varying Parameters," in *Statistical Analysis and Forecasting of Economic Structural Change,* ed. by P. Hackl. Springer, 253-264.

Robinson, P. M. (1991), "Time-Varying Nonlinear Regression," in *Economic Structural Change Analysis and Forecasting,* ed. by P. Hackl. Berlin:Springer, 179-190.

TABLE I. EMPIRICAL LEVELS OF TESTS

	$\varepsilon_t \sim i.i.d\ N(0,1)$			$\varepsilon_t \sim ARCH(1)$			$\varepsilon_t \mid X_t \sim N(0, f(X_t))$		
Test	100	250	500	100	250	500	100	250	500
				Rejection Rates Based on Empirical Critical Values					
LHLF	0,026	0,030	0,026	0,031	0,032	0,031	0,027	0,030	0,027
LW	0,019	0,025	0,027	0,018	0,024	0,026	0,063	0,063	0,060
C	0,048	0,044	0,034	0,065	0,496	0,043	0,236	0,287	0,315
C-robust	0,062	0,049	0,038	0,078	0,594	0,046	0,080	0,063	0,039
H	0,078	0,068	0,046	0,098	0,078	0,059	0,348	0,434	0,046
H-robust	0,098	0,081	0,052	0,117	0,089	0,063	0,129	0,103	0,061
				Rejection Rates Based on Bootstrap Critical Values					
LHLF	0,051	0,050	0,052	0,056	0,052	0,051	0,062	0,051	0,049
LW	0,047	0,054	0,056	0,048	0,052	0,048	0,070	0,066	0,057
C	0,073	0,067	0,053	0,079	0,067	0,057	0,116	0,081	0,082
C-robust	0,069	0,050	0,050	0,073	0,060	0,054	0,083	0,057	0,060
H	0,074	0,063	0,058	0,082	0,061	0,056	0,122	0,086	0,081
H-robust	0,070	0,058	0,053	0,077	0,057	0,054	0,148	0,062	0,058

Note: %5 significance level, 5000 iterations.

TABLE II. EMPIRICAL POWERS OF TESTS

	DGP 2			DGP 3			DGP 4			DGP 5			DGP 6		
Test	100	250	500	100	250	500	100	250	500	100	250	500	100	250	500
	Rejection Rates Based on Empirical Critical Values														
LHLF	0,376	0,786	0,986	0,349	0,683	0,964	0,351	0,728	0,980	0,355	0,777	0,983	0,566	0,986	1,000
LW	0,060	0,059	0,069	0,058	0,060	0,071	0,054	0,058	0,061	0,047	0,056	0,062	0,060	0,084	0,093
C	0,364	0,745	0,972	0,301	0,639	0,951	0,337	0,711	0,977	0,411	0,805	0,992	0,574	0,985	1,000
C-robust	0,318	0,741	0,975	0,277	0,632	0,943	0,310	0,705	0,972	0,380	0,792	0,992	0,556	0,982	1,000
H	0,446	0,873	0,992	0,323	0,676	0,959	0,339	0,775	0,988	0,462	0,889	0,998	0,635	0,987	1,000
H-robust	0,419	0,857	0,990	0,308	0,650	0,955	0,323	0,750	0,990	0,447	0,777	0,998	0,617	0,983	1,000
	Rejection Rates Based on Bootstrap Critical Values														
LHLF	0,379	0,781	0,989	0,353	0,712	0,971	0,332	0,693	0,963	0,348	0,748	0,983	0,572	0,982	1,000
LW	0,067	0,046	0,057	0,053	0,060	0,064	0,056	0,074	0,057	0,057	0,053	0,045	0,059	0,065	0,091
C	0,377	0,738	0,976	0,351	0,736	0,963	0,326	0,639	0,963	0,399	0,827	0,988	0,585	0,974	1,000
C-robust	0,353	0,719	0,977	0,333	0,720	0,957	0,299	0,636	0,957	0,371	0,807	0,988	0,569	0,975	1,000
H	0,450	0,853	0,995	0,355	0,767	0,979	0,334	0,681	0,950	0,481	0,891	0,998	0,660	0,986	1,000
H-robust	0,433	0,836	0,994	0,336	0,744	0,975	0,321	0,663	0,948	0,455	0,886	0,998	0,650	0,985	1,000

Note: %5 significance level, 1000 iterations.