

Real Fluctuations at the Zero Lower Bound*

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Abstract

Does an economy constrained by the zero lower bound respond differently to real shocks? Under standard Taylor (1993)-type policy rules subject to the zero lower bound, government spending multipliers are large, improvements in technology cause large contractions in output, and structural reforms that decrease firm market power are bad for the economy. However, these policy rules imply that the central bank stops responding to the economy at the zero lower bound. This assumption is inconsistent with recent statements and actions by monetary policymakers. Under an alternative history-dependent policy rule, the central bank endogenously responds to current economic conditions at the zero lower bound using expectations about future policy. If the monetary authority continues to respond to the economy, then spending multipliers are much smaller and increases in technology and firm competitiveness can remain expansionary. I provide several examples from the literature that recent Federal Reserve behavior is described by at least a moderate amount of history-dependence.

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1 Introduction

Since the end of 2008, the United States economy has remained stuck at the zero lower bound. In this environment, the Federal Reserve cannot further help stabilize the economy by lowering its short-term nominal policy rate. Since the monetary authority can no longer use its standard policy tool, many economists have argued that the economy is fundamentally different at the zero lower bound. For example, [Krugman \(2011\)](#) argues that improvements in technology that would normally cause output to expand can cause economic contractions at the zero lower bound. Work by [Christiano, Eichenbaum and Rebelo \(2011\)](#) and many others argue that the benefits of increased government spending are much larger when the economy is in a liquidity trap. Since the European Central Bank is also near the zero lower bound, [Eggertsson, Ferrero and Raffo \(2013\)](#) argues that structural reforms in Europe that would decrease firm market power do not support economic activity in the short-run.

In this paper, I argue that these results about the differential nature of the economy at the zero lower bound crucially hinge on particular assumptions about monetary policy. Much of the previous work on the effects of real shocks at the zero lower bound uses dynamic general equilibrium models with nominal rigidities. In almost all studies, the previous literature assumes that monetary policy follows some variant of the following [Taylor \(1993\)](#)-type policy rule subject to the zero lower bound:

$$r_t^d = r + \phi_\pi \pi_t + \phi_x x_t \quad (1)$$

$$r_t = \max(0, r_t^d) \quad (2)$$

where r_t^d is the desired policy rate of the monetary authority, π_t is the inflation rate, x_t is some measure of real economic activity, r_t is the actual policy rate subject to the zero lower bound, and $\phi_\pi \geq 1$ and $\phi_x \geq 0$ are parameters. During normal times, this policy rule prescribes that the central bank responds to contemporaneous fluctuations in inflation and economic activity using its short-term nominal policy rate. When inflation and economic activity fall too low, however, the central bank becomes constrained by the zero lower bound. Under this standard policy rule, government spending multipliers can be very large, and increases in technology and firm competitiveness can cause large declines in output at the zero lower bound.

However, standard [Taylor \(1993\)](#)-type policy rules imply that the central bank stops responding to the state of the economy at the zero lower bound. These policy rules imply that the monetary authority will only respond to the economy once inflation and real economic activity increase enough to allow liftoff from the zero lower bound. Even if the economy is continually hit by bad shocks at the zero lower bound, the central bank will not respond to the economy until conditions improve. This assumption is inconsistent with many recent statements and actions by policymakers. For example, the December 2011 statement from the Federal Open Market Committee states, “The

Committee will continue to assess the economic outlook in light of incoming information and is prepared to employ its tools to promote a stronger economic recovery in a context of price stability.” Since encountering the zero lower bound, many central banks have relied on “unconventional” policy tools such as forward guidance about the future conduct of policy and quantitative easing to help improve macroeconomic outcomes. While the exact effectiveness of these policies remains uncertain, central banks continue to endogenously respond to macroeconomic conditions at the zero lower bound.¹

In this paper, I examine the effects of real shocks at the zero lower bound when the central bank continues to respond to the state of the economy. Through the following history-dependent policy rule, the central bank responds to the economy using expectations about future policy:

$$r_t^d = r + \phi_\pi \pi_t + \phi_x x_t + \phi_d (r_{t-1}^d - r_{t-1}) \quad (3)$$

$$r_t = \max \left(0, r_t^d \right) \quad (4)$$

where $\phi_d > 0$ is a parameter.² When the monetary authority is unconstrained by the zero lower bound, the policy rule in Equation (3) responds exactly as a simple Taylor (1993)-type policy rule. At the zero lower bound, however, the central bank continues to respond to the state of the economy by adjusting its future path of desired policy rates. By including smoothing in the desired nominal policy rate at the zero lower bound, nominal policy rates become a function of current and past economic conditions. Declines in current inflation and real activity automatically lower future desired policy rates, which helps offset the previous higher-than-desired nominal rates caused by the zero lower bound. Agents in the economy internalize the future expansionary monetary policy, which can alter the response of the economy to real shocks.

I show that a moderate amount of history-dependence in the central bank’s policy rule can dramatically alter the effects of real shocks at the zero lower bound. To analyze the quantitative impact of history-dependence, I calibrate and solve a general-equilibrium model with nominal price rigidity and a zero lower bound constraint on the central bank’s nominal policy rate. If monetary policy continues to respond to the economy, government spending multipliers can be smaller than one and improvements in technology and firm competitiveness can increase output. Thus, the assumptions about future monetary policy are crucial in determining the effects of real shocks even when the economy is stuck at the zero lower bound.

Finally, I provide anecdotal evidence and several examples from the literature that recent Federal Reserve behavior is described by at least a moderate amount of history-dependence. The

¹See Bernanke (2012) for a review of the effectiveness of these policies.

²Reifschneider and Williams (2000) originally proposed history-dependent rules like Equation (3) and shows that they can reduce the detrimental effects of the zero lower bound.

history-dependence in central bank’s reaction function can be formulated as a response to the desired nominal interest rate, nominal price-level or nominal income, or the unemployment rate. If actual policy is history-dependent, then the model implies that the responses of the United States economy to real shocks may not look as fundamentally different than an economy away from the zero lower bound.

2 Model

This section outlines the baseline dynamic stochastic general equilibrium model that I use in my analysis. The baseline model shares many features with the models of Ireland (2003) and Ireland (2011). The model features optimizing households and firms and a central bank that systematically adjusts the nominal interest rate to offset adverse shocks in the economy. I allow for sticky prices using the quadratic-adjustment costs specification of Rotemberg (1982). The baseline model considers fluctuations in the discount factor of households, technology, government spending, and firms’ desired markups.

2.1 Households

In the model, the representative household maximizes lifetime expected utility over streams of consumption C_t and disutility over labor N_t . The household receives labor income W_t for each unit of labor N_t supplied in the representative intermediate goods-producing firm. The representative household also owns the intermediate goods firm and receives lump-sum dividends D_t . The household also has access to zero net supply nominal bonds B_t and real bonds B_t^R . A nominal bond pays the gross one-period nominal interest rate R_t while a real bond pays the gross one-period real interest rate R_t^R . The household divides its income from labor and its financial assets between consumption C_t and the amount of the bonds B_{t+1} and B_{t+1}^R to carry into next period. The government levies lump-sum taxes T_t on households each period. The discount factor of the household β is subject to shocks via the stochastic process a_t . An increase in a_t induces households to consume more and work less for no technological reason. Thus, I interpret changes in the household discount factor as demand shocks for the economy.

The representative household maximizes lifetime utility by choosing C_{t+s} , N_{t+s} , B_{t+s+1} , and B_{t+s+1}^R , for all $s = 0, 1, 2, \dots$ by solving the following problem:

$$\max E_t \sum_{s=0}^{\infty} a_{t+s} \beta^s \left(\log(C_{t+s}) - \chi \frac{N_{t+s}^{1+\eta}}{1+\eta} \right)$$

subject to the intertemporal household budget constraint each period,

$$C_t + \frac{1}{R_t} \frac{B_{t+1}}{P_t} + \frac{1}{R_t^R} B_{t+1}^R \leq \frac{W_t}{P_t} N_t + \frac{B_t}{P_t} + \frac{D_t}{P_t} + B_t^R - T_t.$$

Using a Lagrangian approach, household optimization implies the following first-order conditions:

$$a_t C_t^{-1} = \lambda_t \quad (5)$$

$$N_t^\eta = \lambda_t \frac{W_t}{P_t} \quad (6)$$

$$1 = E_t \left\{ \left(\beta \frac{\lambda_{t+1}}{\lambda_t} \right) \left(\frac{R_t P_t}{P_{t+1}} \right) \right\} \quad (7)$$

$$1 = E_t \left\{ \left(\beta \frac{\lambda_{t+1}}{\lambda_t} \right) R_t^R \right\} \quad (8)$$

where λ_t denotes the Lagrange multiplier on the household budget constraint. Equations (5) - (6) represent the household intratemporal optimality conditions with respect to consumption and leisure, and Equations (7) - (8) represent the Euler equations for the one-period nominal and real bonds.

2.2 Final Goods Producers

The representative final goods producer uses $Y_t(i)$ units of each intermediate good produced by the intermediate goods-producing firm $i \in [0, 1]$. The intermediate output is transformed into final output Y_t using the following constant returns to scale technology:

$$\left[\int_0^1 Y_t(i)^{\frac{\theta_t-1}{\theta_t}} di \right]^{\frac{\theta_t}{\theta_t-1}} \geq Y_t$$

θ_t is the elasticity of substitution across different intermediate goods, which is subject to exogenous fluctuations. Each intermediate good $Y_t(i)$ sells at nominal price $P_t(i)$ and each final good sells at nominal price P_t . The finished goods producer chooses Y_t and $Y_t(i)$ for all $i \in [0, 1]$ to maximize the following expression of firm profits:

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

subject to the constant returns to scale production function. Finished goods-producer optimization results in the following first-order condition:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta_t} Y_t$$

The market for final goods is perfectly competitive, and thus the final goods-producing firm earns zero profits in equilibrium. Using the zero-profit condition, the first-order condition for profit maximization, and the firm objective function, the aggregate price index P_t can be written as follows:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_t} di \right]^{\frac{1}{1-\theta_t}}$$

2.3 Intermediate Goods Producers

Each intermediate goods-producing firm i rents labor $N_t(i)$ from the representative household in order to produce intermediate good $Y_t(i)$. Intermediate goods are produced in a monopolistically competitive market where producers face a quadratic cost of changing their nominal price $P_t(i)$ each period. Firm i chooses $N_t(i)$, and $P_t(i)$ to maximize the discounted present-value of cash flows $D_t(i)/P_t(i)$ given aggregate demand Y_t and price P_t of the finished goods sector. The intermediate goods firms all have access to the same constant returns-to-scale Cobb-Douglas production function, subject to a fixed cost of production Φ . Changes in θ_t cause fluctuations in the desired markups of intermediate-goods producing firms over marginal cost.

Each intermediate goods-producing firm maximizes discount cash flows using the household stochastic discount factor:

$$\max \text{E}_t \sum_{s=0}^{\infty} \left(\beta^s \frac{\lambda_{t+s}}{\lambda_t} \right) \left[\frac{D_{t+s}(i)}{P_{t+s}} \right]$$

subject to the production function:

$$\left[\frac{P_t(i)}{P_t} \right]^{-\theta_t} Y_t \leq Z_t N_t(i) - \Phi,$$

where

$$\frac{D_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t} \right]^{1-\theta_t} Y_t - \frac{W_t}{P_t} N_t(i) - \frac{\phi_P}{2} \left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right]^2 Y_t$$

The first-order conditions for the firm i are as follows:

$$\frac{W_t}{P_t} N_t(i) = \Xi_t Z_t N_t(i) \tag{9}$$

$$\begin{aligned} \phi_P \left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right] \left[\frac{P_t}{\Pi P_{t-1}(i)} \right] &= (1 - \theta_t) \left[\frac{P_t(i)}{P_t} \right]^{-\theta_t} + \theta_t \Xi_t \left[\frac{P_t(i)}{P_t} \right]^{-\theta_t - 1} \\ &+ \phi_P \text{E}_t \left\{ \left(\beta \frac{\lambda_{t+1}}{\lambda_t} \right) \frac{Y_{t+1}}{Y_t} \left[\frac{P_{t+1}(i)}{\Pi P_t(i)} - 1 \right] \left[\frac{P_{t+1}(i)}{\Pi P_t(i)} \frac{P_t}{P_t(i)} \right] \right\}, \end{aligned} \tag{10}$$

where Z_t is the level of technology and Ξ_t is the multiplier on the production function, which denotes the real marginal cost of producing an additional unit of intermediate good i .

2.4 Monetary and Fiscal Policy

I assume a cashless economy where the monetary authority sets the one-period net nominal interest rate $r_t = \log(R_t)$. Due to the zero lower bound on nominal interest rates, the central bank cannot lower its nominal policy rate below by zero. I assume that monetary policy follows one of following specifications, which are both subject to the zero lower bound:

$$r_t^d = r + \phi_\pi \log\left(\Pi_t/\Pi\right) + \phi_y \log\left(Y_t/Y\right) \tag{11}$$

$$r_t^d = r + \phi_\pi \log(\Pi_t/\Pi) + \phi_y \log(Y_t/Y) + \phi_d (r_{t-1}^d - r_{t-1}) \quad (12)$$

$$r_t = \max(0, r_t^d). \quad (13)$$

Equation (11) is a standard Taylor (1993)-type policy rule, where the central bank responds only to contemporaneous deviations from output and inflation from their steady state values. However, when the economy encounters the zero lower bound, the central bank is unable to respond to the economy. Equation (12) endogenously adjusts the future path of monetary policy when the economy encounters the zero lower bound. The additional parameter ϕ_d controls the amount of response of the future path of policy when the economy hits the zero lower bound. When the economy is unconstrained by the zero lower bound, the two previous rules will imply the same monetary policy rule. Fiscal policy finances government spending G_t using lump-sum taxes on households.

2.5 Shock Processes

The model features fluctuations in household discount factors, technology, the desired markup of firms, and government spending. The stochastic process for these fluctuations is as follows:

$$a_t = (1 - \rho_a)a + \rho_a a_{t-1} + \sigma^a \varepsilon_t^a \quad (14)$$

$$Z_t = (1 - \rho_Z)Z + \rho_Z Z_{t-1} + \sigma^Z \varepsilon_t^Z \quad (15)$$

$$\theta_t = (1 - \rho_\theta)\theta + \rho_\theta \theta_{t-1} + \sigma^\theta \varepsilon_t^\theta \quad (16)$$

$$G_t = (1 - \rho_G)G + \rho_G G_{t-1} + \sigma^G \varepsilon_t^G \quad (17)$$

The steady state levels of the discount factor and technology shock are normalized to one. In this paper, I am interested in comparing the responses of the model economy both at and away from the zero lower bound. To simulate a large recession that takes the economy to the zero lower bound, I simulate a large increase in the household discount factor. This shock acts like a large decline in aggregate demand, which forces the economy to encounter the zero lower bound. After encountering the zero lower bound, I examine the impact of the three other real shocks conditional on either specification of monetary policy.

2.6 Equilibrium and Solution Method

In the symmetric equilibrium, all intermediate goods firms choose the same price $P_t(i) = P_t$ and employ the same amount of labor $N_t(i) = N_t$. Thus, all firms have the same cash flows and I define gross inflation as $\Pi_t = P_t/P_{t-1}$ and the markup over marginal cost as $\mu_t = 1/\Xi_t$. Therefore, I can model the intermediate-goods firms with a single representative intermediate goods-producing firm.

To formally analyze the impact of the zero lower bound, I solve the model using the policy function iteration method of Coleman (1990) and Davig (2004). This global approximation method, as opposed to local perturbation methods such as linearization, allows me to model the occasionally binding zero lower bound constraint. This method discretizes the state variables on a grid and solves for the policy functions which satisfy all the model equations at each point in the state space. Appendix A contains the details of the policy function iteration algorithm.

2.7 Calibration

Table 1 lists the calibrated parameters of the model. I calibrate the model at quarterly frequency using standard parameters for one-sector models of fluctuations. Since the model shares features with the estimated models of Ireland (2003) and Ireland (2011), I calibrate many of the parameters to match the estimates reported by those papers. I choose χ to normalize output Y to equal one at the deterministic steady state. I calibrate η such that the model has a Frisch labor supply elasticity of one half. The fixed cost of production for the intermediate-goods firm Φ is calibrated to $(\mu - 1)Y$, which eliminates pure profits in the deterministic steady state of the model. To be consistent with national income accounting, I treat the adjustment costs for price adjustment as intermediate inputs in production. Therefore, I plot output as the sum of consumption and government spending in all impulse responses.

The crucial parameter in my calibration is the policy parameter ϕ_d , which controls amount of history-dependence in the policy rule. In my baseline model, I calibrate set $\phi_d = 0.9$, which implies a fair amount of history-dependence when the economy hits the zero lower bound. In addition to the anecdotal evidence from the Introduction, work by Gust, López-Salido and Smith (2013) suggests that the Federal Reserve’s recent behavior can be described by an interest-rate rule with significant history-dependence. In the following simulations, I compare the effects of real shocks under a standard Taylor (1993)-type rule to the history-dependent policy rule. When the economy is away from the zero lower bound, these rules imply the exact same behavior of the monetary authority.

3 Effects of Real Shocks

3.1 Aggregate Demand and Aggregate Supply

Before turning to my quantitative findings, I present the key intuition for my results heuristically using simple aggregate demand and aggregate supply diagrams. Following Eggertsson (2010), Appendix B shows that the following supply and demand relations can be derived using a first-

order approximation of the model at the deterministic steady state:

$$\pi_t = \beta E_t \pi_{t+1} + \psi_y y_t - \omega_t - (1 - \eta) \psi z_t - \psi a_t + \psi_g (g_t - g) \quad (18)$$

$$y_t = E_t y_{t+1} - (1 - s_g) \left(r_t^r + (1 - \rho_a) a_t + s_g (g_t - g) \right) \quad (19)$$

$$r_t = E_t \pi_{t+1} + r_t^r \quad (20)$$

$$r_t = \max \left(0, r + \phi_\pi \pi_t + \phi_y y_t \right) \quad (21)$$

$$(22)$$

Figure 1 plots the aggregate demand and supply curves for the economy both at and away from the zero lower bound. The key difference at the zero lower bound is that the aggregate demand curve becomes upward sloping in output and inflation space. This result emerges from the interactions of the Fisher relation in Equation (20) and the zero lower bound. For illustrative purposes only, hold the technology, markup, and government spending shocks fixed at their steady state values. Then, Equations (18), (20), and (21) can be substituted into Equation (19) to derive the following relation:

$$\left(1 + (1 - s_g) (\phi_y + \beta^{-1} \psi_y) \right) y_t = E_t y_{t+1} - (1 - s_g) (\phi_\pi - \beta^{-1}) \pi_t + (1 - s_g) (1 - \rho_a + \beta^{-1} \psi) a_t \quad (23)$$

During normal times, the central bank follows the Taylor principle $\phi_\pi > \beta^{-1}$ and adjusts its nominal interest rate r_t more than one for one with movements in inflation. If inflation rises by one percent, nominal interest rates rise by more than one percent through the Taylor principle. Thus, real interest rates r_t^r real interest rates rise after an increase in inflation. Higher real interest rates discourage consumption, which causes a decline in output. All else equal, Equation (22) shows that higher inflation causes output to decline when the economy is away from the zero lower bound. The endogenous response of the monetary authority is the key component in generating the downward sloping aggregate demand curve.

At the zero lower bound, however, the slope of the aggregate demand curve becomes positive. Setting $r_t = 0$ and substituting Equations (18) and (20) into (19) implies the following relation:

$$\left(1 + (1 - s_g) \beta^{-1} \psi_y \right) y_t = E_t y_{t+1} + (1 - s_g) \beta^{-1} \pi_t + (1 - s_g) (1 - \rho_a + \beta^{-1} \psi) a_t + (1 - s_g) r \quad (24)$$

Higher inflation is not immediately offset by higher nominal policy rates today. An increase in inflation lowers real interest rates through the Fisher relation, which induces higher consumption and output and the positive slope of the aggregate demand curve. All else equal, Equation (24) shows that higher inflation raises output when the economy is stuck at the zero lower bound. This differential slope of the aggregate demand curve is why the previous work argues that the economy is fundamentally different at the zero lower bound. For example, Figure 2 analyzes a shock which causes a downward shift in the aggregate supply curve holding the aggregate demand curve fixed. During normal times, this shock would normally cause a modest decline in inflation and an increase

in output. All else equal, the same shock at the zero lower bound would imply a larger disinflation and a fall in economic activity. Thus, the inability of the monetary authority to adjust its current nominal policy rate generates the upward sloping demand curve and the potential for changing the response of the economy to shocks.

The point of this paper is to highlight that the central bank can still have a stabilizing role even in the face of the upward sloping demand curve. Although it cannot adjust its current policy rate, the central bank under a history-dependent policy can endogenously adjust its future path of policy in order to offset adverse fluctuations. By committing to a lower path of nominal policy rates for any given level of expected inflation, the central bank can affect expectations of future inflation and output at the zero lower bound. Although the central bank cannot affect the slope of the aggregate demand curve in Figure 2, it can still affect the equilibrium outcomes of inflation and output through shift variables of expected inflation and output in Equations (18) and (24). Expectations of higher inflation in the future cause an additional upward shift in the aggregate supply curve as firms choose higher prices for any given level of current output and inflation. Higher output in the future cause the aggregate demand curve to shift outward.

The simple demand and supply diagrams show that the equilibrium outcomes for inflation and output clearly depend on the magnitude of the shifts. The amount of endogenous response of the monetary authority at the zero lower bound is controlled by the amount of history-dependence in the policy rule. In the next section, I show that a reasonably calibrated model with a fair amount of history dependence is enough to dramatically alter the responses of several real shocks. Thus, the differential nature of the economy at the zero lower bound depends crucially on the assumptions about future policy.

3.2 Technology Shocks

Figure 3 plots the impulse response of a one percent increase in technology both at and away from the zero lower bound. Figure 4 shows the impact of the shock heuristically using the aggregate supply and demand diagrams from the previous section. Away from the zero lower bound, the increase in productivity decreases firm marginal costs. Lower current and future marginal costs cause a shift down in the aggregate supply curve as firms choose lower prices for any given level of current output. Under a standard Taylor (1993)-rule, the central bank aggressively lowers its nominal policy rate. By following the Taylor principle, the central bank lowers its policy rate more than one for one with inflation, which helps lower real interest rates. Since the technology shock is persistent, agents expect higher output throughout the duration of the shock. Higher output expectations shift out the aggregate demand curve for any given level of real interest rates. However, the central bank does not response aggressively enough and the economy experiences some disinflation as a result of the technological improvement. At impact of the shock, output rises by

over one half percent and inflation falls by one percent when the central bank follows a standard policy rule away from the zero lower bound.

Under a standard Taylor (1993) rule at the zero lower bound, the technology shock becomes highly contractionary. Under a positively slope aggregate demand curve, a shift downward in the aggregate supply curve reduces output and generates a large disinflation. Expectations of lower output in the future also cause a leftward shift in the aggregate demand curve as households reduce consumption for a given level of real interest rates. At impact, inflation and output fall by about two percent and remain negative throughout the zero lower bound episode. Under its contemporaneous policy rule, the monetary authority begins responding to the economy only after the zero lower bound episode and doesn't alter its future course of policy after the large contractionary shock.

If monetary policy follows the history-dependent rule, then the central bank responds to the technology shock using forward guidance about future policy. In response to the large contractions in output and inflation, the central bank commits to a lower path of policy in the future. Expectations of future expansionary policy raises inflation expectations and causes an upward shift in the aggregate supply curve. Higher expected output after the zero lower bound episode helps attenuate much of the leftward shift in the aggregate demand curve. For the calibrated model from Section 2, the additional shifts implied by the history-dependent rule are enough to offset much of the contraction in output. After a smaller initial contraction of one half percent, output rises and remains positive throughout the duration of the zero lower bound episode. The time path for inflation starts below and then rises above the the impulse responses away from the zero lower bound. Overall, the results indicate that the equilibrium response of a technology shock depends crucially on future assumptions about monetary policy. If the monetary authority uses forward guidance to stabilize the economy, the response to a technology shock may not look too qualitatively different at the zero lower bound.

3.3 Markup Shocks

Figure 5 plots the impulse response of a one percent decrease in firm desired markups both at and away from the zero lower bound. This shock is also acts as a positive supply shock which causes a downward shift in the aggregate supply curve. Thus, Figure 6 shows the results through the shifts in aggregate supply and demand, which looks qualitatively similar to the technology shock responses. Recent work by Eggertsson, Ferrero and Raffo (2013) argues that a decrease in desired markups can model the proposed structural reforms in Europe designed to increase overall competitiveness. The calibrated model implies that a one percent decrease in desired markups causes a 0.2 percent increase in output and 0.4 percent decline in inflation at impact away from the zero lower bound. If the central bank fails to respond to the economy at the zero lower bound, the

same shock causes declines in output and inflation of over one percent. However, if the central bank tries to stabilize the economy using expectations of future policy, the time path for inflation looks similar to the responses away from the zero lower bound. After an initial small decline, output remains positive throughout the zero lower bound episode. Thus, the equilibrium responses to an increase in firm competitiveness may not be fundamentally different if the central bank continues to respond to the state of the economy.

3.4 Government Spending Shocks

Finally, I examine the effects of increased government spending. Recent work by Christiano, Eichenbaum and Rebelo (2011) and many others argues that government spending multipliers are substantially larger when the economy is constrained by the zero lower bound. Figure 8 plots the impulse responses of the model to a one percent increase in government spending and Figure 9 illustrates the results in aggregate supply and demand space. Higher government spending induces a negative wealth effect on households, which causes them to work more for any given level of the real wage. Higher labor input, with the increased government spending, creates expectations of higher output in the future. Higher expected output shifts out the aggregate demand curve. However, the household's desire to work more increases firm marginal costs. Higher future marginal costs raise inflation expectations and shifts out the aggregate supply curve as well. When the economy is away from the zero lower bound, the negative wealth effect on households causes them to reduce overall consumption. Since the model does not have capital, $Y_t = C_t + G_t$, so a fall in consumption implies a government spending multiplier of less than one.

Under the standard Taylor (1993) rule at the zero lower bound, the outward shift in aggregate demand raises both inflation and output. The increased inflation expectations help further lower real interest rates. These lower real interest rates induce higher consumption and cause consumption to increase after the shock. Thus, the government spending multiplier becomes larger than one when the central bank follows a standard contemporaneous policy rule. While the exact size of the multiplier is subject to specific conditions, the upward sloping aggregate demand curve and the non-responsive monetary policy are crucial elements in getting consumption to rise after a spending shock.

Under the history-dependent policy, the central bank endogenously adjusts its future path in an effort to curb future inflation. Thus, the monetary policy rule implies a tighter policy in the future because of the large increase in government spending. Expectations of higher nominal rates in the future induce lower output and inflation expectations. Expectations of lower output and inflation shift the aggregate demand curve backward and induce a slight downward shift in the aggregate supply curve. By continuing to respond to the economy, the central bank curtails much of the expansion caused by fiscal policy in order to maintain stable inflation. Consumption

now falls under the history-dependent policy, which reduces the spending multiplier to less than one. The results highlight that the model-implied benefits of higher government spending, even when current nominal interest rates are zero, depend crucially on the assumptions about monetary policy.

3.5 Amount of History-Dependence

The previous result highlight that history-dependence in the monetary policy rule can significantly alter the effects of real shocks. In my baseline model, I calibrate the history-dependence parameter ϕ_d to be 0.9. This value is consistent with the empirical evidence of Gust, López-Salido and Smith (2013), which I discuss in the next section. In this section, I show the degree of history-dependence in the policy rule needed to alter the effects of shocks at the zero lower bound. Figure 9 plots the model responses of a positive technology shock at the zero lower bound under alternative level of the history-dependence parameter ϕ_d . The results show that the policy rule needs a history-dependence coefficient of larger than 0.8 in order to eliminate much of the contractionary nature of the technology shock at the zero lower bound. The reason for this result is largely mechanical: The degree of history-dependence controls the length of the memory for the policy rule. Suppose that the economy has been stuck at the zero lower bound for n periods. Then, solving backward using the history-dependent rule implies the following value for the desired nominal policy rate at the zero lower bound:

$$r_t^d = r + \phi_\pi \pi_t + \phi_x x_t + \phi_d r_{t-1}^d$$

$$r_t^d = r + \phi_\pi \pi_t + \phi_x x_t + \phi_d \left(r + \phi_\pi \pi_{t-1} + \phi_x x_{t-1} + \phi_d r_{t-2}^d \right)$$

$$r_t^d = r + \phi_\pi \pi_t + \phi_x x_t + \sum_{i=1}^n \phi_d^i \left(r + \phi_\pi \pi_{t-i} + \phi_x x_{t-i} \right)$$

Thus, the value of ϕ_d controls the weight of current versus past economic conditions in the monetary policy rule. Alternatively, ϕ_d controls how the monetary authority alters the future path of policy when they become constrained by the zero lower bound. Without sufficiently high values for the history-dependence parameter, the amount of memory in the policy rule decays very quickly. Also, the backward solution shows that the product of ϕ_d^i and the policy rule coefficients ϕ_π and ϕ_x determines the overall degree of history-dependence. Policy rules that are closer to optimal policy (Higher values of ϕ_π and ϕ_x) during normal times require less history-dependence in order to achieve a given level of memory in the policy rule.

3.6 Forms of History-Dependence

The previous result highlight that history-dependence in the monetary policy rule can significantly alter the effects of real shocks. In my baseline model, I model this history-dependence as the Reifschneider and Williams (2000) rule. This policy rule acts as a standard contemporaneous policy rule, but adds an additional term $r_{t-1}^d - r_{t-1}$ that only affects the economy during and after a zero lower bound episode. The advantage of this rule is that it nests the standard policy rule, and thus produces the same responses away from the zero lower bound. However, the form of the history-dependence can take many alternative forms. For example, Gust, López-Salido and Smith (2013) suggests that the Federal Reserve’s recent behavior is described by a the following interest-rate rule with significant history-dependence:

$$r_t^d = (1 - \phi_r) r + \phi_r r_{t-1}^d + \phi_\pi \pi_t + \phi_x x_t \quad (25)$$

$$r_t = \max \left(0, r_t^d \right). \quad (26)$$

This policy rule implies history-dependence by smoothing through the desired nominal policy rate of the central bank. At the zero lower bound, the policy rule is operationally similar to the Reifschneider and Williams (2000) rule in my baseline model. However, the smoothing in the desired policy rate will imply a more moderately sloped exit from the zero lower bound. Gust, López-Salido and Smith (2013) estimates the 95% confidence interval for ϕ_r to be (0.79, 0.92), which implies at least a moderate amount of history-dependence in recent Federal Reserve Policy. Thus, my calibrated amount of history-dependence in my baseline model is consistent with their empirical evidence.

In addition to smoothing in the desired nominal policy rate, the history-dependence policy could also target nominal income, the nominal price-level, or any other backward-looking state variable. Even small weights on these backward-looking variables in the policy rule would imply significant history-dependence. For example, suppose the central bank places weight on the nominal price level in its policy rule, which Gorodnichenko and Shapiro (2007) argues is consistent with the Federal Reserve behavior in the late 1990s. Agents in the economy understand that any deflation caused by the zero lower bound will eventually be offset with higher inflation in the future to stabilize the price level. Thus, the central bank continues to adjust its future path of policy in response to current economic conditions at the zero lower bound. Regardless of the form of the history-dependence, the key insight is that the central bank continues to respond to the economy at the zero lower bound. In the next section, I explore the implications of a specific-type of history-dependence, which is of particular interest to policymakers.

3.7 Unemployment and History-Dependence

The previous section shows that the history-dependence in the policy rule can take a number of forms. In this section, I show that a response of monetary policy to unemployment automatically implies history-dependence in a model of labor search frictions. Recent work by Blanchard and Galí (2010) embeds a Diamond-Mortensen-Pissarides search and matching model of unemployment into a standard model of nominal price rigidity. In their framework, the number of employed workers N_t evolves according to the following law of motion:

$$N_t = (1 - \psi) N_{t-1} + m_t \quad (27)$$

where m_t denotes a matching function that depends on the amount of unemployed workers and the number of vacancies, and ψ denotes the job destruction rate. Under the assumption of full participation, the unemployment rate u_t can be written as follows:

$$u_t = 1 - N_t \quad (28)$$

Solving backward, we can write the unemployment rate as a backward looking state variable with $\psi > 0$

$$u_t = 1 - \sum_{i=0}^{\infty} (1 - \psi)^i m_{t-i} \quad (29)$$

If monetary policy reacts to fluctuations in unemployment in its policy rule, the backward-looking nature of the unemployment rate automatically implies history-dependence in the monetary policy rule. At the zero lower bound, agents internalize the central bank's commitment to stabilizing the labor market. This link between current and past labor market conditions implies that the central bank continues to respond to the state of the economy even when constrained by the zero lower bound. This response to unemployment is consistent with recent Federal Open Market Committee statements and actions. Since the beginning of the Great Recession, the Federal Reserve consistently cites the elevated unemployment rate as justification for using its policy tools to improve labor market conditions. Since December 2011, the Federal Reserve has explicitly cited a numerical target for the unemployment rate as a necessary condition for raising the federal funds rate. This response to unemployment, through the lens of a standard search and matching model, automatically implies history-dependence in the central bank's reaction function.

3.8 Discussion and Connections with Existing Literature

This paper aims to extend results from the literature on monetary policy in models of nominal price rigidities. Prior work such as Basu and Kimball (2005), Woodford (2003), and many others shows that the effects of real shocks crucially depend on the assumptions about monetary policy. This paper argues that the assumptions about monetary policy continue to matter greatly even

the economy is stuck at the zero lower bound. Zero nominal policy rates alone are not a sufficient definition to pin down the conduct of monetary policy.

This work is related to independent work by Erceg and Lindé (2012), which also examines the effects of government spending multipliers at the zero lower bound. Under a standard Taylor (1993)-type policy rule, Erceg and Linde show that the government spending multiplier can be very large under a variety of assumptions about the structure of fiscal policy and the frictions and agents in the real economy. In an Appendix, however, they show numerically that government spending multipliers are considerably smaller at the zero lower bound if monetary policy responds to the nominal price-level. The point of my paper is to highlight why government spending multipliers differ greatly under the history-dependent policy. By continuing to respond to the state of the economy at the zero lower bound, the monetary authority endogenously adjusts future policy in response to current economic conditions. This response offsets much of the inflation generated by the fiscal expansion and limits the “free lunch” for the fiscal authority. In addition, I highlight the inconsistency of the standard contemporaneous policy rule assumption and the recent actions and statements by policymakers.

This paper is also related to work by Eggertsson and Woodford (2004). Under optimal monetary and fiscal policy, they show that the gains from optimal monetary policy are smaller when fiscal policy is conducted optimally in a liquidity trap. This paper aims to show the converse of this result in a positive rather than normative setting. The benefits from fiscal expansions in a liquidity trap are smaller if monetary policy acts in a history-dependent manner.

4 Conclusions

Even when current nominal policy rates are zero, this paper argues that assumptions about monetary policy are crucial in determining the equilibrium effects of real shocks. Alternative assumptions about policy can generate government spending multipliers that are greater or less than one and cause increases in technology and firm competitiveness to become expansionary or highly contractionary. Given the importance of the assumed policy rule, future versions of this work will aim to identify the amount of history-dependence in the recent Federal Reserve actions. I plan to use both direct and indirect methods to argue that this recent behavior is characterized by at least a moderate amount of history-dependence.

Technical Appendix

A Numerical Solution Method

To analyze the impact of real shocks at the zero lower bound, I solve the model using the policy function iteration method of Coleman (1990). This global approximation method allows me to model the occasionally-binding zero lower bound constraint. This section provides the details of the algorithm when monetary policy follows a standard Taylor (1993)-type policy rule. The algorithm is implemented using the following steps:

1. Discretize the state variables of the model: $\{a_t \times Z_t \times \theta_t \times G_t\}$
2. Conjecture initial guesses for the policy functions of the model $N_t = N(a_t, Z_t, \theta_t, G_t)$, $\Pi_t = \Pi(a_t, Z_t, \theta_t, G_t)$, and $R_t^R = R^R(a_t, Z_t, \theta_t, G_t)$.
3. For each point in the discretized state space, substitute the current policy functions into the equilibrium conditions of the model. Use interpolation and numerical integration over the exogenous state variables a_t , Z_t , θ_t , G_t to compute expectations for each Euler equation. This operation generates a nonlinear system of equations. The solution to this system of equations provides an updated value for the policy functions at that point in the state space. The solution method enforces the zero lower bound for each point in the state space and in expectation.
4. Repeat Step (3) for each point in the state space until the policy functions converge and cease to be updated.

I implement the policy function iteration method in FORTRAN using the nonlinear equation solver DNEQNF from the IMSL numerical library. When monetary policy follows an alternative specification, the state variables and Euler equations are adjusted appropriately.

B Derivation of Linearized Model

[To be written]

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Table 1: Calibration of Model Parameters

Parameter	Description	Calibrated Value
β	Household Discount Factor	0.99
ϕ_P	Adjustment Cost to Changing Prices	160.0
Π	Steady State Inflation Rate	1.000
η	Inverse of the Frisch Labor Supply Elasticity	2.0
χ	Scalar Affecting Steady State Hours	0.72
θ	Steady State Elasticity of Substitution Intermediate Goods	6.0
G/Y	Steady State Government Spending Over Output	0.2
ρ_a	Preference Shock Persistence	0.90
ρ_Z	Technology Shock Persistence	0.90
ρ_G	Government Spending Shock Persistence	0.90
ρ_θ	Markup Shock Persistence	0.90
σ^a	Preference Shock Volatility	0.005
σ^A	Government Spending Shock Volatility	0.0025
σ^G	Government Spending Shock Volatility	0.0025
σ^θ	Markup Shock Volatility	0.0025

Figure 1: Aggregate Supply and Demand and the Zero Lower Bound

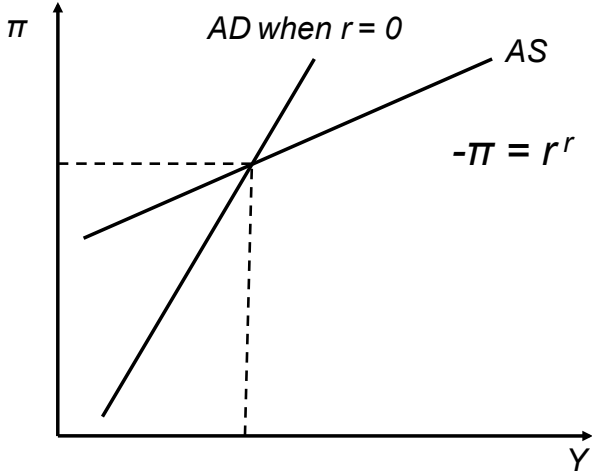
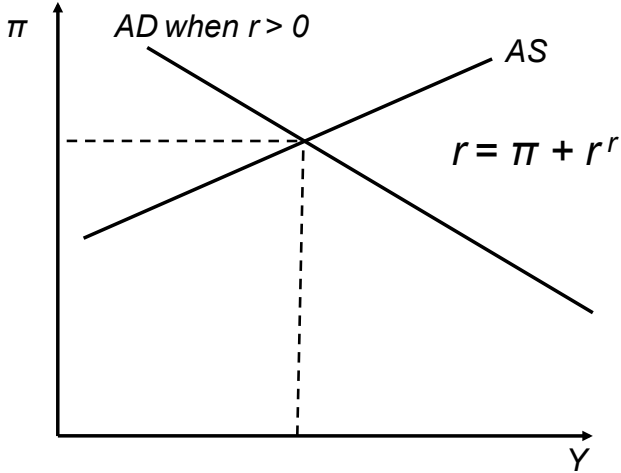


Figure 2: Downward Shift in Aggregate Supply

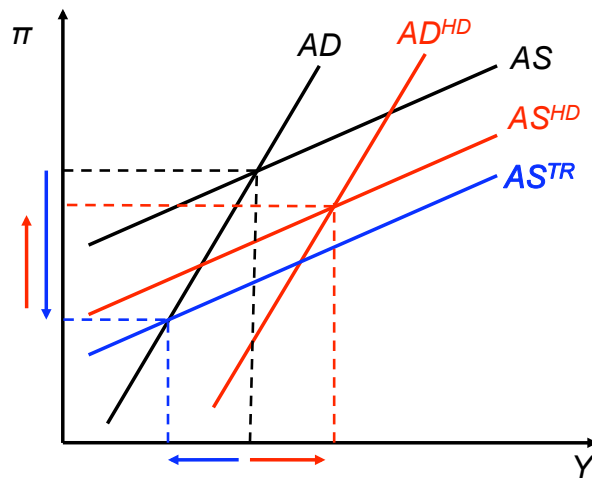
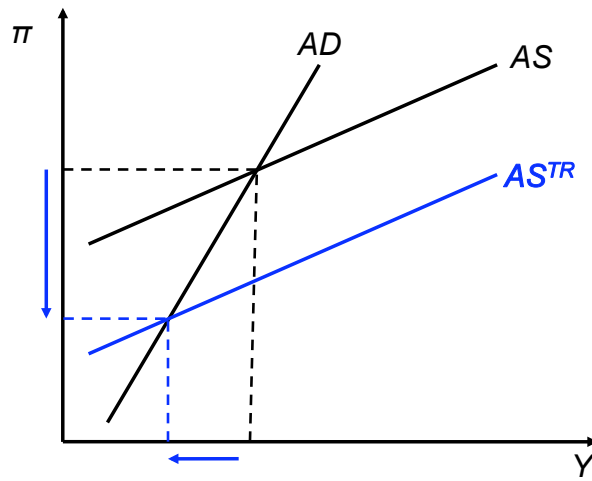
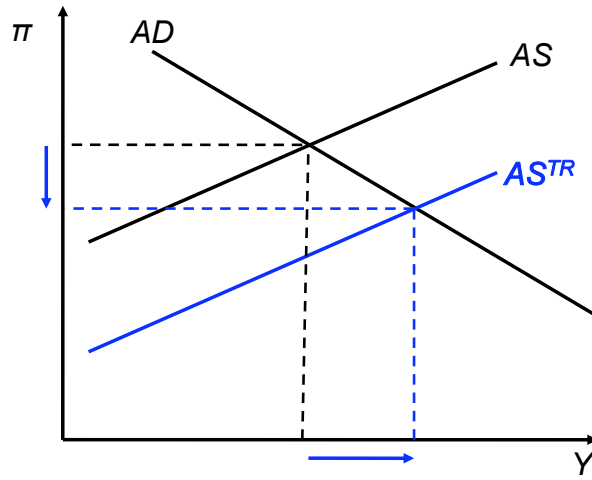
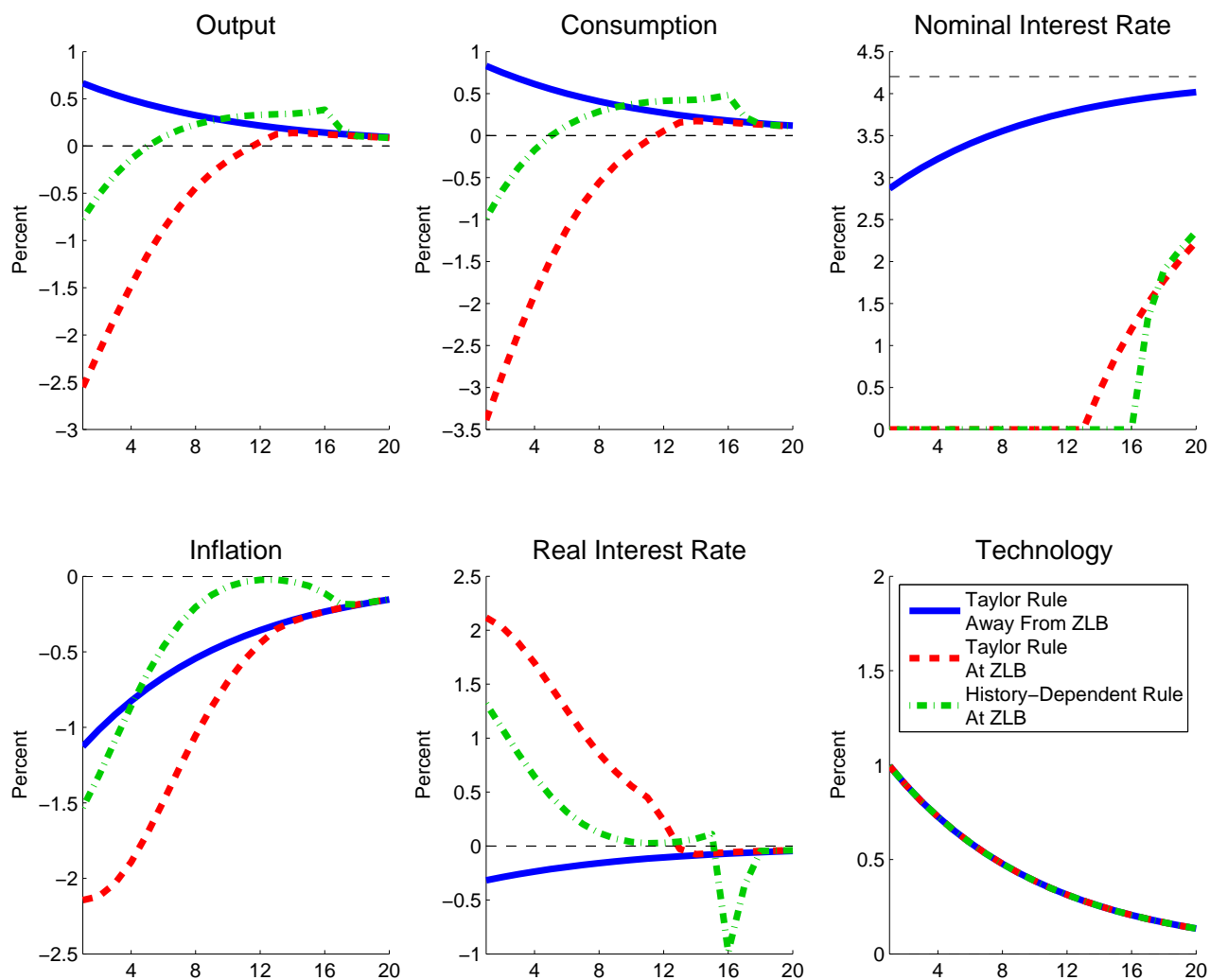


Figure 3: Model Responses to Positive Technology Shock



Note: Output, consumption, and technology are plotted as percent deviations. The nominal interest rate is plotted in annualized percent, while the real interest rate and inflation are plotted in annualized percent deviations.

Figure 4: Effects of Positive Technology Shock

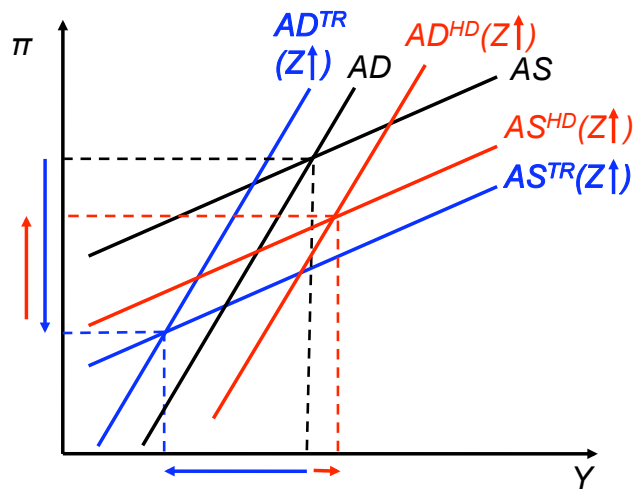
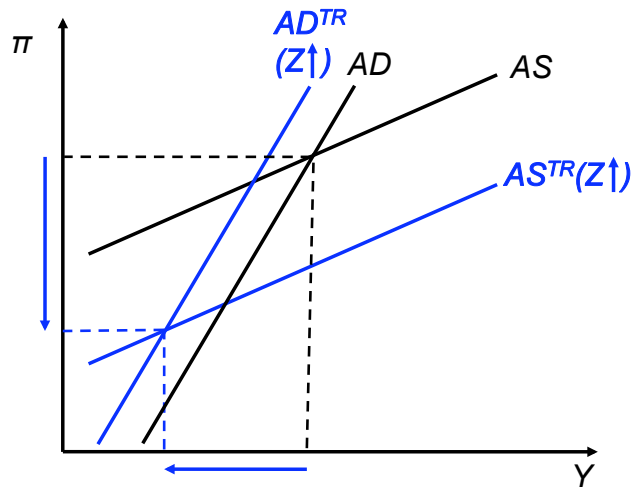
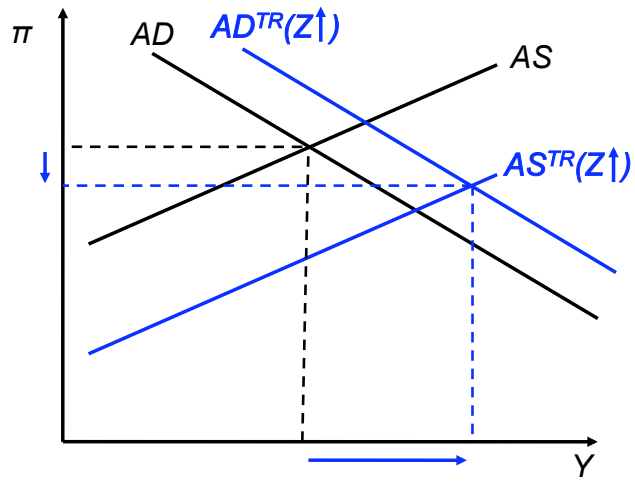
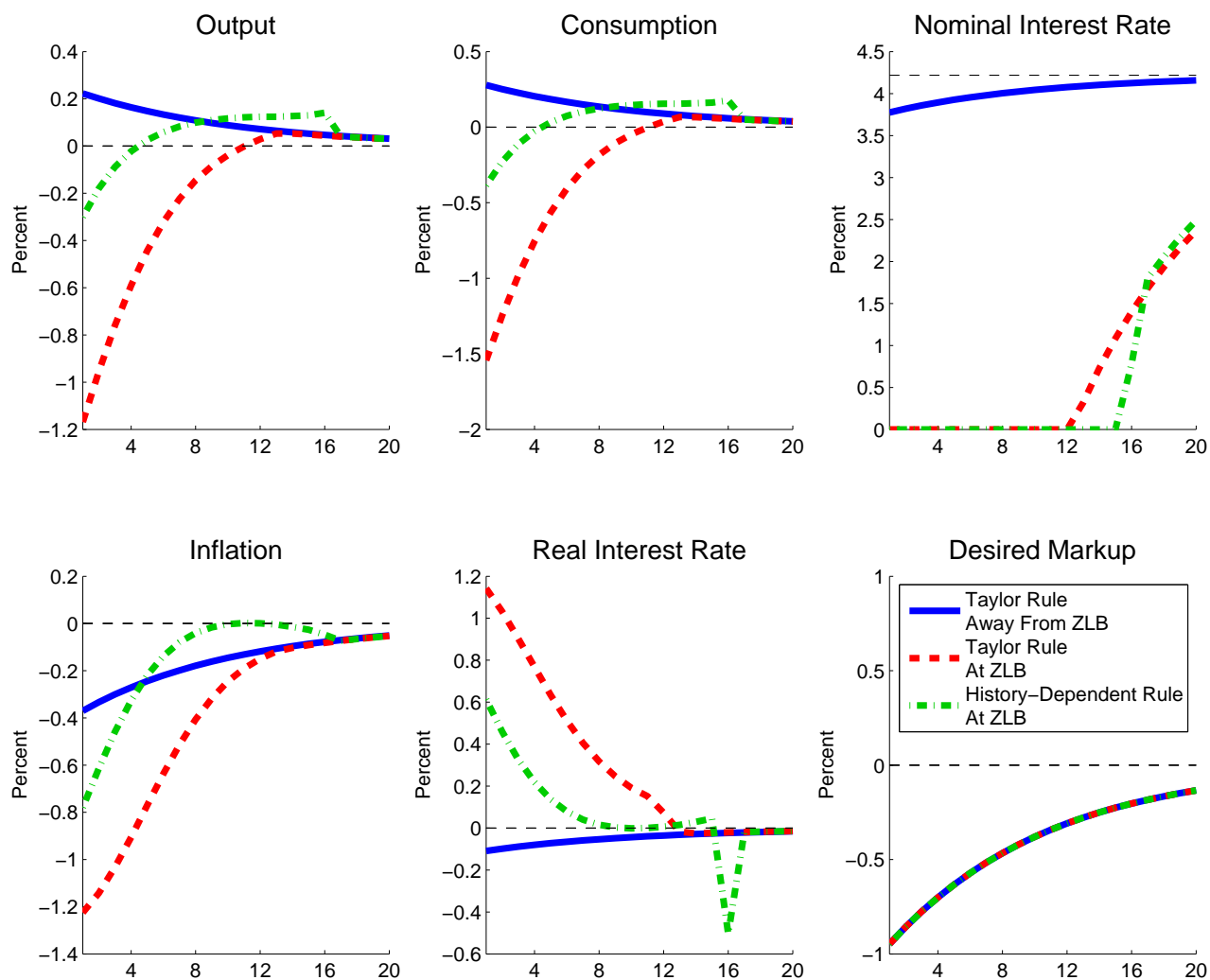


Figure 5: Model Responses to Decrease in Firm Desired Markups



Note: Output, consumption, and desired markups are plotted as percent deviations. The nominal interest rate is plotted in annualized percent, while the real interest rate and inflation are plotted in annualized percent deviations.

Figure 6: Effects of Decrease in Firm Desired Markups

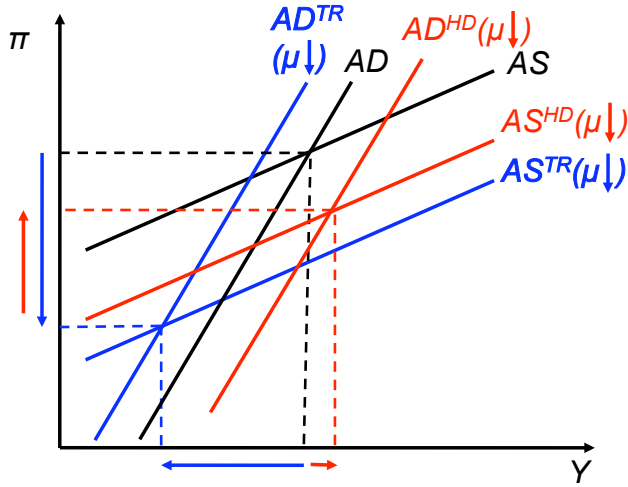
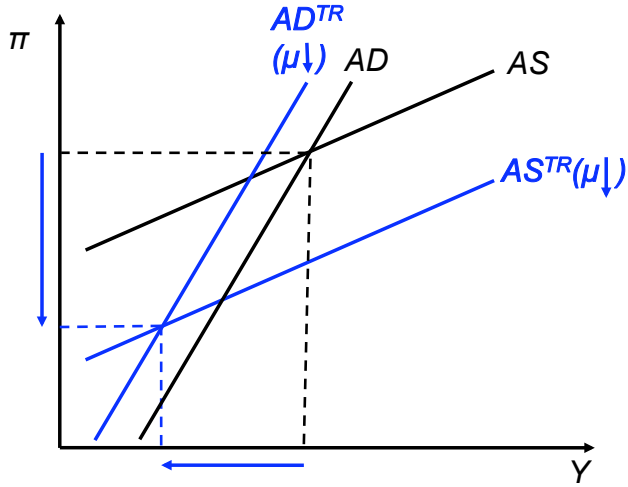
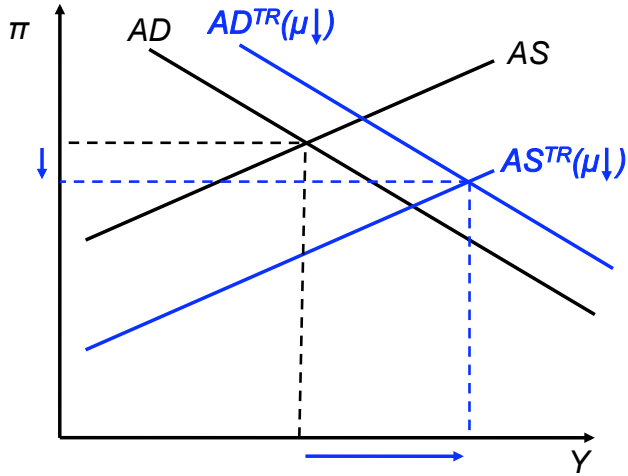
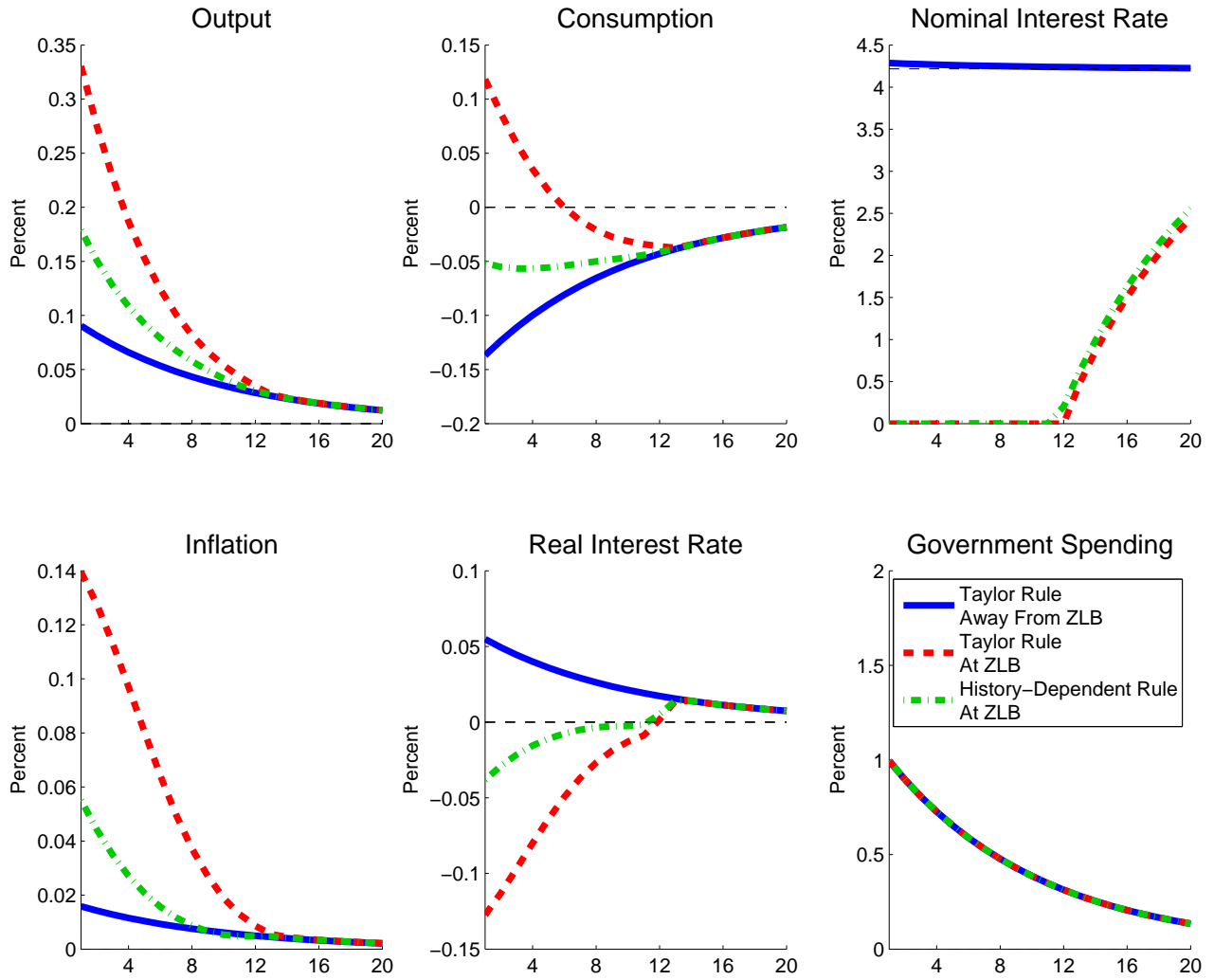


Figure 7: Model Responses to Increase in Government Spending



Note: Output, consumption, and government spending are plotted as percent deviations. The nominal interest rate is plotted in annualized percent, while the real interest rate and inflation are plotted in annualized percent deviations.

Figure 8: Effects of Increase in Government Spending

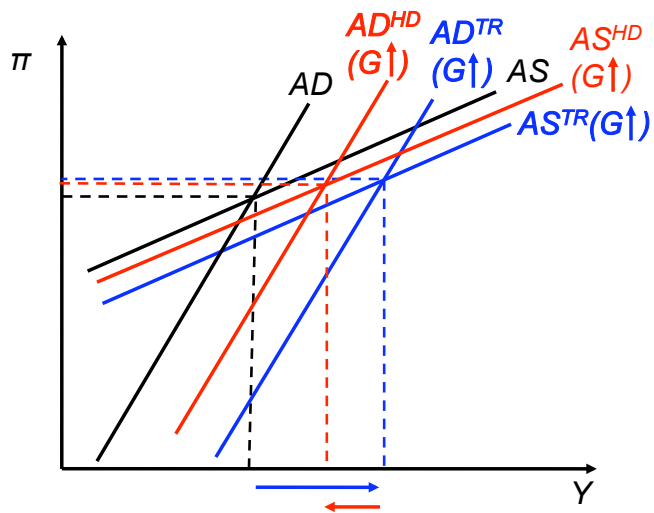
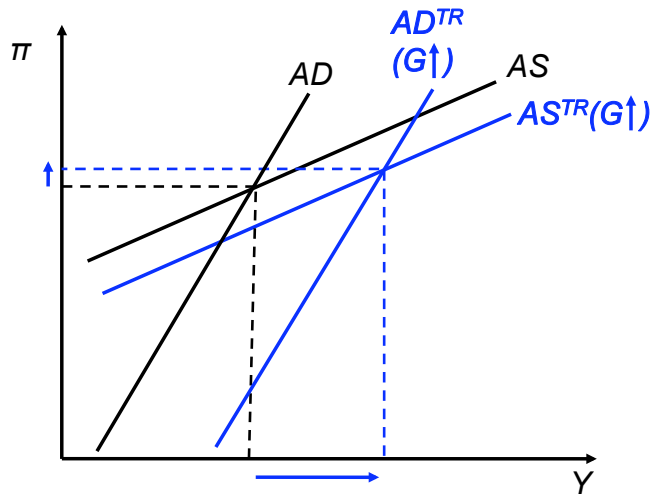
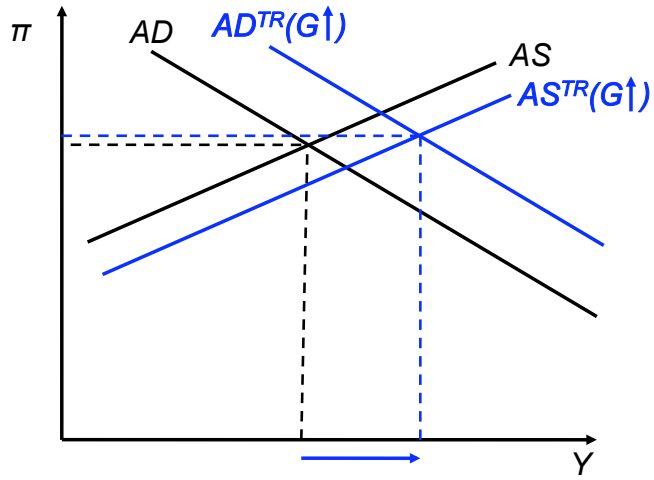
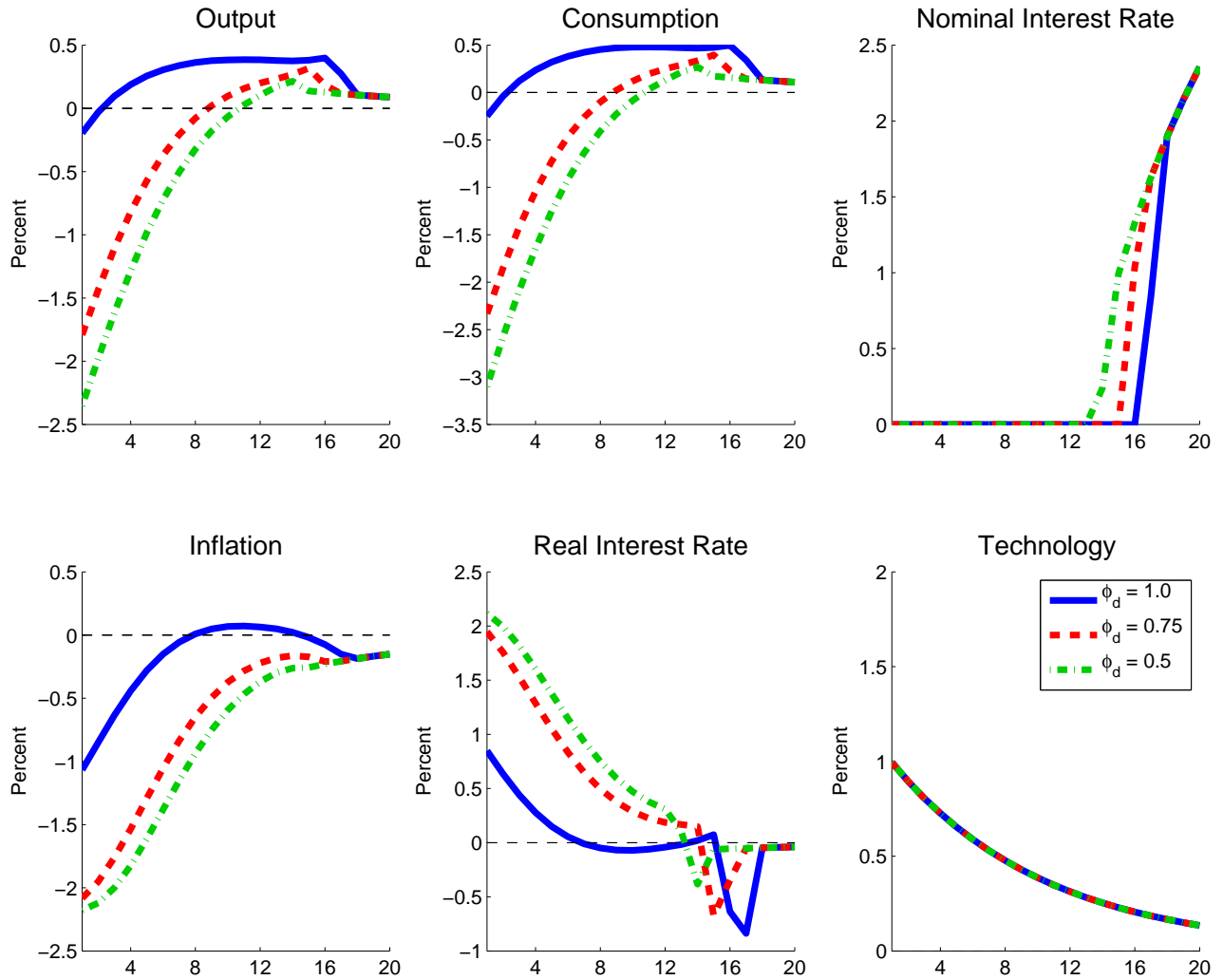


Figure 9: Model Responses Under Alternative Calibrations of History-Dependence



Note: Output, consumption, and government spending are plotted as percent deviations. The nominal interest rate is plotted in annualized percent, while the real interest rate and inflation are plotted in annualized percent deviations.