Banking Competition, Capital Accumulation, and Monetary Policy

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Abstract

In recent years, the increased concentration of activity in the banking system has received much attention. This paper studies the implications of concentration in the banking sector for economic activity. Changes in the competitive structure affect investment, risk-sharing, and social welfare. A key aspect of our analysis is that banks in more concentrated systems allocate a lot of resources towards cash reserves rather than investing in productive assets. However, perfect competition should not be a regulatory goal for the banking system. The model also demonstrates that the efficacy of monetary policy varies with the degree of concentration. Finally, the optimal money growth rule is increasing in the degree of concentration. Empirical analysis among countries in the OECD is consistent with the main predictions of the model. Such observations are important as regulators and monetary policy authorities confront the challenges of an evolving competitive landscape out of the crisis.

1 Introduction

There have been considerable changes in the concentration of the banking sector across the globe. In particular, during the recent financial crisis, a number of institutions grew so large that they were deemed "too big to fail." Furthermore, it is widely acknowledged that such institutions generally became even larger after the crisis.[1] Yet, such trends have been part of a long-term pattern of consolidation activity. In particular, the Bank for International Settlements (2001) provides a thorough review of consolidation in the banking sector across countries. In the United States, for example, there were nearly 19,000 financial institutions in 1989. Just a decade later, only 10,000 were active in the sector. Moreover, Janicki and Prescott (2006) provide evidence indicating that the largest banks in the United States (which consist of less than 1% of active institutions) held over 75% of assets in the banking system prior to the crisis. With this large of a role in economic activity, it is hard to believe that these institutions would not take advantage of their market power in the financial system.

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[1] See Gongloff et al. (2013) and Gandel (2013).

Such dramatic observations raise some very important questions for regulatory and monetary policy authorities to address. What is the optimal competitive structure of the financial system? Should perfect competition in the banking system be a regulatory goal? How does the degree of concentration affect investment and capital accumulation? Does the efficacy of monetary policy depend on the degree of concentration?

The objective of this paper is to study the implications of concentration in the banking sector for economic activity. In particular, we demonstrate that changes in the competitive structure affect the level of investment, risk-sharing, and social welfare. A key aspect of our analysis is that banks in more concentrated systems allocate a lot of resources towards money balances. That is, they have a tendency to hoard cash reserves. The implications of the competitive structure in our model lie at the core of the role of financial intermediation for economic activity. Notably, Bencivenga and Smith (1991) stress that an active intermediary sector promotes risk-pooling services and "eliminates excessive investment in unproductive liquid assets...[the absence] leads towards unfavorable levels of capital accumulation." Yet, in stark contrast, we demonstrate that more concentrated banking systems considerably deviate from this function.

In order to study the role of financial sector competition for investment and capital accumulation, we follow Bencivenga and Smith (1991) by studying an overlapping generations version of Diamond and Dybvig (1983) with production. As in Schreft and Smith (1997, 1998), limited communication and restrictions on asset portability generate a transactions role for fiat money. Physical capital and money balances are the only two assets in the economy. In contrast to Schreft and Smith, the banking sector is imperfectly competitive with fixed entry. In particular, intermediaries engage in Cournot competition in the market for capital. As a result, differing degrees of concentration in the banking sector affect the provision of risk-pooling and investment.

Notably, banks engage in strategic behavior and exploit their market power in the capital sector. That is, in contrast to perfectly competitive banks lacking market power, banks take into account that they face a downward-sloping demand for capital by firms. As a result, the industrial organization of the banking sector has serious consequences for real activity. In particular, intermediaries exploit their market power by holding back resources available to firms and devoting more funding to unproductive money balances. Consequently, highly concentrated sectors provide a large amount of insurance against liquidity risk. Thus, the model presents a trade-off between the provision of productive resources to firms and risk-pooling to depositors as the competitive structure of the banking system varies.

We turn to the questions posed above. For example, does the efficacy of monetary policy depend on the degree of concentration? Yes, but it's not straightforward. And, it doesn't bode well for the current economic climate. Monetary policy in the model produces a classic asset-substitution channel from liquid to productive assets. However, changes in the degree of concentration can render policy less effective in stimulating investment and capital accumulation. Notably, at initial levels of consolidation, increased concentration enhances the

efficacy of monetary policy because banks hold a larger amount of reserves. This would make monetary policy highly effective because it is easy to exploit the asset substitution channel of policy when income losses from inflation take place. As a result, the amount of investment would be very high. Nevertheless, as concentration increases (if institutions become very large in size) it leads to reduced efficacy because banks would respond more to the desire to exploit their market power and would acquire even more money balances at higher inflation rates as concentration increases. In this manner, increased concentration "clogs" the standard transmission channels of monetary policy. Therefore, it should not be difficult to understand why central banks have needed to resort to aggressive methods of policy intervention in recent years.

Next, should policymakers unambiguously strive for increased competition in the banking sector? Should perfect competition be a regulatory goal? No. Moreover, the optimal competitive structure depends on the amount of liquidity risk that individuals face. If the degree of liquidity risk is high, the amount of capital accumulation is unfavorably low and distortions from market power in the capital market are relatively high. Thus, an increase in competition would lead to a welfare gain as the capital stock would be significantly higher. However, there are limits to the gains from promoting competition. It is possible that the degree of competition can be too high because excessive amounts of competition would lead to less income derived from the capital sector as institutions less aggressively exploit their market power. Therefore, policymakers should carefully consider recent policy initiatives in many countries to promote competition. For example, in the U.K. there have been changes in banking requirements to lower barriers to entry in order to encourage credit funding.[2,3]

Moreover, if liquidity risk is low, a lot of resources are devoted to investment. As a result, the returns from the capital are too low and the banking sector would benefit from a high degree of concentration. That is, in some environments, a highly concentrated, monopolistic sector would produce the best social outcome.

Finally, we address the implications of the competitive structure for optimal monetary policy. Interestingly, irrespective of agents' degree of exposure to liquidity risk, the welfare-maximizing rate of money growth is inversely related to the degree of banking competition. In other words, in response to the recent trend towards concentration of firms in the banking sector, the model would call for higher rates of money growth. One of the reasons is that higher inflation rates would offset the incentives for firms with market power to hold excessively large amounts of money balances. A second reason applies to economies with a low incidence of liquidity risk where the return to capital is too low. In such an environment, the rate of return to money would be too high and some additional inflation would be valuable. In this manner, the model suggests that

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[2] Financial Services Authority (2013) describes the changes in banking requirements. One of them is a reduction in capital requirements for new entrants.

[3] Both Hannan (1991) and Corvoisier and Gropp (2002) contend that interest rates on loans are higher in markets with higher concentration ratios. Furthermore, Beck, Demirguc-Kunt and Maksimovic (2003) document that credit rationing occurs more often in concentrated banking systems.

central banks should consider the possibility of raising their inflation targets as the sector becomes increasingly concentrated towards a small number of large institutions. For example, during the past couple of years in the United States, the Federal Open Market Committee has stated that it is willing to tolerate a higher core-inflation rate.

The remainder of the paper is as follows. Section 2 provides a brief overview of the related literature. Section 3 outlines the physical environment of the model. Section 4 studies a monetary steady-state equilibrium. Section 5 addresses the welfare implications of the model, including the optimal competitive structure and the optimal monetary policy rule under different degrees of concentration of the banking sector. In addition to our analysis of behavior in the steady-state, Section 6 studies transitional dynamics to show that both the short-run and long-run effects of monetary policy are weaker in concentrated banking systems. Section 7 provides empirical analysis to study the role of banking concentration on economic activity across countries in the OECD. In particular, we find that results from regression analysis are consistent with the main predictions of our model. Finally, Section 8 concludes.

2   Related Literature

Our paper contributes to recent work which examines the implications of the industrial organization of the financial system for economic activity. In particular, Paal, Smith, and Wang (2005) construct an endogenous growth framework to study the effects of different competitive structures of the banking system for economic growth. Their model demonstrates that a monopolistic banking system may produce a higher rate of growth than a competitive banking system. The closest paper to our work is Ghossoub (2012) who studies the implications of the competitive structure for prices in capital markets.

In a model of credit market activity, Ghossoub, Laosuthi, and Reed (2012) study the effects of monetary policy and find that there are important qualitative differences in the effects of policy across competitive structures. Under perfect competition, the transmission of monetary policy is straightforward in which higher rates of money growth lead to an increase in credit market activity. However, in a distorted monopolistic setting, policy produces the opposite effect. Building on the structure of Ghossoub, Laosuthi, and Reed (2012), Matsuoka (2011) examines optimal policy across competitive structures.

All of these papers focus exclusively on financial markets under price competition. Consequently, they are unable to address the implications of changes in the degree of concentration. Importantly, we show that there are significant non-monotonicities in the relationship between the degree of competition and welfare and the relationship between the efficacy of monetary policy and the degree of concentration. Obviously, these asymmetries pose significant challenges for banking regulators and monetary policy authorities.

There are a few other papers that deserve mentioning. Ghossoub and Reed

(2013) develop a model of banks that differ in size to investigate the optimal size distribution of the banking system and the strength of the impact of monetary policy. However, in contrast to our framework, they study an endowment economy and focus on the role of credit markets to promote consumption smoothing. Laosuthi and Reed (2013a) develop a model of equilibrium entry into the banking system to show that inflation distorts the ability of institutions to promote risk-sharing and thereby deters firm entry. In this manner, inflation exacerbates distortions in the credit market due to barriers to entry.[4] Alternatively, Laosuthi and Reed (2013b) develop a model of imperfectly competitive behavior in the market for transactions-demand financial services.

There are also other noteworthy papers that introduce imperfect competition in real economies. For example, Cetorelli and Peretto (2010) construct a general equilibrium model with capital accumulation in which financial intermediaries engage in Cournot competition. However, as money does not circulate in their framework, they do not consider the connections between monetary policy and the competitive structure of the banking system. Interestingly, Corbae and D'Erasmo (2013, 2014) study dynamics of the banking sector in the presence of a size distribution of firms. In contrast to our work, their focus is to study activity across the cycle including bank failures.

Gerali et al. (2010) develop a dynamic stochastic general equilibrium model with an imperfectly competitive banking system to study the effects of shocks from the banking sector on macroeconomic activity.[5] In addition, Olivero (2010) introduces imperfect competition in the banking sector in an open economy Real Business Cycle model to study the transmission of productivity shocks. In a similar manner, Mandelman (2010) studies the propagation of shocks in an open economy New Keynesian model.

In addition to theoretical work, Cechetti (1999) and Kashyap and Stein (1997) look at how the effectiveness of policy depends on the industrial organization of the financial system. In particular, Cechetti shows that the impact of monetary policy shocks on output varies across countries. As our model shows at relatively high degrees of concentration, output responds more to monetary policy in countries with banking systems that are more competitive. Peersman's (2004) evidence focusing on countries in Europe is also consistent with the work of Cechetti and Kashyap and Stein (1997). Kashyap and Stein (2000) find that the impact of monetary policy is stronger among smaller banks in the financial system.

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[4]In a similar vein, Williamson (1986) studies a model of Cournot competition in credit markets. However, financial intermediaries do not perform risk-pooling services as in our setup.

[5]By comparison, Andreasen et al. (2013) offer a RBC model with financial intermediaties that perform maturity transformation to study the implications of productivity shocks through the banking sector.

3 Environment

Consider a discrete-time economy with two geographically separated locations or islands. Let $t = 1, 2, .., \infty$, index the time period. On each location, there are two types of agents that live for two periods: workers (potential depositors) and bankers. At the beginning of each time period, a unit mass of ex-ante identical workers and N financial intermediaries (or bankers) are born on each island. Each bank is indexed by j, where $j = 1, 2, ..., N$.

Workers are born with one unit of labor effort which they supply inelastically when young and are retired when old. In comparison to workers, bankers have no endowments. Furthermore, all agents derive utility from consuming the economy's single consumption good when old (c_{t+1}). The preferences of a typical worker are expressed by $u(c_{t+1}) = \frac{c_{t+1}^{1-\theta}}{1-\theta}$, where $\theta < 1$ is the coefficient of risk aversion.[6] On the other hand, bankers are risk-neutral agents.

There are two types of assets in this economy: money (fiat currency) and physical capital. Denote the aggregate nominal monetary base and capital stock available in period t by M_t and K_t, respectively. One unit of goods invested by a young worker in period t yields one unit of capital in $t + 1$. Moreover, at the initial date 0, the generation of old workers at each location is endowed with the initial aggregate stocks of capital and money $(K_0$ and $M_0)$. Since the population of workers is equal to one, these variables also represent their values per worker. Assuming that the price level is common across locations, we refer to P_t as the number of units of currency per unit of goods at time t.

The consumption good is produced by a representative firm using capital and labor as inputs. The production function is of the Cobb-Douglas form, $Y_t = AK_t^\alpha L_t^{1-\alpha}$, where Y_t and L_t are period t aggregate output and labor, respectively. In addition, A is a technology parameter and $\alpha \epsilon (0, 1)$ reflects the capital intensity. Further, we assume that the capital stock depreciates completely in the production process.

Following previous work such as Schreft and Smith (1997, 1998), private information and limited communication between locations require workers to use cash if they move to a different location. The communication friction provides a rationale for the circulation of fiat money – due to incomplete information across islands, individuals will not accept privately issued liabilities. Thus, in some trades between agents, only fiat money will be accepted. Moreover, workers in the economy are subject to relocation shocks. After exchange occurs in the first period, a fraction $\pi \in (0, 1)$ of agents is randomly chosen to relocate. These agents are called "movers." By comparison, individuals who remain on the same location are "non-movers." While π is known at the beginning of the

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[6]Bhattacharya et al. (1997), Smith (2003), and Ghossoub and Reed (2010) also study economies in which $\theta < 1$. Bhattacharya et al. (2009) discuss that both $\theta < 1$ and $\theta > 1$ are common in the literature. It simply depends on the application studied. As we describe in Section 7, money growth is generally associated with a Tobin effect to real economic activity in low inflation countries. This only occurs in the model if $\theta < 1$. Otherwise, the model generates the prediction that money demand is increasing in the inflation rate. Hence, our desire to study banking systems where $\theta < 1$.

period, agents are privately informed about their types at the end of period.[7]

Unlike workers, bankers are not subject to relocation shocks. Moreover, as in Diamond and Dybvig (1983), all of workers' savings are intermediated as financial intermediaries are able to provide their depositors with insurance against idiosyncratic liquidity risk. Each banker j allocates its deposits into money and capital. By construction, there is a fixed number of firms in the banking sector. Moreover, in contrast to previous work such as Schreft and Smith, banks are Cournot competitors in capital markets.

The final agent in this economy is a government (or central bank) that adopts a constant money growth rule. The real value of money balances held by a given intermediary is denoted as m_t. However, there are a total of N imperfectly competitive institutions active in each location. Therefore, the real *aggregate* money stock in period t is $\tilde{m}_t = N m_t$. The evolution of real money balances between periods $t-1$ and t is expressed as:

$$\tilde{m}_t = \sigma \frac{P_{t-1}}{P_t} \tilde{m}_{t-1} \tag{1}$$

where $\sigma > 0$ is the constant gross rate of money creation (or destruction when $\sigma < 1$) and $\frac{P_{t-1}}{P_t}$ is the gross rate of return on money balances between period $t-1$ and t. The government rebates seigniorage income to young workers through lump-sum transfers. Denote the total amount of transfers at the beginning of period t by τ_t, where

$$\tau_t = \frac{\sigma - 1}{\sigma} \tilde{m}_t \tag{2}$$

We now describe the timing of events that occur in every period. At the beginning of each date, a new generation of agents is born. Workers provide labor effort and banks supply capital to firms for production. Subsequently, workers receive wages and banks obtain their rental earnings. Banks pay returns from capital to non-movers. Workers follow by depositing labor income with intermediaries.

The process of intermediation continues as banks acquire assets. First, old movers show up at banks to acquire consumption goods in exchange for their holdings of money balances. Banks accept the money so they can insure the new generation of depositors against relocation shocks. Thus, intermediaries facilitate intergenerational trade.

Banks acquire additional money balances by trading with the monetary authority. Based upon its money growth rule, the central bank injects additional currency into the banking system. With their remaining deposit income, banks invest in capital.

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[7]There are also numerous papers in which the relocation probability is stochastic. Models generally introduce aggregate uncertainty in in order to study banking crises and lender of last resort policies. For example, see Antinolfi, Huybens and Keister (2001) who show that introducing a discount window may achieve complete risk sharing but also subjects the economy to a continuum of inflationary equilibria. In addition, Boyd, de Nicoló, and Smith (2004) study the probabilities of banking crises across monopolistic and perfectly competitive banking systems.

Finally, depositors learn their location status for the next date. Individuals who learn they will transact in the opposite location liquidate their deposits to obtain cash. After relocation takes place, the period ends.

4 Trade

4.1 Factor Markets

In period t, a representative firm rents capital and hires workers in perfectly competitive factor markets at rates r_t and w_t, respectively. The inverse demand functions for labor and capital by a typical firm are expressed by:

$$w_t = (1 - \alpha) A K_t^{\alpha} L_t^{-\alpha} \tag{3}$$

and

$$r_t = \alpha A K_t^{\alpha-1} L_t^{1-\alpha} \tag{4}$$

where $L_t = 1$ in equilibrium.

4.2 A Typical Worker

In period t, each worker receives τ_t units of goods in real injections from the government. The injections are combined with labor income (w_t) to constitute total income. Given that agents only value old agent consumption, all income is saved. Furthermore, as agents are subject to relocation shocks, all savings are intermediated.

4.3 A Typical Bank's Problem

At the beginning of period t, each banker announces deposit rates taking the announced rates of return of other banks as given. A bank promises a gross real return on deposits, r_t^m if a young individual is relocated and a gross real return r_t^n if not. Given that banks offer similar financial services, each bank receives the same market share in the deposit market, attracting $1/N$ depositors. Each bank allocates its deposits towards cash reserves and capital goods. Let m_t and k_{t+1}, respectively, denote the real amount of cash balances and capital goods held by each bank.[8]

Furthermore, unlike previous work such as Schreft and Smith (1997) and Ghossoub (2012), the rental market is characterized by Cournot (quantity) competition. That is, each bank recognizes that its own decisions about the amount of capital supplied will affect the market rental rate but that its choice does not affect that of other banks.

In equilibrium, price competition among banks for depositors will force them to choose return schedules and portfolio allocations to maximize the expected utility of a representative depositor. A bank's objective function is:

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[8] Given that banks solve the same problem, we omit the indexation for each bank.

$$\underset{r_t^m, r_t^n, m_t, k_{t+1}}{Max} \frac{\pi \left[r_t^m \left(w_t + \tau_t\right)\right]^{1-\theta} + \left(1 - \pi\right) \left[r_t^n \left(w_t + \tau_t\right)\right]^{1-\theta}}{1 - \theta} \qquad (5)$$

subject to the following constraints.

First, a bank's balance sheet at the beginning of period t is expressed by:

$$\frac{1}{N} \left(w_t + \tau_t\right) = m_t + k_{t+1} \qquad (6)$$

Furthermore, as relocated agents need cash to transact, total payments made to movers satisfy:

$$\frac{\pi}{N} r_t^m \left(w_t + \tau_t\right) = m_t \frac{P_t}{P_{t+1}} \qquad (7)$$

As we choose to study equilibria in which money is dominated in rate of return, banks will not hold excess reserves. A bank's total payments to non-movers are therefore paid out of its revenue from renting capital to firms in $t + 1$. The constraint on payments to non-movers is such that:

$$\frac{1 - \pi}{N} r_t^n \left(w_t + \tau_t\right) = r \left(K_{t+1}\right) k_{t+1} \qquad (8)$$

In addition, the contract between a bank and its depositors needs to be incentive compatible to prevent agents from misrepresenting their realizations of the relocation shock. That is:

$$r_t^m \le r_t^n \qquad (9)$$

Finally, as stated above, each bank faces the market's inverse demand for capital:

$$r \left(k_{t+1}\right) = \alpha A \left(K_{t+1} \left(k_{t+1}\right)\right)^{\alpha-1} L_{t+1}^{1-\alpha} \qquad (10)$$

where $K_{t+1} = \sum_{j=1}^{N} k_{t+1}^j$.

The role of market power by an intermediary enters in the choice of investment by an individual firm, k_{t+1}. Under a perfectly competitive capital market, intermediaries do not have any market power. Consequently, the marginal revenue from supplying resources to the capital market would simply be equal to r_{t+1}. For example, the rental rate represents marginal income earned from investment in Schreft and Smith (1997).

However, in an imperfectly competitive market, each intermediary is aware that they face a downward-sloping demand curve for capital. As a result, marginal revenue is given by $r' \left(K_{t+1}\right) k_{t+1} + r \left(K_{t+1}\right)$. Therefore, due to the distortions from market power, the marginal income earned in the capital market is lower than under perfect competition.

The solution to the problem yields the demand for money balances by a single financial institution:

$$m_t = \frac{\frac{w_t + \tau_t}{N}}{1 + \frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right]^{\frac{1}{\theta}} I_t^{\frac{1-\theta}{\theta}}} \tag{11}$$

where $I_t = r_{t+1}\frac{P_{t+1}}{P_t}$ is the nominal return to capital between period t and $t+1$.

Equivalently, each bank allocates a fraction (γ_t) of its deposits towards cash reserves:

$$\gamma_t = \frac{1}{1 + \frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right]^{\frac{1}{\theta}} I_t^{\frac{1-\theta}{\theta}}} \tag{12}$$

For a given level of deposits, the demand for money balances is strictly decreasing in the return to capital. As a higher rate of money growth would be expected to raise the nominal return, the model generates a standard Tobin asset-substitution channel of monetary policy. That is, higher nominal returns induce intermediaries to substitute out of liquid assets.

However, standard asset substitution arguments generally do not recognize the limitations from imperfectly competitive behavior in physical asset markets. Notably, $\left[1 - \frac{1-\alpha}{N}\right] \in [0, 1]$ reflects the degree of distortions in financial markets stemming from imperfect competition. In particular, banks hold more liquid portfolios when the banking system becomes less competitive (lower N). As a benchmark, under a perfectly competitive banking system ($N \to \infty$), the money demand equation would resemble that in Schreft and Smith (1998).

Substituting (6) and (11) into (7) and (8), the relative return to depositors is such that:

$$\frac{r_t^n}{r_t^m} = \left[1 - \frac{(1-\alpha)}{N}\right]^{\frac{1}{\theta}} I_t^{\frac{1}{\theta}} \tag{13}$$

Given that banks hold relatively less cash when the banking system becomes more competitive, this translates into lower insurance against liquidity risk to their depositors. That is, the distortions from market power and concentration in the capital market lead to higher amounts of risk-sharing provided to depositors. In this manner, the degree of concentration of the banking system would involve a trade-off between capital accumulation and risk-sharing.

Analogously, depositors receive less insurance when the return to capital is higher. Moreover, it is evident from (13) that the incentive compatibility constraint holds when the nominal return is above some threshold level, $\underline{I} = \frac{1}{1 - \frac{(1-\alpha)}{N}}$. Unlike standard random relocations models with perfect competition, the lower bound on the nominal net return to capital is above zero. That is, a banking equilibrium is sustainable when

$$I_t \geq \underline{I} = \frac{1}{1 - \frac{(1-\alpha)}{N}} > 1 \tag{14}$$

In this manner, the Friedman rule where money and capital yield the same real return cannot support a banking equilibrium. Such an equivalence is not possible in the presence of distortions from market power among intermediaries.

4.4   General Equilibrium

In equilibrium, all markets will clear. In particular, labor receives its marginal product, (3), and the labor market clears with $L_t = 1$. Furthermore, using (2) and (11), the demand for money balances by one financial intermediary can be expressed as:

$$m_t = \frac{w_t}{N \left[1 + \frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right]^{\frac{1}{\theta}} I_t^{\frac{1-\theta}{\theta}} - \frac{\sigma-1}{\sigma} \right]} \tag{15}$$

where I_t satisfies (14). Aggregating over the banking sector, the total demand for money in period t is $\tilde{m}_t = Nm_t$. Upon substituting (15) into the expression for the evolution of money, (1), equilibrium in the money market generates the evolution of the nominal return to capital:

$$I_{t+1} = \left(\frac{\left[\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right]^{\frac{1}{\theta}} I_t^{\frac{1-\theta}{\theta}} + \frac{1}{\sigma} \right] \frac{1}{\sigma} \frac{I_t}{\alpha A} \frac{K_{t+1}}{K_t^{\alpha}} - \frac{1}{\sigma}}{\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right]^{\frac{1}{\theta}}} \right)^{\frac{\theta}{1-\theta}} \tag{16}$$

The price level, P_{t+1}, may be obtained from the following goods market clearing condition:

$$Y_t = K_{t+1} + r_t K_t + \frac{M_{t-1}}{P_t}$$

which can be related to the growth rate of money:

$$P_t = \frac{\sigma^{t-1} M_0}{Y_t - r_t K_t - K_{t+1}} \tag{17}$$

Furthermore, the total amount of capital traded in a particular period t is $K_t = Nk_t$. Using (2), (6), and (15), the law of motion of capital (supply of capital in $t + 1$) is:

$$K_{t+1} = \left[1 - \frac{1}{\sigma} \frac{1}{\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right]^{\frac{1}{\theta}} I_t^{\frac{1-\theta}{\theta}} + \frac{1}{\sigma}} \right] (1 - \alpha) A K_t^{\alpha} \tag{18}$$

Finally, using (4) and the definition of I_t, the inverse demand for capital is such that:

$$I_t = \frac{P_{t+1}}{P_t} \alpha A K_{t+1}^{\alpha-1} \tag{19}$$

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The loci defined by $(16) - (19)$ characterize the behavior of the economy over time. Equations (16) and (18) govern the supply of capital while (19) represents the inverse demand. However, the main focus of this manuscript is on the steady-state behavior of the economy which we address in the following section.

4.5 Steady-State Analysis

Imposing steady-state on the evolution of I, (16), the steady-state gross inflation rate is pinned down by the rate of money growth with $\frac{P_{t+1}}{P_t} = \sigma$. Furthermore, from (18), the steady-state aggregate supply of capital, K^S is such that:

$$K^S = \left((1 - \alpha) A \left[1 - \frac{1}{\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right]^{\frac{1}{\theta}} I^{\frac{1-\theta}{\theta}} \sigma + 1} \right] \right)^{\frac{1}{1-\alpha}} \tag{20}$$

while from (19), the demand demand for capital, K^D is:

$$K^D = \left(\frac{\sigma \alpha A}{I} \right)^{\frac{1}{1-\alpha}} \tag{21}$$

The two loci above characterize the stationary behavior of the economy. It is trivial to show that banks economize on cash holdings when the return to capital is higher. Therefore banks supply more capital when its return increases as illustrated in Figure 1 below. By comparison, firms demand less capital goods when the cost of capital is higher. We provide a full characterization of (20) and (21) in the following Lemma.

Lemma 1.
    a. The locus defined by (20) satisfies: $\frac{dK}{dI} > 0$, $\lim_{I \to \infty} K \to [(1 - \alpha) A]^{\frac{1}{1-\alpha}}$,
and $K = \left[(1 - \alpha) A \left(1 - \frac{1}{\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right] \sigma + 1} \right) \right]^{\frac{1}{1-\alpha}} \equiv \underline{K}^S$ *when* $I = \underline{I}$.
    b. The locus defined by (21) satisfies: $\frac{dK}{dI} < 0$, $\lim_{I \to \infty} K \to 0$, $\lim_{I \to 0} K \to \infty$,
and $K = \left(\sigma \alpha A \left[1 - \frac{1-\alpha}{N} \right] \right)^{\frac{1}{1-\alpha}} \equiv \bar{K}^D$ *when* $I = \underline{I}$.

We proceed to establish existence and uniqueness of steady-state equilibria in the following proposition.

Proposition 1. *Suppose* $\sigma \geq \underline{\sigma} = \frac{\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi}}{1 - \frac{(1-\alpha)}{N}}$. *Under this condition, a steady-state banking equilibrium exists and is unique.*

Given the characterization of (20) and (21), both loci always intersect once. However, from our discussion of the self-selection constraint, (13), a banking equilibrium exists if the equilibrium nominal return to capital (I^*) is above the threshold level $\underline{I} > 1$ to prevent depositors from misrepresenting their types (a

mover or non-mover) ex-post. Therefore, as illustrated in Figure 1 below, an excess demand for capital has to occur at $\underline{I}$. This takes place when inflation is above a threshold level, $\underline{\sigma}$.
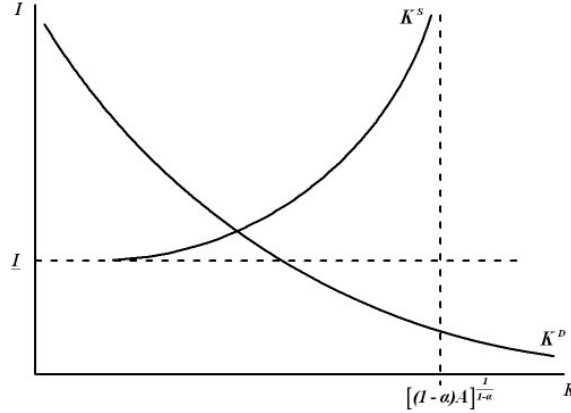

Figure 1: Equilibrium in the Capital Market

We proceed to examine how the degree of banking competition affects the economy in the following Proposition.

Proposition 2.
i. $\frac{d\gamma}{dN} > 0$, $\frac{dK}{dN} > 0$, $\frac{dR}{dN} < 0$, $\frac{d\frac{r^n}{r^m}}{dN} > 0$.
ii. $\frac{dk}{dN} \geq (<)\, 0$ $\textit{if}$ $N \leq (>)\, \hat{N}$.

A change in the number of financial intermediaries has two primary direct effects on banks' portfolios. First, when the degree of concentration in capital markets weakens, each intermediary recognizes that it has less market power and therefore allocates a larger fraction of its deposits towards capital investment. The change in the composition of banks' portfolios towards less liquid assets results in less insurance to depositors against random relocation shocks.

In turn, at the aggregate level, an increase in competition stimulates capital investment. The higher stock of capital puts downwards pressure on its return and upward pressure on wages and deposits. The increase in deposits further contributes towards capital accumulation.

However, there is a second, opposing factor which hinders the amount of capital investment by an individual institution as concentration is lower. Although the entire deposit base is larger under a more competitive financial sector, each bank receives a smaller share of the deposit base. The reduced access to aggregate deposits interferes with a bank's ability to expand its balance sheet.

Consequently, when the sector is initially highly concentrated (small N), the increase in competition is dominated by the increase in aggregate deposits. That is, each bank's balance sheet can expand and allocate more funding towards

capital investment. As a result, both investment by an individual intermediary (k) and total investment (K) increase if competition improves from a highly concentrated sector.

However, once the degree of competition rises above some threshold level, $\hat{N}$, competition crowds out banks' access to deposit funding. Due to the lower amount of access, individual investment would be decreasing in the competitive structure but aggregate investment is higher. The aggregate result deviates from the individual result because each institution allocates more funding to capital as a fraction of deposits received. This occurs because the pricing distortion $\left[1 - \frac{1-\alpha}{N}\right]$ is weaker as N is higher.

In the following Proposition, we discuss the effects of monetary policy.

Proposition 3. $\frac{dK}{d\sigma} > 0$, $\frac{dR}{d\sigma} < 0$, $\frac{dI}{d\sigma} > 0$, $\frac{d\frac{r^n}{r^m}}{d\sigma} > 0$.

The intuition behind the result in Proposition 3 is simple. In this environment, monetary policy operates through two primary channels. First, a higher rate of money creation lowers the value of money which drives banks to allocate a larger fraction of their deposits towards real capital goods. This is a standard asset-substitution channel. Second, depositors receive a higher amount of transfers from the government when money creation expands. This raises the amount of deposits and the size of banks' balance sheets. As more capital is available to rent, its real return falls.

Furthermore, driven by higher inflation, the nominal return to capital is higher. Finally, as banks hold less liquid portfolios, they provide less insurance to their depositors against relocation shocks. Using (20) and (21), and differentiating with respect to σ, we get the following expression for the effects of monetary policy on capital formation:

$$\frac{dK}{d\sigma} = \frac{K}{(1-\alpha)\,\theta\sigma\left[\frac{1}{1-\Omega(K(N))} + \frac{1-\theta}{\theta}\right]} > 0 \qquad (22)$$

where $\Omega\left(K(N)\right) = \frac{(K(N))^{1-\alpha}}{(1-\alpha)A}$ is the capital-wage ratio. We next move to studying the role of concentration in the monetary transmission channels previously discussed.

How do the effects of monetary policy depend on the degree of concentration of the financial sector? Does an increase in concentration render monetary transmission to be more or less effective in stimulating investment? Upon differentiating (22) with respect to N, we get the following result:

Proposition 4. $\frac{d}{dN}\left(\frac{dK}{d\sigma}\right) \geq (<)\,0$ *for all* $N \leq (>)\,\tilde{N}_0$, *where* $\tilde{N}_0 : 1 + \frac{1-\theta}{\theta}\left(1 - \Omega\left(K(N)\right)\right) - \frac{(1-\alpha)\Omega(K(N))}{(1-\Omega(K(N)))} = 0$.

Notably, the relationship between the efficacy of monetary policy and the capital stock is not straightforward. In particular, it is non-monotonic. This

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creates a serious challenge for policymakers and banking regulators. Understanding the connections can be observed from:

$$\frac{d}{dN}\left(\frac{dK}{d\sigma}\right) = \frac{\left(\frac{1}{(1-\Omega(K))}\right)\left[1 + \frac{1-\theta}{\theta}\left(1 - \Omega\left(K(N)\right)\right) - \frac{(1-\alpha)\Omega(K(N))}{(1-\Omega(K(N)))}\right]}{(1-\alpha)\sigma\theta\left[\frac{1}{1-\Omega(K(N))} + \frac{1-\theta}{\theta}\right]^2}\left(\frac{dK}{dN}\right)$$

In particular, the efficacy of monetary policy as the degree of concentration changes is proportional to the impact of competition on capital accumulation in the economy. In addition, we have already shown that the aggregate capital stock is higher if the banking sector is more competitive. So, $\frac{dK}{dN} > 0$. Yet, the sign of $\frac{d}{dN}\left(\frac{dK}{d\sigma}\right)$ depends on $1 + \frac{1-\theta}{\theta}\left(1 - \Omega\left(K(N)\right)\right) - \frac{(1-\alpha)\Omega(K(N))}{(1-\Omega(K(N)))}$. Recall that $\Omega\left(K(N)\right)$ is the capital-wage ratio. The inverse, $1 - \Omega\left(K(N)\right)$, reflects the relative amount of money balances that are held by a bank. The ability of monetary policy to promote capital accumulation depends on the severity of the distortion from market power in the capital sector. For highly concentrated economies, the level of the capital stock is likely to be very low.

The twin distortions from inefficient levels of capital accumulation and excessive risk pooling complicate the relationship between the efficacy of monetary policy and the degree of concentration of the banking sector. If the sector initially has a relatively large number of firms and consolidates, Proposition 4 suggests that monetary policy would be more effective. As long as the sector is not too concentrated, banks' money holdings would be fairly extremely high as each institution distorts activity in the capital sector. Therefore, at moderate levels level of money holdings, the income losses from inflation would take effect. As a result, there is a strong role for inflation to promote investment and capital accumulation. That is, *the asset substitution channel of monetary policy works well if intermediaries have relatively high levels of money balances.*

Alternatively, in a highly concentrated economy, further increases in concentration would lead to increased hoarding of money balances even if inflation were to rise. In such systems, banks internalize that they have a lot of market power. As a result, the impact of monetary policy on economic activity is weaker. Thus, asset substitution arguments may not apply if the sector consolidates from a sector with a small number of large institutions. In such financial systems, banks would be "too big" for standard asset substitution channels of policy to be effective.

5 Welfare Analysis

We proceed to examine how the degree of competition and monetary policy affect economic welfare. In particular, we study the interaction between optimal

monetary policy and banking competition. Following previous work such as Williamson (1986) and Ghossoub and Reed (2010), we use the expected utility of a typical generation of depositors as a proxy for welfare.

As we demonstrate in the appendix, the expected utility of a typical depositor in the steady-state can be expressed as:

$$
u = \frac{\pi^{\theta}}{1-\theta} \left[\frac{1}{\frac{1-\pi}{\pi} (\alpha A)^{\frac{1-\theta}{\theta}} \left[1 - \frac{(1-\alpha)}{N} \right]^{\frac{1}{\theta}}} \right]^{1-\theta} \frac{K^{\frac{[1-\alpha+\alpha\theta](1-\theta)}{\theta}}}{\sigma^{\frac{1-\theta}{\theta}}} + \frac{(1-\pi)^{\theta} [\alpha A]^{1-\theta}}{1-\theta} K^{\alpha(1-\theta)}
$$

(23)

We begin by answering the following question. *Does more competition in the banking sector promote welfare?* The following Proposition sheds some light on this issue.

Proposition 5. *Suppose $\pi > \hat{\pi}$, where $\hat{\pi} = (1-\alpha) \left[1 - \left(\frac{\alpha}{1-\alpha} \right)^2 \right]$. Under this condition, $\frac{du}{dN} \geq (<) 0$ if $N \leq (>) \tilde{N}_1$. By comparison, suppose $\pi < \hat{\pi}$. Under this condition, $\frac{du}{dN} < 0$.*

Proposition 5 provides an interesting result and continues to point out the problems regarding banking regulation. The welfare effects of increasing the degree of competition are far from transparent. Should policymakers aim to promote competition in the financial system? Not necessarily – the model demonstrates that there can be "too much competition" in the banking sector. Engineering the optimal degree of concentration in the financial sector is likely to be a challenge.

Notably, the welfare effects of the degree of concentration in the financial sector hinge on the degree of liquidity risk. As we explain below, the degree of liquidity risk ties into the allocation of funding to the capital market. Moreover, the severity of distortions in the capital market depend on the degree of concentration.

From our discussion of Proposition 3, as the banking system becomes more competitive, banks hold less liquid portfolios and provide less insurance to their depositors. The reduction in insurance against liquidity risk would adversely affect their welfare.

However, an increase in competition leads to a higher aggregate capital stock as the distortions from market power in the capital sector are lower. The increase in wages resulting from a higher stock of capital promotes welfare.

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Given this trade-off between higher income and less risk sharing, the net effect of an increase in the competitive structure depends on the degree of banking concentration and the need for liquidity in the economy.

In particular, if the degree of liquidity risk is high, capital investment is inefficiently low and agents' wage income is also low. In this type of environment, concentration in the financial sector aggravates problems in the economy because firms hold back more resources from the capital market. One might view that this problem is one of the reasons that central banks had to resort to unconventional monetary policies during and immediately following the crisis. For example, see commentary by Fisher and Rosenblum (2009) and Rosenblum et al. (2010). Therefore, the marginal gains in income that come about from higher competition in the banking sector are significant. As a result, higher degrees of banking competition stimulate welfare if the sector is highly concentrated.

As an example, such a setting would apply to economies in which there is a lot of risk in the labor market. In economies with high unemployment rates, the risk of job destruction and employment loss is high. As a result, individuals are more likely to have their savings plans interrupted and to need to liquidate their asset holdings. In particular, Ghossoub and Reed (2010) develop a framework in which the incidence of liquidity risk responds to the overall level of income in the economy. They argue that there is less liquidity risk in economies with more economic activity. Analogously, as we explain below, the model indicates that economies with low risk would benefit from consolidation because banks over-invest in productive resources.

As the banking system becomes more competitive, less risk sharing is provided to depositors since the slope of the demand curve for capital is less steep if banks have less market power. Once the number of firms crosses a threshold level, $\tilde{N}_1$, the gains in income are outweighed by the loss of risk sharing because banks do not internalize how their portfolio choices affect the rental rate for capital as aggressively in the presence of more firms. In this manner, there can be excessive levels of competition in the banking system.

In contrast, if the need for liquidity in the economy is low, workers (depositors) earn high wages as the capital stock is relatively large. However, the high amount of capital accumulation suppresses the income earned from the capital market. In order to boost the income earned from capital, depositors would be better off if the sector became less competitive. That is, higher degrees of concentration are favored in environments with low liquidity risk. A highly concentrated, monopolistic banking system may produce the best social outcome.

In summary, the welfare effects of the degree of concentration largely depend on the degree of liquidity risk. If liquidity risk is low, a lot of resources are devoted to investment. The capital stock is very high as a result. Welfare would be higher if the sector were highly concentrated so that firms would aggressively internalize their impact on returns in the capital market. In stark contrast, promoting competition can be socially valuable in economies with high liquidity risk. Promoting competition would alleviate distortions from market power which lead to a low capital stock. Wages and deposit funding would also

improve.

We proceed to study the effects of monetary policy on welfare. Proposition 5 provides an answer to the following two questions: What rate of money growth maximizes social welfare? More importantly, how does optimal monetary policy vary with the degree of banking competition?

Proposition 6. *Suppose* $\pi > \tilde{\pi} = 1 - \left(\frac{\alpha}{1-\alpha}\right)^2$. *Under this condition,* $\frac{du}{d\sigma} \geq (<) \, 0$ *for all* $\sigma \leq (>) \, \tilde{\sigma}$, *and* $\sigma^* = \tilde{\sigma} > \underline{\sigma}$. *By comparison, suppose* $\pi \leq \tilde{\pi}$, *we have* $\frac{du}{d\sigma} < 0$ *and* $\sigma^* = \underline{\sigma}$. *Finally, under both cases, we have* $\frac{d\sigma^*}{dN} < 0$.

From our discussion of Proposition 3 in the previous section, higher inflation rates stimulate capital formation and wages which can promote welfare. However, depositors receive less insurance against idiosyncratic risk when inflation is higher. As we demonstrate in the appendix, the latter effect dominates and inflation adversely affects total welfare when the probability of relocation is relatively low.

Following from the analysis of Proposition 5, banks heavily distort the capital market in economies with large amounts of liquidity risk. Consequently, inflation can play an important role in alleviating the severity of the distortion through exploiting a Tobin-type asset substitution channel. However, once the inflation rate passes the threshold σ^*, the welfare gains from additional capital accumulation are outweighed by losses from less risk-sharing.

By comparison, we obtain a different result in economies with low levels of liquidity risk. As the capital stock is favored when liquidity risk is low, the marginal gains in capital formation that result from a lower value of money are small compared to the loss in risk sharing. Therefore, the optimal rate of money creation is the lower bound required for existence in Proposition 1. If the rate of money creation is too low, money would not be dominated in rate of return because the rental rate in the capital sector would also be too low. Consequently, the optimal inflation rate is the rate which achieves the highest level of risk-sharing that is possible in the presence of distortions from imperfectly competitive institutions.[9]

Interestingly, irrespective of agents' degree of exposure to liquidity risk, the welfare-maximizing rate of money growth is inversely related to the degree of banking competition. In response to the recent trend towards concentration of firms in the banking sector, the model would call for higher rates of money growth. In order to offset the incentives for firms with market power to hold excessively large amounts of money balances, the optimal policy for a central bank is to raise the opportunity cost of holding reserves. Higher inflation rates are needed for two possible reasons: (1) one possibility is that the costs of holding money balances need to be raised so that intermediaries allocate more funding towards physical productive assets or (2) the return to capital is too low which provides institutions with incentives to hoard cash (because the rate of return

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[9] As explained in the discussion of Proposition 1, the Friedman Rule cannot be achieved under an imperfectly competitive financial sector.

to money is too high). Therefore, central banks should consider the possibility of raising their inflation targets as the sector becomes increasingly concentrated towards a small number of large institutions. For example, during the past couple of years in the United States, the Federal Open Market Committee has stated that it is willing to tolerate a higher core-inflation rate.

6   Stability Analysis

While the previous section focuses on the stationary behavior of the economy, we proceed to study the transitional dynamics. We first derive a phase diagram to examine the global information on the stability properties of the steady-state. Subsequently, we study the local stability properties of the system in the neighborhood of the steady-state equilibrium.

6.1   A Phase Diagram

The dynamic behavior of the economy is summarized by the system of equations, (16) and (18). Moreover, it can be easily verified that the laws of motion for capital and the nominal return to capital can be respectively expressed as:

$$\Delta K_t = K_{t+1} - K_t = \left[1 - \frac{1}{\sigma} \frac{1}{\left[\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right]^{\frac{1}{\theta}} I_t^{\frac{1-\theta}{\theta}} + \frac{1}{\sigma} \right]} \right] (1-\alpha) A K_t^{\alpha} - K_t \tag{24}$$

and

$$\Delta I_t = I_{t+1} - I_t = \frac{\left(\frac{1-\alpha}{\alpha} I_t^{\frac{1}{\theta}} - \frac{1}{\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right]^{\frac{1}{\theta}}} \right)^{\frac{\theta}{1-\theta}}}{\sigma^{\frac{\theta}{1-\theta}}} - I_t \tag{25}$$

In addition, the locus defined as $\Delta K_t = 0$ satisfies: $\frac{dI_t}{dK_t} > 0$, $\lim_{I_t \to \infty} K_t \to [(1-\alpha) A]^{\frac{1}{1-\alpha}}$, and $K_t = \frac{[(1-\alpha)A]^{\frac{1}{1-\alpha}}}{\left(1 + \frac{1}{\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right] \sigma} \right)^{\frac{1}{1-\alpha}}}$ when $I_t = \frac{1}{1 - \frac{(1-\alpha)}{N}}$. As illustrated in Figure 2 below, the capital stock is rising (falling) over time if the economy lies above (below) the $\Delta K_t = 0$ locus. By comparison, the locus $\Delta I_t = 0$ is such that $\frac{dI_t}{dK_t} = 0$ and has the configuration illustrated in Figure 2.

It is clear from the Figure that the steady-state is saddle-path stable. In particular, for a given rate of money creation and any initial value of the capital

stock, there exists a unique value of the nominal return to capital that brings the economy to its steady-state on the stable manifold. By definition of I_t, (19), as the economy converges to its steady-state level at E_1 from an initial stock of capital, K_0, inflation monotonically increases to its stationary level, σ. In this manner, the short run relationship between inflation and the real level of economic activity is consistent with our long-run analysis.
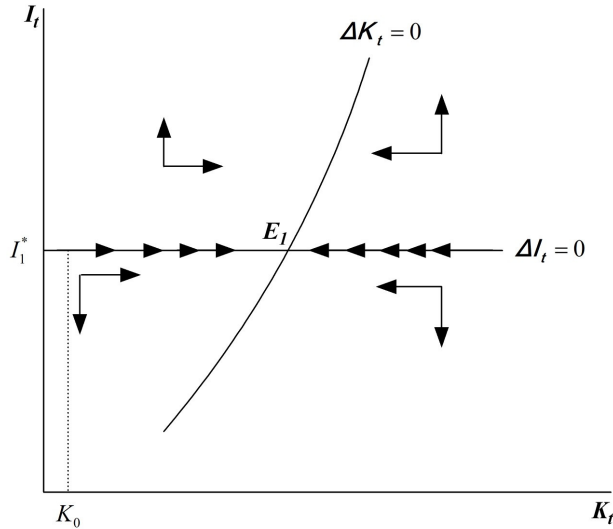

Figure 2. Transitional Dynamics

We proceed to examine the short-run effects of monetary policy. To begin, it is trivial to show that the $\Delta K_t = 0$ locus shifts to the right under a higher rate of money growth. This reflects the short-term growth effect from higher money growth for a given I.

Furthermore, the $\Delta I_t = 0$ locus shifts up under a higher rate of money growth, indicating that the nominal return to capital has to increase in order to maintain a constant interest rate at a higher value of σ. The impact of an expansionary monetary policy is illustrated in Figure 3 below. Overall, the economy initially jumps from E_1 to E_2 and asymptotically reaches the new steady-state, E_3, using the stable manifold.

A higher rate of money creation raises the amount of real money balances in $t+1$. In turn, this causes inflation to rise and the nominal return to capital to jump. In addition, at the level of capital stock corresponding to E_2, a higher nominal return to capital discourages banks from holding money balances which stimulates capital formation and wages in $t+1$.[10] In this manner, the economy converges monotonically to its new resting point, E_3.

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[10] Inflation initially rises by less than the rate of money growth. As the economy growth to its new resting point, inflation will converge monotonically to its new long-run level that corresponds to the higher rate of money growth.
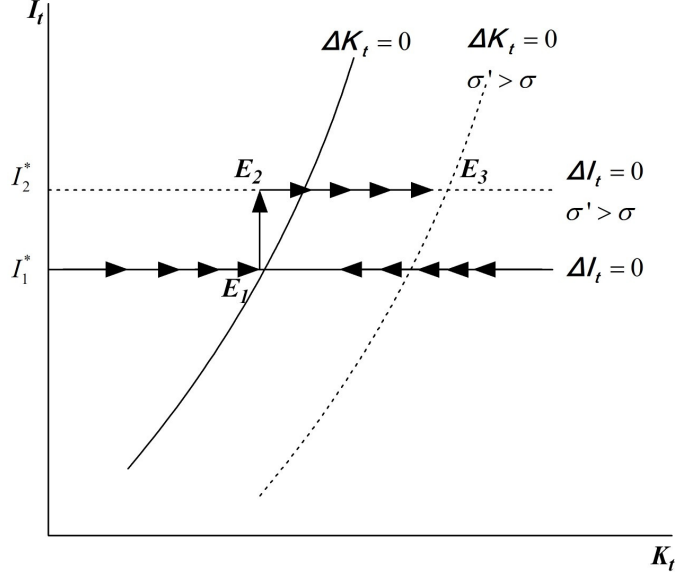
Figure 3. The Short-Run Effects of an Expansionary Monetary Policy

In order to show how the short-run effects of monetary policy depend on the degree of concentration, we begin by considering the dynamic behavior of the capital stock from the law of motion of the capital stock in equation (24):

$$\frac{K_{t+1}}{K_t} = \left[1 - \frac{1}{\frac{(1-\alpha)}{\alpha} \frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right]^{\frac{1}{\theta}} I_t^{\frac{1}{\theta}}} \right] (1-\alpha) A K_t^{\alpha-1}$$

However, as shown in the phase diagram, we know that the nominal interest rate immediately adjusts in response to a change in monetary policy. Therefore, we impose that $\Delta I_t = 0$ on the adjustment path for the capital stock:

$$\frac{K_{t+1}}{K_t} = \frac{\sigma}{I_t} \alpha A K_t^{\alpha-1}$$

It is easy to see that an increase in the rate of money growth will cause the capital stock to increase along the transition to the new steady-state. It can also be observed from the $\Delta I_t = 0$ locus that the nominal interest rate would be higher in a more concentrated sector. Therefore, controlling for the level of economic activity (i.e., the capital stock), the short-run effect of an increase in the money growth rate will be stronger if the banking sector is more competitive.

In this manner, we can see that *both the short-run and long-run effects of monetary policy are weaker if the banking sector is more concentrated.*

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6.2 Local Dynamics

We proceed to study the local stability properties of the economy. From our global analysis above, the dynamic behavior of the economy is summarized by the system of equations, (16) and (18). The stability properties of a steady state are generated from the eigenvalues of the Jacobian matrix J,

$$J = \begin{bmatrix} \frac{\partial K_{t+1}}{\partial K_t} & \frac{\partial K_{t+1}}{\partial I_t} \\ \frac{\partial I_{t+1}}{\partial K_t} & \frac{\partial I_{t+1}}{\partial I_t} \end{bmatrix}_{ss}$$

where the subscript "ss" designates values evaluated in the steady-state. Denote the determinant and trace of J by D and T, respectively.

Moreover, the discriminant, Δ, is $\Delta = T^2 - 4D$. The elements of the Jacobian are given by:

$$\frac{\partial K_{t+1}}{\partial K_t}\Big|_{ss} = \alpha > 0 \tag{26}$$

$$\frac{\partial K_{t+1}}{\partial I_t}\Big|_{ss} = \frac{\frac{1-\theta}{\theta}}{\left[\sigma \frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N}\right]^{\frac{1}{\theta}} I^{\frac{1-\theta}{\theta}} + 1\right]} \frac{K}{I} > 0 \tag{27}$$

$$\frac{\partial I_{t+1}}{\partial K_t}\Big|_{ss} = 0 \tag{28}$$

$$\frac{\partial I_{t+1}}{\partial I_t}\Big|_{ss} = 1 + \frac{1}{\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N}\right]^{\frac{1}{\theta}} I^{\frac{1-\theta}{\theta}} \sigma} + \frac{\theta}{1-\theta} (1-\alpha) A K^{\alpha-1} > 0 \tag{29}$$

Furthermore, it can be easily verified that:

$$T = \alpha + 1 + \frac{1}{\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N}\right]^{\frac{1}{\theta}} I^{\frac{1-\theta}{\theta}} \sigma} + \frac{\theta}{1-\theta} (1-\alpha) A K^{\alpha-1} > 0$$

$$D = \alpha \left(1 + \frac{1}{\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N}\right]^{\frac{1}{\theta}} I^{\frac{1-\theta}{\theta}} \sigma} + \frac{\theta}{1-\theta} (1-\alpha) A K^{\alpha-1}\right) > 0$$

and

$$\Delta = \left[\alpha - \left(1 + \frac{1}{\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N}\right]^{\frac{1}{\theta}} I^{\frac{1-\theta}{\theta}} \sigma} + \frac{\theta}{1-\theta} (1-\alpha) A K^{\alpha-1}\right)\right]^2 > 0$$

The eigenvalues of J may be obtained by solving the following equation:

$$p(\lambda) = |J - \lambda I| = 0$$

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where

$$|J - \lambda I| = \begin{vmatrix} \frac{\partial K_{t+1}}{\partial K_t}|_{SS} - \lambda & \frac{\partial K_{t+1}}{\partial I_t}|_{SS} \\ \frac{\partial I_{t+1}}{\partial K_t}|_{SS} & \frac{\partial I_{t+1}}{\partial I_t}|_{SS} - \lambda \end{vmatrix}$$

with

$$\lambda_i = \frac{T \pm \sqrt{\Delta}}{2} \text{ with } i = 1, 2$$

Using the information above, it can be easily verified that the eigenvalues are:

$$\lambda_1 = \alpha < 1$$

and

$$\lambda_2 = 1 + \frac{1}{\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N}\right]^{\frac{1}{\theta}} I^{\frac{1-\theta}{\theta}} \sigma} + \frac{\theta}{1-\theta} \left(1 - \alpha\right) A K^{\alpha-1} > 1$$

which suggests that the fixed point is a saddle path solution. Therefore, there are some initial points that will converge to the steady-state, consistent with our previous findings from the global dynamics.

7   Empirical Results

We now seek to describe the empirical analysis which will be used to test the main predictions of our model. There are three key predictions of our framework. First, banks in more concentrated sectors hold more cash reserves. Second, more competitive banking systems lead to higher capital accumulation. Finally, in order to alleviate the distortions from market power in the banking sector, the welfare-maximizing rate of money growth is inversely related to the degree of banking competition. As we explain below, we concentrate our efforts in studying economic activity among the countries in the OECD.

We begin by discussing the primary dependent variables of interest. For example, we seek to test the prediction that concentrated sectors hold more cash reserves. Therefore, one of the key variables that we study is the extent of cash holdings across banking systems. To that end, we look at the behavior of bank liquid reserves to bank assets (LRBA) from the World Development Indicators of the World Bank. It measures the ratio of domestic currency holdings and deposits with monetary authorities relative to bank assets such as claims on the private sector.[11] As observed in Table 1, the availability of the data varies across countries. For example, in terms of the United States, it is only available from 2001-2011. There is also a fair amount of variation in liquidity holdings across countries. Around one-third of the countries have liquidity ratios less

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[11] This measure is listed as variable FD.RES.LIQU.AS.ZS in the World Development Indicators of the World Bank. It is available at: http://data.worldbank.org/data-catalog/world-development-indicators. Accessed May 2014.

than 2%. The lowest ratio is observed in Canada at .311. At the other extreme, numerous other countries have cash ratios above 10% with Slovakia being the largest at 41%.

Second, we are also interested in examining the role that concentration plays in capital formation across countries. In order to ensure that our results are invariant to scale effects across countries, we choose to study gross capital formation as a percentage of GDP (CF). This variable contains information on additions to fixed assets across countries, including inventories. Data for each country is listed in Table 2.[12] The average ratio is 22.9% across all countries over time. Only five countries lie below the 20% number.

We next look at the primary control variables in the regressions. The primary measure of money growth that we use is the growth rate of M1 (ΔM). However, we also find it useful to conduct various types of robustness exercises. To that end, at various points in the estimation, we alter the conditioning information set. One way to do this in terms of money growth would be to look at the growth rate of M2 ($\Delta M2$). Both variables are obtained from first differences of the money stocks from the World Bank World Development Indicators.[13]

In addition, the level of income is likely to be correlated with the level of cash reserves and capital formation. Therefore, we use GDP growth (ΔGDP) as an important control variable. As a robustness check, we also study how the growth rate of deposits ($\Delta Deposits$) affects economic activity.[14]

The concentration ratio (CR) is the primary control variable of interest in our regression analysis. Though the data is available from the World Bank, it is derived from data available in Bankscope.[15] The particular measure of the concentration ratio that is reported is the three-firm concentration ratio. That is, it consists of assets of the three largest commercial banks as a fraction of total commercial banking assets.

The average concentration ratio across countries is equal to 71.7%. However, there are substantial differences in the competitive structure of the banking system across countries. According to the three-firm concentration ratio, the United States has the most competitive banking system in the OECD (27.6%). Japan ranks second in terms of the degree of competition at 38.8%. At the other extreme, many banking systems are highly concentrated. Six countries (Estonia, Finland, Iceland, New Zealand, Norway, and Sweden) have banking sectors in which the largest three firms hold over 90% of banking assets.

The earliest that the data is available is 1997, but for numerous countries, there are a few years after the beginning of the sample where the variable is not available. Summary statistics for all variables are provided in Table 4. As shown in Tables 1-3, there is substantial variability of data across countries.

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[12] World Bank Development Indicators, Source Code: NE.GDI.TOTL.ZS.

[13] Alternatively, from lines lines 34 and 35 in the International Financial Statistics (IFS).

[14] The relative amount of bank deposits (bank deposits to GDP) comes from the World Bank Global Development dataset, Source Code: GFDD.OI.02. The real value of deposits in each year is based upon the ratio multiplied by real GDP in 2005 dollars from the World Bank Development Indicators (Source Code NY.GDP.MKTP.KD) The growth rate of deposits is obtained by first differencing the series in log levels.

[15] World Bank Global Development, Source Code: GFDD.OI.01.

Consequently, the standard approach of constructing averages over time as an approximation of long-run activity in cross-country studies may be spurious. That is, constructing a "long-run" average for banking concentration would not be a true measure of the average degree of concentration in many countries as the variable is missing for a number of years.

Also, the timeframe in which the data is available also varies across countries. Thus, it would not be appropriate to suggest that it would be possible to obtain estimates from "contemporaneous regressions" as described by King and Levine (1993). Further, given the relatively short length of the sample, the bias in the relationship may not be trivial. As a result, "long-run" estimates of the role of concentration across a large sample of countries could be incorrect.

Moreover, we are particularly concerned that the level of development may impact how banking concentration and monetary policy affect economic activity. Thus, conclusions generated from a large heterogeneous group of countries may be misleading because they aggregate across vastly different banking systems and development patterns. Notably, Ghossoub and Reed (2010, 2012) develop models with multiple steady-states where the effects of monetary policy vary across the level of capital formation. The advanced steady-state is associated with a Tobin effect from monetary policy while the opposite occurs in the steady-state with a low level of activity. Schreft and Smith (1997) also demonstrate that the effects of money growth should be asymmetric across countries.

While previous empirical cross-country studies such as Boyd et al. (2001) and Fisher (1993) provide evidence that inflation is associated with lower levels of economic activity, other papers that focus on either a single country or a small number of similar countries suggest that the effects of monetary policy do vary across countries. For example, Bullard and Keating (1995) point out that permanent shocks to the money growth rate increase output in low inflation countries. Moreover, using annual data for the United States, Ahmed and Rogers (2000) also find evidence of a long-run Tobin effect. Thus, it is reasonable to conclude that the relationship observed in large cross-country studies such as Boyd et al. is dominated by relatively high inflation countries at lower stages of development. Consequently, failing to consider the role of economic development in the transmission of monetary policy is inappropriate.

For example, among observations where there is data on banking concentration available, the average rate of money growth in the OECD is under 9.5%. By comparison, in the lowest income countries in the World Development Indicators, the rate of money growth is nearly twice as high at 18.5%. Interestingly, despite the excessively high rate of money growth in the poorest countries, banks tend to hold more liquid assets than in the developed world. The liquidity ratio is nearly 25% in the poor countries while it is only around 6.5% in the OECD. As argued by Ghossoub and Reed (2010), such differences are likely due to the higher degree of liquidity risk facing individuals in poor countries. Thus, in trying to understand the allocations of intermediaries, it is also important to consider countries at similar stages of economic and financial development.

As the model has a unique steady-state in which monetary policy leads to a Tobin effect, the predictions that emerge from our framework most directly

apply to advanced countries. Therefore, we focus our attention on activity across countries in the OECD. Since we study activity across a relatively small number of countries, we use panel regressions to study the role of banking concentration. Studying the data from a panel perspective allows us to take into account important time-series variation over the sample period as well as country-specific information that limits omitted-variable bias.

The first prediction from the model is that banks in more concentrated sectors hold more cash reserves. Table 5 presents our regression results which seek to test this prediction. Heteroskedasticity-consistent standard errors are reported in parentheses below each coefficient estimate. The first two columns provide the benchmark regression results in which ΔGDP, ΔM, and CR are the primary control variables. Column (I) accounts for country-specific unobservables while column (II) includes both country fixed-effects and year fixed-effects. Interestingly, only the concentration ratio is a significant determinant of cash holdings and enters with the correct sign as suggested by our model. Columns (III) - (VI) present several robustness exercises in which we adjust the conditioning information set across specifications. Columns (III) and (IV) use the growth rate of deposits instead of the growth rate of GDP while columns (V) and (VI) consider the growth rate of M2 rather than M1. The estimate for the concentration ratio is quite robust across specifications, generally indicating that if the concentration ratio were to increase by ten percentage points, the liquidity ratio would increase by around eight percentage points. Consequently, the competitive structure of the banking system is a significant determinant of cash reserves across countries.

We next consider the second prediction of our framework: more competitive banking systems lead to higher capital accumulation. Table 6 presents the results. The organization of the table is the same as Table 5. As one would expect, GDP growth is a significant determinant of capital formation across countries. Following the predictions of the model, countries with more concentrated banking systems exhibit less capital formation. Based upon the coefficient estimates, an increase in concentration from the U.S. level (27.6%) to Icelandic concentration (100%) would cause capital formation to decline by over twenty percentage points. Thus, banking concentration is not only a statistically significant factor in capital formation, it is also an economically significant determinant of capital formation and economic growth. The coefficient estimate is robust across all six specifications in the table.

Finally, the model predicts that central banks will pursue higher money growth rates in countries with concentrated banking systems. Table 7A presents the benchmark results. First, monetary authorities adopt higher rates of money growth in countries with higher GDP growth. For example, higher rates of money growth would be required in order to achieve price stability. Second, consistent with the predictions of the model, observed rates of money growth are higher if the concentration ratio is higher. The results in the table indicate that a ten percentage point increase in concentration would contribute to around a two percentage point increase in the rate of money growth. Results in Table 7B using the growth rate of deposits as a control variable instead of GDP growth

are very close to the estimates from Table 7A.

We also seek to verify that our results are driven by supply behavior of central banks rather than variation in money demand. To do so, Table 7C reports results from regressions that look at the determinants of the growth rate of M2 instead of M1 across countries. The regressions indicate that the growth rate of GDP is an important factor in the growth rate of M2, but the concentration ratio is not. Consequently, we conclude that the results in Tables 7A and 7B are driven by the supply behavior of central banks rather than money demand in concentrated sectors. Therefore, we contend that the empirical evidence across all specifications is consistent with the predictions of our model.

8   Conclusions

In recent years, the increased concentration of activity in the banking system has received much attention. However, such consolidation is part of a long-term pattern of consolidation activity. For example, Janicki and Prescott (2006) present evidence that the largest banks which made up less than 1% of active institutions in the United States held over 75% of assets in the banking system prior to the crisis. With this large of a role in economic activity, it is hard to believe that these institutions would not take advantage of their market power in the financial system. Moreover, institutions grew to be so large that they were deemed "too big to fail" during the crisis. Banking regulators and monetary policy authorities must confront the challenges associated with an increasing concentration of activity in the banking sector.

Our analysis demonstrates that banks in more concentrated systems have a tendency to hoard cash reserves in order to exploit their strategic advantages in the market for capital. However, Bencivenga and Smith (1991) argue that one of the key roles of intermediaries is to facilitate risk-sharing and promote investment activity. In stark contrast, concentrated banking systems deviate from this function. Yet, that does not mean that perfect competition should be a regulatory goal. In some settings, depositors would benefit from consolidation. Nevertheless, the model also demonstrates that the optimal money growth rule should be more aggressive as the banking sector consolidates. In this manner, we view that our work contributes to recent important research that examines the implications of the industrial organization of the banking system.[16]

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[16] Tarullo (2011) calls for further research into the industrial organization of the financial system.

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Technical Appendix

1. Derivation of a bank's money demand function. We begin by solving a typical bank's problem and deriving an expression for money demand, (11). From the text, the bank makes its portfolio choice by maximizing (5) subject to $(6) - (10)$. Upon substituting the binding constraints into objective function, the problem can be reduced into a choice of k_{t+1}. Specifically, the bank solves:

$$
\underset{k_{t+1}}{Max} \frac{\pi \left[\frac{N}{\pi} \frac{P_t}{P_{t+1}} \left(\frac{1}{N} (w_t + \tau_t) - k_{t+1} \right) \right]^{1-\theta} + (1-\pi) \left[\frac{N}{1-\pi} \alpha A K_{t+1}^{\alpha-1} k_{t+1} \right]^{1-\theta}}{1-\theta}
$$

Differentiating the objective function and some simplification, capital is rented to firms up to the point where marginal cost equals marginal benefit:

$$
\pi \left(\frac{1}{\pi} \frac{P_t}{P_{t+1}} \right)^{1-\theta} \left(\frac{1}{N} (w_t + \tau_t) - k_{t+1} \right)^{-\theta} = (1-\pi) \left(\frac{1}{1-\pi} \alpha A \right)^{1-\theta} \left[k_{t+1}^{-1} - (1-\alpha) K_{t+1}^{-1} \right] K_{t+1}^{(\alpha-1)(1-\theta)} k_{t+1}^{1-\theta}
$$

Next, we impose equilibrium where $K_{t+1} = N k_{t+1}$. Using the bank's balance sheet condition and some simplification yields:

$$
m_t = \frac{k_{t+1}}{\frac{1-\pi}{\pi} \left[1 - \frac{(1-\alpha)}{N} \right]^{\frac{1}{\theta}} I_t^{\frac{1-\theta}{\theta}}}
$$

Using the bank's balance sheet, (6), along with the derived expression for m_t above, yields the expression for money holding in the text, (11).

2. Proof of Proposition 1. The result follows directly from our discussion in the text and from our work in Lemma 1. In particular, an excess demand is obtained at $\underline{I}$ if: $\bar{K}^D \geq \underline{K}^S$. Using the information in Lemma 1 and simplifying, yields the existence condition in the Proposition. This completes the proof of Proposition 1.

3. Proof of Proposition 2. We begin by showing that $\frac{dK}{dN} > 0$. In order to do so, we first use simple algebra to re-write the supply of capital, (20) as:

$$
\frac{K}{w(K)} = 1 - \frac{1}{\frac{1-\alpha}{\alpha} \left[1 - \frac{(1-\alpha)}{N} \right]^{\frac{1}{\theta}} \frac{1-\pi}{\pi}} \frac{1}{I^{\frac{1}{\theta}}} \tag{30}
$$

Using (21) and (30), along with a functional form for the production function, the equilibrium aggregate capital stock is a solution to the following polynomial:

$$
\frac{K^{1-\alpha}}{(1-\alpha) A} = 1 - \frac{1}{[\alpha A]^{\frac{1}{\theta}} \frac{1-\alpha}{\alpha} \left[1 - \frac{(1-\alpha)}{N} \right]^{\frac{1}{\theta}} \frac{1-\pi}{\pi}} \frac{K^{\frac{1-\alpha}{\theta}}}{\sigma^{\frac{1}{\theta}}} \tag{31}
$$

31

By differentiating (31) with respect to N and some simplification, we get:

$$\frac{dK}{dN} = \frac{\frac{1}{\theta}\left[1 - \frac{(1-\alpha)}{N}\right]^{-1}\frac{(1-\alpha)}{N}\frac{K}{N}}{\frac{1-\alpha}{\theta} + \frac{K^{1-\alpha}}{(1-\alpha)A}\frac{(1-\alpha)}{\left[1 - \frac{K^{1-\alpha}}{(1-\alpha)A}\right]}} > 0 \tag{32}$$

which also implies that $\frac{dR}{dN} < 0$ by diminishing marginal productivity.

We proceed to show that $\frac{d\gamma}{dN} < 0$ and $\frac{d\frac{r^n}{r^m}}{dN} > 0$. Using the definition of τ, where $\tau = \frac{\sigma-1}{\sigma}Nm$ into a bank's balance sheet, (6), along with some algebra, yields:

$$\gamma = \frac{1}{\frac{1}{\sigma}\frac{1}{1-\frac{K}{w}} + \frac{\sigma-1}{\sigma}} \tag{33}$$

where $\frac{K}{w} = \frac{K^{1-\alpha}}{(1-\alpha)A}$ is strictly increasing in K, which implies that $\frac{\partial \gamma}{\partial K} < 0$. In this manner, $\frac{d\gamma}{dN} = \frac{\partial \gamma}{\partial K}\frac{dK}{dN} < 0$. Furthermore, solving for $I^{\frac{1}{\theta}}$ from (30) and using the expression into (13), the relative return to depositors is such that:

$$\frac{r^n}{r^m} = \frac{1}{\frac{1-\alpha}{\alpha}\frac{1-\pi}{\pi}}\frac{1}{1 - \frac{K}{w(K)}}$$

which is increasing in K and N.

Finally, we examine how N affects capital investment by an individual bank. In equilibrium, $K = Nk$. Therefore, $\frac{dK}{dN} = k + N\frac{dk}{dN}$. Equivalently, $\frac{dk}{dN} = \frac{K}{N^2}\left(\frac{N}{K}\frac{dK}{dN} - 1\right)$. In this manner, $\frac{dk}{dN} \geq 0$ if $\frac{N}{K}\frac{dK}{dN} \geq 1$. Using the (32), and some simplification, this condition can be written as:

$$\frac{1}{[N - (1-\alpha)]} \geq 1 + \frac{\theta}{\frac{(1-\alpha)A}{K^{1-\alpha}} - 1} \tag{34}$$

Given that $\frac{dK}{dN} > 0$, it is easy to verify that their exists an $\hat{N}$ such that the condition above holds with equality. The result ii in Proposition 2 directly follows. This completes the proof of Proposition 2.

4. Proof of Proposition 3. The proof that $\frac{dK}{d\sigma} > 0$ follows directly from (22) in the text. By diminishing marginal productivity, we have $\frac{dR}{d\sigma} < 0$. Furthermore, using the capital demand equation, (21), the equilibrium condition, (31) can be written as:

$$\frac{\alpha\sigma}{(1-\alpha)I} + \frac{1}{\frac{1-\alpha}{\alpha}\left[1 - \frac{1-\alpha}{N}\right]^{\frac{1}{\theta}}\frac{1-\pi}{\pi}}\frac{1}{I^{\frac{1}{\theta}}} = 1 \tag{35}$$

It is trivial to show that $\frac{dI}{d\sigma} > 0$ and given that $\frac{d\frac{r^n}{r^m}}{dI} > 0$ we also have that $\frac{d\frac{r^n}{r^m}}{d\sigma} > 0$. This completes the proof of Proposition 3.

5. Proof of Proposition 4. By differentiating (22) with respect to N, we get:

$$\frac{d}{dN}\left(\frac{dK}{d\sigma}\right) = \frac{\left[1 + \frac{1-\theta}{\theta}\left(1 - \Omega\left(K\right)\right) - \frac{\left(1-\alpha\right)\Omega\left(K\right)}{\left(1-\Omega\left(K\right)\right)}\right]\frac{\frac{dK}{dN}}{\left(1-\Omega\left(K\right)\right)}}{\left(1-\alpha\right)\sigma\theta\left[\frac{1}{1-\Omega\left(K\right)} + \frac{1-\theta}{\theta}\right]^2} \tag{36}$$

Given that the numerator is positive and that $\frac{dK}{dN} > 0$, the sign of $\frac{d}{dN}\left(\frac{dK}{d\sigma}\right)$ depends on the sign of the term in solid brackets in the numerator. More specifically, $\frac{d}{dN}\left(\frac{dK}{d\sigma}\right) \geq 0$ if:

$$\chi\left(K\right) \equiv 1 + \frac{1-\theta}{\theta}\left(1 - \Omega\left(K\right)\right) - \frac{\left(1-\alpha\right)\Omega\left(K\right)}{\left(1-\Omega\left(K\right)\right)} \geq 0$$

where $\chi'\left(K\right) < 0$. Implicitly, $\frac{d\chi}{dN} > 0$ and it is easy to verify that their exists an $\tilde{N}_0$ at which $\chi = 0$. For all $N \leq (>) \tilde{N}_0$, $\chi\left(K\right) \geq (<) 0$ and the result in the Proposition follows. This completes the proof of Proposition 4.

6. Proof of Proposition 5. We begin by deriving an expression for the expected utility of depositors in the steady-state, (23). Using (7), (8), (10), and the fact that $K = Nk$ in equilibrium into (5), the steady-state expected utility of depositors is:

$$u = \frac{\pi\left[\frac{N}{\pi}\frac{1}{\sigma}m\right]^{1-\theta} + \left(1 - \pi\right)\left[\frac{1}{1-\pi}\alpha A K^\alpha\right]^{1-\theta}}{1 - \theta} \tag{37}$$

Furthermore, using the expression for government transfers, (2) into a bank's balance sheet, (6), we have: $K = w - \frac{1}{\sigma}Nm$. Using this information along with the expression for capital demand, (21) into the capital supply function, (30), simple algebra yields:

$$\frac{1}{\sigma}Nm = \frac{\left(1-\alpha\right)AK^{\frac{1-\alpha}{\theta}+\alpha}}{\left[\alpha A\right]^{\frac{1}{\theta}}\frac{1-\alpha}{\alpha}\left[1 - \frac{\left(1-\alpha\right)}{N}\right]^{\frac{1}{\theta}}\frac{1-\pi}{\pi}\sigma^{\frac{1}{\theta}}} \tag{38}$$

Finally, we use (38) into (37), to obtain the indirect welfare function in the text, (23).

Next, we differentiate with respect to N to obtain:

$$\frac{du}{dN} = \frac{\left(\left[1 - \alpha + \alpha\theta + \frac{\alpha\theta\left(\frac{1-\pi}{\pi}\right)\left[\alpha A\right]^{\frac{(1-\theta)}{\theta}}\left[1 - \frac{(1-\alpha)}{N}\right]^{\frac{1-\theta}{\theta}}\sigma^{\frac{1-\theta}{\theta}}}{K^{\frac{[1-\alpha](1-\theta)}{\theta}}}\right]K^{-1}\frac{dK}{dN} - \frac{\left(1-\alpha\right)}{\left[1 - \frac{(1-\alpha)}{N}\right]N^2}\right)X}{\theta\left(\left(\alpha A\right)^{\frac{1-\theta}{\theta}}\frac{1-\pi}{\pi}\sigma^{\frac{1}{\theta}}\right)^{1-\theta}\left[1 - \frac{(1-\alpha)}{N}\right]^{\frac{1-\theta}{\theta}}}$$

$$\pi^\theta K^{\frac{[1-\alpha+\alpha\theta](1-\theta)}{\theta}}$$

33

It is easy to verify that: $\frac{du}{dN} \geq 0$ if:

$$\frac{N}{K}\frac{dK}{dN} \geq \frac{\frac{1-\theta}{\theta}\left[1 - \frac{(1-\alpha)}{N}\right]^{-1}\frac{(1-\alpha)}{N}}{\frac{[1-\alpha+\alpha\theta](1-\theta)}{\theta} + \frac{\alpha(1-\theta)\frac{1-\pi}{\pi}\sigma^{\frac{1-\theta}{\theta}}\left[1 - \frac{(1-\alpha)}{N}\right]^{\frac{1-\theta}{\theta}}[\alpha A]^{\frac{(1-\theta)(1-\theta)}{\theta}}}{K^{\frac{(1-\alpha)(1-\theta)}{\theta}}}} \tag{39}$$

Using (32) into (39), along with the equilibrium condition, (31), the condition can be written as:

$$\Gamma(N,\sigma) \equiv 1 + \left(\frac{1-\pi}{\pi}\right)^{\theta}\left[\frac{\alpha}{(1-\alpha)}\right]^{1-\theta}\left[1 - \frac{K^{1-\alpha}}{(1-\alpha)A}\right]^{\theta} - \frac{1}{\alpha}\frac{K^{1-\alpha}}{(1-\alpha)A} \geq 0 \tag{40}$$

where implicitly $K \equiv K(N,\sigma)$. It is clear that $\frac{d\Gamma}{dN} < 0$. Therefore, there exists an $\tilde{N}_1 : \frac{du}{dN} \geq (<)0$ for $N \leq (>)\tilde{N}_1$. Moreover, the optimal level of banking competition is decreasing in sigma given that $\frac{dK}{d\sigma} > 0$. Finally, given that the degree of banking competition affects the amount of risk sharing offered by the banks, we need examine the parameter space where a banking equilibrium exists. From our work in Proposition 1, a banking equilibrium exists if: $\sigma \geq \frac{\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi}}{1 - \frac{(1-\alpha)}{N}}$, which can be written as: $N \geq \frac{(1-\alpha)}{1 - \frac{\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi}}{\sigma}} = \underline{N}$, where at $\underline{N}$, $I = \underline{I}$. Evaluating (31) at $\underline{N}$, $\frac{K^{1-\alpha}}{(1-\alpha)A} = \left(1 - \frac{\alpha}{1-\alpha}\frac{\pi}{1-\pi}\right)$. Moreover, $\underline{N} > \tilde{N}_1$ if $\Gamma(\underline{N},\sigma) < 0$. Upon using the information above into the expression for Γ, $\underline{N} > \tilde{N}_1$ if $\pi < (1-\alpha)\left[1 - \left(\frac{\alpha}{1-\alpha}\right)^2\right]$. The result in the Proposition directly follows. This completes the proof of Proposition 5.

7. Proof of Proposition 6. By differentiating the utility function, (23), with respect to σ and some simplification, we get:

$$\frac{du}{d\sigma} = \frac{\left[\left(\frac{1-\alpha+\alpha\theta}{\theta} + \frac{\alpha\frac{1-\pi}{\pi}(\alpha A)^{\frac{1-\theta}{\theta}}\left[1 - \frac{(1-\alpha)}{N}\right]^{\frac{1-\theta}{\theta}}\sigma^{\frac{1-\theta}{\theta}}}{K^{\frac{(1-\alpha)(1-\theta)}{\theta}}}\right)\frac{1}{K}\frac{dK}{d\sigma} - \frac{1}{\theta\sigma}\right]}{\left[\frac{(\alpha A)^{\frac{1-\theta}{\theta}}\left[1 - \frac{(1-\alpha)}{N}\right]^{\frac{1}{\theta}}\frac{1-\pi}{\pi}\sigma^{\frac{1}{\theta}}}{K^{\frac{[1-\alpha+\alpha\theta]}{\theta}}}\right]^{1-\theta}}\pi^{\theta}$$

where the sign of $\frac{du}{d\sigma}$ depends on the sign of the term in the numerator. By using the equilibrium condition, (31) into the expression for $\frac{du}{d\sigma}$, it is easy to verify that $\frac{du}{d\sigma} \geq 0$ if:

$$\frac{\sigma}{K}\frac{dK}{d\sigma} \geq \frac{1}{[1 - \alpha + \alpha\theta] + \left(\frac{1-\pi}{\pi}\right)^{\theta}\alpha\theta\left[\frac{1}{\frac{1-\alpha}{\alpha}\left[1 - \frac{K^{1-\alpha}}{(1-\alpha)A}\right]}\right]^{1-\theta}} \tag{41}$$

34

Furthermore, by substituting for $\frac{\sigma}{K}\frac{dK}{d\sigma}$ from (22), the condition can be written as:

$$\Psi\left(\sigma\right) \equiv \left[1 - \frac{K^{1-\alpha}}{(1-\alpha)A}\right] + \frac{\left(\frac{1-\pi}{\pi}\right)^{\theta}\left[1 - \frac{K^{1-\alpha}}{(1-\alpha)A}\right]^{\theta}}{\left[\frac{1-\alpha}{\alpha}\right]^{1-\theta}} - \frac{1-\alpha}{\alpha} \geq 0$$

It is clear that Ψ is strictly decreasing in σ. Moreover, it is easily verified that there exists rate of money growth, $\tilde{\sigma}$ such $\Psi\left(\tilde{\sigma}\right) = 0$. For all $\sigma \leq (>)\tilde{\sigma}$, $\frac{du}{d\sigma} \geq (<)0$. Finally, $\underline{\sigma} > \tilde{\sigma}$ if $\Psi\left(\underline{\sigma}\right) < 0$. Using the definition of $\underline{\sigma}$ and (31), $\underline{\sigma} > \tilde{\sigma}$ if $\pi < 1 - \left(\frac{\alpha}{1-\alpha}\right)^{2}$. $\Psi\left(\sigma\right) < 0$ in the feasible range ($\sigma \geq \underline{\sigma}$) and we have $\frac{du}{dN} < 0$. In addition, it is trivial to show that $\tilde{\sigma}$ and $\underline{\sigma}$ are both falling in N. This completes the proof of Proposition 6.