Firm and Job Creation Policies during Recessions in Emerging Economies*

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Abstract

Many emerging economies implemented cyclical labor-market policies in response to the 2008-2009 financial crisis, but high-frequency data limitations impede a detailed empirical assessment of the effectiveness of these policies. We develop a business cycle model with frictional labor markets, endogenous self-employment, and endogenous small firm creation and expansion consistent with the employment and firm structure of developing countries. We use the model to assess the quantitative effects of several cyclical labor-market policies within such market structure. Hiring subsidies and better job intermediation services for employment in large firms provide partial income protection, bolster faster output recoveries, and limit the rise and persistence of unemployment. The small-firm counterpart of these policies initially protect employment, but have limited medium-term aggregate benefits. Cyclical subsidies for non-salaried firm creation generate more sluggish recoveries with little income protection, while cyclical wage subsidies have very limited effectiveness. Model-implied fiscal multipliers suggest that cyclical hiring subsidies and better employment matching for large firms are particularly effective in fostering faster recoveries and boosting employment.

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1 Introduction

In response to the 2008-2009 financial crisis, many developing countries introduced policies to support employment (Table 1). Some of the most prominent policies included temporary wage and hiring subsidies to foster job creation, the temporary expansion of job intermediation services, and the expansion of public expenditures to support the creation of micro and small firms via credit facilities, among others (Table 2).[1] Limitations on the frequency and availability of data on labor flows for most emerging economies pose a serious challenge for in-depth empirical assessment of the benefits, costs, and aggregate effects of these policies on employment and economic activity. As developing countries continue to improve their ability to implement countercyclical policy, it is increasingly important to understand the impact of different policy tools in response to adverse shocks. Yet, extrapolating conclusions from similar policies implemented in advanced economies is ultimately inadequate given several distinctive characteristics of developing countries' labor market and firm structure.

To get around these limitations, we develop a novel business cycle model with frictional labor markets and firm heterogeneity that is consistent with the employment and firm structure of emerging economies. We exploit the increasingly rich evidence on the employment and firm structure of several Latin American economies to guide our modeling choices and use the region as a representative example. Then, we use the model as a tractable laboratory to study the aggregate consequences and effectiveness of several cyclical labor demand and firm creation policies that were implemented by developing countries during the crisis.

The structure of the model is based on the following facts. First, relative to developed countries, the share of self-employment in Latin American economies is substantial, ranging anywhere from 20 to 40 percent of total employment and reaching even higher rates in other developing countries (Perry et al., 2007; Loayza and Rigolini, 2011). Second, a larger fraction of firms in these emerging economies are micro or small, and they employ a larger share of salaried employment relative to developed countries (Table 3).[2] Third, while bank credit

[1] ILO and World Bank (2012) and Banerji, Newhouse, Paci, and Robalino (2014) offer an excellent summary of the policy responses in a broad set of developing countries. Cho and Newhouse (2013) analyze the impact of the financial crisis on different types of workers in several developing countries.

[2] For additional evidence on the distribution of employment across firm size for other countries, as well as the relationship between GDP per capita and firm size, see Poschke (2014) and Figure 3.3 in the Global Financial Development Report (2014), among others.

and other formal financing sources are more often available for larger firms in developing countries, micro and small firms tend to be informal and face severe difficulties in obtaining capital. They also often cite access to finance as their biggest obstacle (Kantis, Ishida, and Komori, 2002; IDB, 2005a, 2005b; Global Financial Development Report, 2014).[3] As a consequence of the lack of formal external financing, many of these firms exhibit a smaller scale, must search for support from suppliers and customers, and hence must rely on alternative, informal financing sources generally based on relationship lending (Tables 4 and 5; Farazi, 2014).[4] Importantly, as shown in Table 5, as small firms grow older their reliance on alternative financing sources such as credit from suppliers, customers, and the purchase of used machinery and equipment—all being relationship-based sources of input credit—remains important.[5] These are key distinctions relative to developed countries, where small firms, while also constrained, usually have better access to formal financing. Finally, as a simple example, Table 6 shows the allocation of resources by firm size for Mexico, which suggests that a very small share of the total capital in the economy is allocated among small salaried firms.[6]

Our model differentiates between self-employed (or micro) firms, small salaried firms, and large firms based on capital intensity, productivity, and the reliance of input credit by micro and small firms in a tractable way. Following Finkelstein Shapiro (2014), large salaried firms act as input credit suppliers to self-employed firms through frictional capital markets.[7] We

[3]For a recent overview of (formal) firm financing in developing countries, see Ayyagari, Demirgüç-Kunt, and Maksimovic (2012), as well as the Global Financial Development Report (2014). For example, only 12 percent of informal firms (which make up the majority of small firms in developing countries) have access to a loan (Figure 1.10, Global Financial Development Report, 2014).

[4]For additional evidence on the importance of trade credit for young and small firms, see Chavis, Klapper, and Love (2011), Demirgüç-Kunt and Maksimovic (2001) and Beck, Demirgüç-Kunt, and Maksimovic (2008).

[5]While firms in East Asia tend to rely less on these financing sources as they age, input credit remains a relevant source of financing (IDB, 2005b).

[6]For more evidence on resource allocation in Latin America, see Busso, Madrigal, and Pagés (2012). The evidence in Table 4 provides a lower bound for the share of small firms, small-firm salaried employment, and self-employment since the Mexican census only considers firms with a fixed location and hence excludes a non-negligible share of employment and firms in the economy. See Busso, Fazio, and Levy (2012) for more details.

[7]This capital search structure successfully generates countercyclical transitions from unemployment to self-employment as well as counteryclical self-employment, as in the data, while simultaneously capturing the notion of capital constraints among small firms. Others, including den Haan, Ramey and Watson (2003), Wasmer and Weil (2004, 2013), Kurmann and Petrosky-Nadeau (2007) and Arseneau, Chugh, and Kurmann (2008), have introduced search frictions in capital markets in various ways to model the prevalence of financial imperfections. However, none of these papers have introduced self-employment.

expand this framework on two fronts. First, owner-only (self-employed) firms can choose to expand to become small salaried firms. The notion of size in the model is related to a restriction on variable capital usage by small salaried firms and their continued reliance on external financing from larger firms. Second, small firms can hire additional workers. Thus, the model incorporates both endogenous small firm creation and expansion. Overall, this structure is consistent with the small allocation of capital among small firms in developing countries (Busso, Fazio, and Levy, 2012), and establishes an important link between large firms, self-employed, and small firms through capital markets.

We consider the aggregate consequences of introducing cyclical hiring subsidies for large and small salaried firms, hiring subsidies for self-employed firms (which foster the creation of small salaried firms), subsidies for the creation of self-employed firms, and wage subsidies. We also consider policies that improve the matching process between salaried firms and the unemployed during downturns, where these policies are meant to capture the expansion of government-provided job intermediation services during recessions. All these policies were introduced in one way or another during the 2008-2009 financial crisis (ILO and World Bank, 2012).

By analyzing cyclical policies that generate the same fiscal cost at the onset of a downturn, we show that the type of firm targeted by the policy (owner-only, small, or large) matters for the effectiveness of cyclical labor market interventions of fostering employment and output recoveries. Moreover, certain policies can generate tradeoffs between limiting the rise in unemployment as the recession hits and fostering a faster recovery in the medium term, while other policies yield gains along both margins. Importantly, we find that the reallocation of capital across firms—and hence the functioning of informal capital markets and their interaction with the labor market —and the effect that this has on the incentives to hire workers in both small and large firms play an important role in the effectiveness of policy. The differential response of market tightness for each employment type also plays a key role in explaining the response of the economy to each of the policies, as well as the benefits from hiring subsidies and increased job intermediation centered on large firms in terms of partial labor income protection relative to an economy with no policy.

We obtain four main results. First, hiring subsidies for large firms and improved inter-

mediation between large firms and the unemployed during downturns can yield gains across the board: a reduction in aggregate volatility, a faster recovery, a smaller contraction in earnings for all types of employment, a lower rise in unemployment, lower unemployment persistence after a recession, and fiscal multipliers above one in the medium term. Second, hiring subsidies for small firms can yield non-negligible positive results for employment—in particular, employment among small salaried firms—but these effects are short-lived and the policy slows down the recovery process in the medium term. Third, fostering the creation of self-employed firms during downturns can be detrimental for the recovery process and the economy as a whole, even as these firms may ultimately lead to future salaried employment creation. This last result is particularly important in light of the policies that many developing countries implemented to support the creation of micro and small enterprises. Fourth, wage subsidies are relatively ineffective in bolstering output when compared to other alternatives, but do provide partial income protection.

Our work is related to the growing literature on business cycles and search frictions, as well as the literature on the impact of fiscal policy in the context of the recent crisis. Nicoletti and Pierrard (2006) present a model where small salaried firms must first match with a financial intermediary to obtain capital and then search for a single worker to produce. However, our model differs from theirs in three key aspects. First, while small firms in our framework are also created endogenously, they can employ multiple workers and, once established, can also expand by hiring additional workers. Second, we introduce self-employed (one-person) firms, which can also ultimately become small salaried firms. Finally, we use capital search in such a way that self-employment is countercyclical, but small firm owners (small firms) are procyclical, in line with the data.[8] Less directly related to our work, but among those exploring the effects of labor market policies during the financial crisis, are studies that focus on advanced economies. Kitao, Şahin, and Song (2010) study the impact of job creation policies introduced in the U.S. in 2010 using a partial-equilibrium search and matching model. Campolmi, Faia, and Winkler (2011) analyze the impact of

[8]Kumar and Schuetze (2007) present a search model with own-account workers and employers. In their framework, transitions from own-account to employer status are exogenous, and they abstract from business cycle dynamics. However, the friction that generates entry into self-employment in their model would yield procyclical entry. See Hobijn and Şahin (2007) for a related model with endogenous entrepreneurship.

fiscal policy and hiring subsidies on employment in general equilibrium setting with labor search frictions and show that hiring subsidies can yield large multipliers.[9] Relatedly, Totzek and Winkler (2010) explore the role of fiscal policy in an environment with endogenous firm entry, but abstract from studying labor market dynamics.[10] Lee and Mukoyama (2013) build a model of industry dynamics that captures the cyclical dynamics of entry and exit in the U.S. manufacturing sector to analyze the role of entry subsidies during recessions, although they abstract from labor market frictions.

We contribute to the existing literature on firm and labor market dynamics over the business cycle, and to recent studies on the consequences of cyclical labor market policies in two ways. First, we place particular attention on the labor market and firm structure of developing countries. Among other things, our model accounts for the large shares of self-employment in these economies in a way that is consistent with the cyclical dynamics of self-employment and salaried employment in the data, and do so in an environment with firm heterogeneity based on capital intensity and access to external finance among small firms. Existing models of firm entry over the business cycle generally abstract from the fact that many new firms, particularly in developing countries, start as being one-person firms that either remain without workers or slowly expand via salaried employment creation. Our framework tractably accommodates this feature. With regards to the policies we consider, a key difference relative to the existing business cycle literature is that we analyze interventions that explicitly support the creation and expansion of self-employment ventures during downturns, which have been common in several emerging economies and are particularly relevant in economies with both large self-employment shares and large employment shares among small firms.

The remainder of this paper is organized as follows. Section 2 describes the model. Section 3 describes the calibration of the model. Section 4 presents the main results from the policy experiments. Section 5 discusses the fiscal implications of different policies. Section 6 concludes.

[9] Also, see Faia, Lechthaler, and Merkl (2013).

[10] For related work on firm entry and business cycles, see Shao and Silos (2008). For studies that merge labor market and firm dynamics in a business cycle context, see Sedláček (2011), Sedláček and Sterk (2013) and Siemer (2013) for the U.S., among others.

2 The Model

We assume a closed economy to focus on employment and firm dynamics.[11] The economy is inhabited by an infinitely-lived representative household with a large number of members whose population consists of a unit mass. All household members participate in the labor force. Following related literature, there is perfect risk-pooling across household members. Final output is produced using as intermediate inputs the output of large firms, small firms, and owner-only firms.[12] We assume that the product market is perfectly competitive. Large firms are more capital intensive compared to small firms, they undertake capital accumulation internally, and they make capital allocation decisions— they choose the share of capital used in their own production and consequently the share available as input credit for self-employed individuals and small firms. Large firms also hire salaried workers in frictional labor markets. Self-employed (or own-account) individuals operate owner-only firms. These firms produce using a single unit of capital and a fixed (normalized) unit of self-employment labor. Importantly, these firms cannot accumulate capital internally. As in Finkelstein Shapiro (2014), obtaining capital to enter self-employment is subject to search frictions so that production capital is obtained by establishing an input credit relationship with large firms (capital suppliers). Owner-only firms also post vacancies and therefore have the possibility to become small salaried firms by hiring employees. If at least one of these vacancies is successfully filled, a self-employed individual becomes a small salaried firm owner.

Small firms cannot accumulate capital internally either, they are less capital intensive than large firms, and they continue to rely on a capital relationship with large firms in order to be operational. For aggregation purposes (discussed below) we assume that each small firm also needs one, and only one, unit of capital to be operational.[13] Thus, each small salaried firm uses a unit of capital and salaried workers to produce. These salaried workers are hired through frictional labor markets. Search frictions are such that the wages for salaried workers in both large and small firms as well as the capital rental rates for self-employed and small business owners are negotiated via Nash bargaining. Taken together, our

[11] Assuming a small open economy does not change the main results.

[12] We use the terms small firms and small businesses interchangeably.

[13] This implies that the measure of self-employed individuals and the measure of small business owners yield the measure of capital used outside the large firm sector.

assumptions on large and small firms yield a larger concentration of capital in the large-firm sector, which is consistent with the evidence in Busso, Fazio, and Levy (2012).

We consider the cyclical impact of four types of policies in response to a recession: hiring subsidies for large and small firms, subsidies for the creation of owner-only firms and small salaried firms, improved job intermediation services for salaried firms, and wage subsidies. We assume that these policies do not affect the steady state and only become operative in response to productivity shocks.[14]

2.1 Households, Self-Employment and Small Firms

2.1.1 Self-Employment

Total profits from self-employed individuals are:

$$\Pi_{SE,t} = \left(p_{SE,t} z_{SE,t} - r_{SE,t} - \tau_t^{v_{SE}} \psi_{SE} v_{SE,t}\right) n_{SE,t}, \tag{1}$$

where $p_{SE,t}$ is the relative price of self-employment output; $z_{SE,t}$ is self-employment productivity; $r_{SE,t}$ is the endogenous Nash rental rate of capital paid by small businesses; ψ_{SE} is the fixed, exogenous flow cost of posting vacancies that allows self-employed firms to endogenously transition into becoming small firms if a match with unemployed workers materializes; $v_{SE,t}$ is the number of vacancies posted by each self employed individual in order to expand and become a small firm owner; and $n_{SE,t}$ is the mass of self-employed individuals. Total output from self employment is given by $y_{SE,t} = z_{SE,t} n_{SE,t}$. $\tau_t^{v_{SE}}$ is a policy that equals one in steady state and decreases (increases) when output is below (above) trend. Thus, it acts as a hiring subsidy during downturns.

[14] Even though 58 percent of the developing countries surveyed in ILO and World Bank (2012) introduced training for the unemployed (Table 1), we abstract from analyzing training policies since the model we present would become substantially more complex.

2.1.2 Small Salaried Firms

Total profits from small firms are given by

$$\Pi_{SB,t} = \left[p_{SB,t} z_{SB,t} F\left(\frac{n_{SB,t}}{o_{SB,t}}, 1 \right) - \tau_t^{w_{SB}} w_{SB,t} \frac{n_{SB,t}}{o_{SB,t}} \right] o_{SB,t} \tag{2}$$
$$- r_{SB,t} o_{SB,t} - \tau_t^{v_{SB}} \psi_{SB} v_{SB,t} o_{SB,t},$$

where $p_{SB,t}$ is the relative price of output in the small business sector; $z_{SB,t}$ is the exogenous productivity of small firms; the production function of an individual small firm, $F(\cdot, \cdot)$, is increasing and concave in each of its arguments (the first argument is salaried labor used per small firm and the second argument is capital usage per small firm—we assume that all operating small firms are identical, which, as shown below, leads to straightforward sectoral aggregation); $n_{SB,t}$ is the total mass of individuals employed by small firms; $w_{SB,t}$ is the wage paid to small-firm employees; $o_{SB,t}$ is the total mass of small firm owners.[15] $r_{SB,t}$ is the endogenous rental rate of capital paid by small businesses; and ψ_{SB} is a fixed and exogenous flow cost of posting vacancies. $\tau_t^{v_{SB}}$ and $\tau_t^{w_{SB}}$ are time-varying policies (hiring and wage subsidies, respectively) that equal one in steady state and decrease (increase) when total output is below (above) trend.[16] Given our assumptions on the production function of individual small firms, equation (2) can be restated as:

$$\Pi_{SB,t} = p_{SB,t} z_{SB,t} F(n_{SB,t}, o_{SB,t}) - \tau_t^{w_{SB}} w_{SB,t} n_{SB,t} \tag{3}$$
$$- r_{SB,t} o_{SB,t} - \tau_t^{v_{SB}} \psi_{SB} v_{SB,t} o_{SB,t}.$$

It follows that, from a sectoral perspective, total output from small businesses is increasing and concave in small firm employees and small firm capital. Indeed, because each individual production entity uses one and only one unit of capital, o_{SB} represents both the mass of small business owners and the total mass of capital rented by small firms. Total output from

[15] It follows that $n_{SB,t}/o_{SB,t}$ denotes the number of workers per small firm.

[16] Given the prevalence of labor informality among small firms in most developing countries, wage subsidies may be difficult to implement in practice since many of these firms do not comply with payroll taxes. We explore the implications of introducing wage subsidies that only affect a fraction of salaried workers in small firms—where this fraction is calibrated to the share of formal employment among large firms—in our quantitative analysis.

small firms is given by $y_{SB,t} = z_{SB,t}F(n_{SB,t}, o_{SB,t})$.

2.1.3 Employment-State Evolution

All job finding and filling probabilities, discussed below, depend on market tightness and are therefore endogenous. Job destruction probabilities, also discussed below, are assumed to increase (decrease) when total output is below (above) trend. This assumption is broadly in line with the introduction of countercyclical job-destruction rates in Shimer (2005). We make this assumption on job destruction in order to keep the model tractable while still being able to get at the issue of how labor market policies may affect the pace of recoveries and hence of job destruction itself.

An unemployed job seeker finds a job in a large firm with probability $f_{L,t}$ and these jobs are destroyed with probability ρ_t^L. Therefore, from the household's labor supply perspective, the mass of individuals employed by large firms $n_{L,t}$ evolves in the following way:

$$n_{L,t+1} = (1 - \rho_t^L)\left(n_{L,t} + f_{L,t}u_t\right), \tag{4}$$

where u_t is the mass of unemployed individuals.

For a self-employment opportunity to arise the household must form a match with a capital supplier (Finkelstein Shapiro, 2014). In order to form such matches, the household spends resources on capital search given by $s_{K,t}$. Matches with capital suppliers are formed with probability $f_{K,t}$. Once a match with a capital supplier is formed, an unemployed individual is assigned to self-employment status. Self-employed individuals post vacancies $v_{SE,t}$, which allows them to expand and become small firm owners by hiring workers. Transitioning from being a self-employed individual into a small firm owner occurs with endogenous probability $q_{SE,t}$. Both existing and newly formed matches with capital suppliers are destroyed with probability ρ_t^K. In addition, a small firm is destroyed with probability ρ_t^O, but in that instance the former small firm owner retains the capital supplier relationship and transitions back to self-employment. Finally, the probability that a salaried position is destroyed in a surviving small firm is ρ_t^S. It follows that, from the household's perspective, the mass of

self-employed individuals $n_{SE,t}$ satisfies:

$$n_{SE,t+1} = (1-\rho_t^K)\left\{\left[1-(1-\rho_t^O)(1-\rho_t^S)v_{SE,t}q_{SE,t}\right]n_{SE,t} + \rho_t^O o_{SB,t} + s_{K,t}f_{K,t}\right\}, \tag{5}$$

and the mass of small firm owners evolves as follows:

$$o_{SB,t+1} = (1-\rho_t^K)(1-\rho_t^O)\left[o_{SB,t} + (1-\rho_t^S)v_{SE,t}q_{SE,t}n_{SE,t}\right]. \tag{6}$$

From a labor demand perspective, at the start of a period $n_{SB,t}^d$ individuals are employed by small firms. Self-employed individuals seeking to become small firms post a total of $v_{SE,t}n_{SE,t}$ vacancies and existing small firms post a total of $v_{SB,t}o_{SB,t}$ vacancies. Therefore,

$$n_{SB,t+1}^d = (1-\rho_t^K)(1-\rho_t^O)(1-\rho_t^S)\left(n_{SB,t}^d + n_{SE,t}v_{SE,t}q_{SE,t} + o_{SB,t}v_{SB,t}q_{SB,t}\right) \tag{7}$$

gives the evolution of salaried employment in small firms from the labor demand perspective. From a labor supply perspective, at the start of a period $n_{SB,t}^s$ individuals are employed by small firms. Unemployed individuals find a job with a newly-formed small firm with probability $f_{SE,t}$ and find a job with an existing small firm with probability $f_{SB,t}$. Therefore,

$$n_{SB,t+1}^s = (1-\rho_t^K)(1-\rho_t^O)(1-\rho_t^S)\left[n_{SB,t}^s + u_t\left(f_{SE,t} + f_{SB,t}\right)\right]. \tag{8}$$

In equilibrium, $n_{SB,t}^s = n_{SB,t}^d = n_{SB,t}$. Finally, unemployment at time t is given by

$$u_t = 1 - n_{L,t} - n_{SE,t} - n_{SB,t} - o_{SB,t}.$$

Because the labor force is normalized to unity, u_t is also the aggregate unemployment rate.

2.1.4 Household Optimization

In solving its optimization problem, the household takes all job finding, filling, and destruction probabilities as given. Also, we assume that from the perspective of small firms the household maximizes utility as a demander of labor in small firms. This assumption keeps

the solution to the household's labor supply and demand problems explicitly decentralized.

The household's problem is to choose sequences of consumption c_t, the desired measure of self-employed individuals (achieved by spending resources on capital search) $n_{SE,t+1}$, the desired demand for salaried workers by small firms (achieved by vacancy posting) $n_{SB,t+1^d}$, the desired measure of small firm owners (also achieved by vacancy posting) $o_{SB,t+1}$, total expenditures on capital search $s_{K,t}$, vacancies posted per small firm $v_{SB,t}$, and vacancies posted per self-employed individual $v_{SE,t}$, in order to maximize $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$, where $\mathbb{E}$ is the expectation operator; $\beta \in (0,1)$ is the constant subjective discount factor; and $u'(c) > 0$ and $u''(c) < 0$, subject to equations (1), (3), (5), (6), (7), and to the household's budget constraint

$$c_t + \tau_t^{\kappa} \kappa\left(s_{K,t}\right) + T_t = \Pi_{SB,t} + \Pi_{SE,t} + \Pi_{L,t} + w_{SB,t} n_{SB,t}^s + w_{L,t} n_{L,t} + b u_t.$$

Note that because there is no labor force participation or sectoral search-intensity decision, the household has no explicit choice over $n_{SB,t+1}^s$ and $n_{L,t+1}$. Instead, the household simply takes as given the labor supply side equations (4) and (8).

The household's budget constraint entails the following. The price of consumption is normalized to unity and the household must pay lump-sum taxes T_t. The large firm, whose problem is described below, is owned by the household, and its profits $\Pi_{L,t}$ are taken as given. Individuals employed by the large firm earn the real wage $w_{L,t}$ and individuals employed by small businesses earn the real wage $w_{SB,t}$. Unemployed individuals obtain the constant and exogenous per-period unemployment flow benefit b. In addition, the household spends resources $s_{K,t}$ at total cost $\kappa\left(s_{K,t}\right)$ to search for capital suppliers (large firms) that provide capital for self-employment ventures, with $\kappa'\left(s_{K,t}\right) > 0$ and $\kappa''\left(s_{K,t}\right) \geq 0$. τ_t^{κ} is a time-varying policy that equals one in steady state and decreases (increases) when total output is below (above) trend. Thus, when output is below trend, the policy acts as a subsidy that reduces the cost of finding external financing to start a self-employment venture (i.e., this policy acts as a self-employment startup subsidy).

2.1.5 Household Optimality Conditions

As shown in the Appendix, the household's decision on $s_{K,t}$ to search for a capital supplier is given by

$$\frac{\tau_t^{\kappa}\kappa'(s_{K,t})}{f_{K,t}} = (1-\rho_t^K)\mathbb{E}_t\Xi_{t+1|t}\left\{p_{SE,t+1}z_{SE,t+1} - r_{SE,t+1} + \frac{\tau_{t+1}^{\kappa}\kappa'(s_{K,t+1})}{f_{K,t+1}}\right\}, \tag{9}$$

where $\Xi_{t+1|t} = \beta u'(c_{t+1})/u'(c_t)$ is the stochastic discount factor. This expression equates the expected marginal cost of spending resources to find capital to the expected marginal benefit. The latter is given by individual earnings in self-employment if the match takes place and becomes productive next period, $p_{SE}z_{SE} - r_{SE}$, plus the continuation value.[17]

The household's optimal choice of transforming a self-employed firm into a small salaried firm is

$$\begin{aligned}
&\left(\frac{\tau_t^{v_{SE}}\psi_{SE}}{q_{SE,t}} - \frac{\tau_t^{v_{SB}}\psi_{SB}}{q_{SB,t}}\right)\frac{1}{(1-\rho_t^K)(1-\rho_t^O)(1-\rho_t^S)} \\
= \;& \mathbb{E}_t\Xi_{t+1|t}\left\{\begin{array}{c} p_{SB,t+1}z_{SB,t+1}F_{o_{SB}}(n_{SB,t+1}, o_{SB,t+1}) \\ -r_{SB,t+1} - (p_{SE,t+1}z_{SE,t+1} - r_{SE,t+1}) \\ + \left(\frac{\tau_{t+1}^{v_{SE}}\psi_{SE}}{q_{SE,t+1}} - \frac{\tau_{t+1}^{v_{SB}}\psi_{SB}}{q_{SB,t+1}}\right)\end{array}\right\}.
\end{aligned}$$

The left-hand side of this expression captures the expected marginal cost, which in this case is given by the expected cost of posting a vacancy—a necessary condition to transition from being a self-employed individual to a small business owner—net of the cost of posting a vacancy that existing small firms face. The latter embodies the value of having an additional salaried worker as an existing small firm. The right-hand side of this expression gives the expected marginal benefit of becoming a small business owner and is comprised of two terms: the continuation value, and the net benefit of being a small business owner.[18] In

[17] Note that the cost of posting vacancies during self-employment does not appear in the optimality condition since the household internalizes the fact that a self-employed individual may be able to become a small business owner in the future, and the only relevant objects that affect the decision to enter self-employment is the revenue net of the cost of capital plus the continuation value of a capital relationship. The cost of posting vacancies will in turn affect the decision to transform a self-employment venture into a small salaried firm (shown below).

[18] Put differently, a self-employed individual is not only taking into account the net value of being a small business owner, but is also taking into account the value of having a salaried worker as a small salaried firm, which is implicitly given by the term $\tau_t^{v_{SB}}\psi_{SB}/q_{SB,t}$.

particular, the latter term is given by the difference between the marginal product of having a household member running a small business net of the cost of the matched unit of capital, $p_{SB}z_{SB}F_{o_{SB}}(n_{SB}, o_{SB}) - r_{SB}$, and self-employment revenue net of the cost of matched capital, $p_{SE}z_{SE} - r_{SE}$.

Finally, the optimal choice of vacancies posted per *existing* small firm seeking to expand yields a standard job creation condition

$$\frac{\tau_t^{v_{SB}}\psi_{SB}}{q_{SB,t}} = (1-\rho_t^K)(1-\rho_t^O)(1-\rho_t^S)\mathbb{E}_t\Xi_{t+1|t}\left\{\begin{array}{c} p_{SB,t+1}z_{SB,t+1}F_{n_{SB}}(n_{SB,t+1}, o_{SB,t+1}) \\ -\tau_{t+1}^{w_{SB}}w_{SB,t+1} + \frac{\tau_{t+1}^{v_{SB}}\psi_{SB}}{q_{SB,t+1}} \end{array}\right\}.$$

This expression simply equates the expected marginal cost of posting a vacancy to hire workers in a small firm to the expected marginal benefit. The latter includes the marginal product of labor net of the wage and the continuation value from the employment relationship.

2.2 Large Firms

A representative large firm chooses desired large-firm employment (achieved by vacancy posting) $n_{L,t+1}$, total vacancies devoted to hiring workers $v_{L,t}$, total capital $k_{L,t+1}$, a target for the capital to be lent out to the self-employed $n_{SE,t+1}$, a target for the capital to be lent out to small businesses $o_{SB,t+1}$, and the fraction of capital used within the firm ω_t, in order to

$$\max \mathbb{E}_0\sum_{t=0}^{\infty}\Xi_{t|0}\left\{\begin{array}{c} p_{L,t}z_{L,t}\mathbb{F}(n_{L,t}, \omega_t k_{L,t}) + r_{SE,t}n_{SE,t} + r_{SB,t}o_{SB,t} \\ -\tau_t^{w_L}w_{L,t}n_{L,t} - \tau_t^{v_L}\psi_L v_{L,t} - i_t - \frac{\varphi_k}{2}\left(\frac{k_{L,t+1}}{k_{L,t}} - 1\right)^2 k_{L,t} \end{array}\right\},$$

where $p_{L,t}$ is the price of sectoral output; $z_{L,t}$ is an exogenous sectoral productivity parameter; $\mathbb{F}(\cdot,\cdot)$ is the production function, which is increasing and concave in each of its arguments; i_t is investment; ψ_L is the fixed and exogenous flow cost of posting employment vacancies; and the last term is a capital adjustment cost. Also, similar to the case of small businesses, we introduce a time-varying policy that affects large-firm wages $\tau_t^{w_L}$, and another policy that affects the cost of hiring $\tau_t^{v_L}$, where both policies equal one in steady state and decrease (increase) when total output is below (above) trend.

The firm's optimization problem is subject to the perceived law of motion for the evolution

of large-firm employment:

$$n_{L,t+1} = (1-\rho_t^L)(n_{L,t} + v_{L,t}q_{L,t}),$$

the perceived evolution of capital used in the self-employment sector:

$$n_{SE,t+1} = (1-\rho_t^K)\left[\left(1-(1-\rho_t^O)(1-\rho_t^S)v_{SE,t}q_{SE,t}\right)n_{SE,t} + \rho_t^O o_{SB,t} + (1-\omega_t)k_{L,t}q_{K,t}\right],$$

the perceived evolution of capital used by small businesses:

$$o_{SB,t+1} = (1-\rho_t^K)(1-\rho_t^O)\left[o_{SB,t} + (1-\rho_t^S)v_{SE,t}q_{SE,t}n_{SE,t}\right],$$

and the evolution of the firm's total capital stock (following Finkelstein Shapiro, 2014, and similar to Kurmann and Petrosky-Nadeau, 2007):

$$\begin{aligned} k_{L,t+1} = (1-\delta)\omega_t k_{L,t} + (\rho_t^K - \delta)n_{SE,t} + \left[(1-(1-\rho_t^K)(1-\rho_t^O)) - \delta\right]o_{SB,t} + i_t \\ + \left[(1-\delta) - (1-\rho_t^K)q_{K,t}\right](1-\omega_t)k_{L,t}, \end{aligned}$$

where δ is the exogenous depreciation rate of capital.

2.2.1 Large-Firm Optimality Conditions

The firm's choice of total capital satisfies a standard capital Euler equation:

$$\left[1+\varphi_k\left(\frac{k_{L,t+1}}{k_{L,t}}-1\right)\right] = \mathbb{E}_t\Xi_{t+1|t}\left\{\begin{array}{c} p_{L,t+1}z_{L,t+1}\mathbb{F}_{\omega_L k_L}(n_{L,t+1},\omega_t k_{L,t+1}) \\ +(1-\delta) - \frac{\varphi_k}{2}\left(\frac{k_{L,t+2}}{k_{L,t+1}}-1\right)^2 + \varphi_k\left(\frac{k_{L,t+2}}{k_{L,t+1}}-1\right)\frac{k_{L,t+2}}{k_{L,t+1}} \end{array}\right\},$$

A standard job creation condition for large firm salaried employment also obtains:

$$\frac{\tau_t^{v_L}\psi_L}{q_{L,t}} = (1-\rho_t^L)\mathbb{E}_t\Xi_{t+1|t}\left\{p_{L,t+1}z_{L,t+1}\mathbb{F}_{n_L}(n_{L,t+1},\omega_t k_{L,t+1}) - \tau_{t+1}^{w_L}w_{L,t+1} + \frac{\tau_{t+1}^{v_L}\psi_L}{q_{L,t+1}}\right\}.$$

Finally, the firm's optimal decision to devote capital to forming new capital matches satisfies

$$\frac{p_{L,t}z_{L,t}\mathbb{F}_{\omega_L k_L}(n_{L,t},\omega_t k_{L,t})+(1-\rho_t^K)q_{K,t}}{q_{K,t}}=(1-\rho_t^K)\mathbb{E}_t\Xi_{t+1|t}\mathbf{J}_{SE,t+1},$$

where $\mathbf{J}_{SE}$ is the large firm's capital gains from a new capital match (see the Appendix). This last expression equates the expected marginal cost of devoting an additional unit of capital to matching to the expected marginal benefit. As in Finkelstein Shapiro (2014), the former is comprised of two terms: the expected marginal cost of that unit of capital within the large firm sector, $\frac{p_{L,t}z_{L,t}\mathbb{F}_{\omega k_L,t}}{q_{K,t}}$, and the opportunity cost of keeping a matched unit of capital within the firm until it becomes active in the self-employment sector next period, $(1-\rho_t^K)$. The expected marginal benefit is given by the value of having a capital relationship next period. As shown in the Appendix, this value not only includes the value of self-employment next period, but also the probability of transitioning from being a self-employed firm to being a small business next period.

2.3 Matching Processes

Let $m_{SE,t}=m_{SE}\left(v_{SE,t}n_{SE,t},u_t\right)$, $m_{SB,t}=\tau_t^{m_{SB}}m_{SB}\left(v_{SB,t}o_{SB,t},u_t\right)$, $m_{L,t}=\tau_t^{m_L}m_L\left(v_{L,},u_t\right)$ be standard matching functions that are increasing and concave in each of their arguments. $\tau_t^{m_L}$ and $\tau_t^{m_{SB}}$ are cyclical policies that equal one in steady state, but improve (reduce) matching efficiency for large and existing small firms, respectively, relative to trend during downturns (upturns). This policy specification tractably captures the expansion (contraction) of government-provided job intermediation services during recessions (expansions). The matching functions imply the following job-finding probabilities: $f_{SE,t}=\frac{m_{SE,t}}{u_t}$, $f_{SB,t}=\frac{m_{SB,t}}{u_t}$, $f_{L,t}=\frac{m_{L,t}}{u_t}$, and the following job-filling probabilities: $q_{SE,t}=\frac{m_{SE,t}}{v_{SE,t}n_{SE,t}}$, $q_{SB,t}=\frac{m_{SB,t}}{v_{SB,t}o_{SB,t}}$, and $q_{L,t}=\frac{m_{L,t}}{v_{L,t}}$. In addition, the capital matching function is $m_{K,t}=m_K((1-\omega_t)k_{L,t},s_{K,t})$. We define the probability of finding a capital supplier per unit of capital search expenditures as $f_{K,t}=\frac{m_{K,t}}{s_{K,t}}$ and $q_{K,t}=\frac{m_{K,t}}{(1-\omega_t)k_{L,t}}$ as the large firm's probability of matching its unused capital.

Labor market tightness for potential new small firms is $\theta_{SE,t}\equiv\frac{v_{SE,t}n_{SE,t}}{u_t}$, and labor market tightness for existing small firms is $\theta_{SB,t}\equiv\frac{v_{SB,t}o_{SB,t}}{u_t}$. Large-firm labor market tightness is

$\theta_{L,t} \equiv \frac{v_{L,t}}{u_t}$. In turn, capital market tightness is $\theta_{K,t} \equiv \frac{s_{K,t}}{(1-\omega_t)k_{L,t}}$. All salaried job-finding probabilities are increasing in market tightness, while all salaried job-filling probabilities are decreasing in market tightness. Given our definition of capital market tightness $\theta_{K,t}$, $f_{K,t}$ ($q_{K,t}$) is decreasing (increasing) in capital market tightness.

2.4 Nash Bargaining

From the perspective of a large firm, the value of having an additional salaried worker is $\mathbf{J}_{L,t}$; the value of having a capital relationship with a self-employed individual is $\mathbf{J}_{SE,t}$; and the value of having a capital relationship with a small firm is $\mathbf{J}_{SB,t}$. From the perspective of the small firm sector, the value of having an additional salaried worker is $\mathbf{J}_{OSB,t}$; and the value of having an additional capital relationship with large firms is $\mathbf{W}_{OSB,t}$. From the perspective of the household, the values of having a household member in salaried employment in a large firm, in salaried employment in a small firm, in self-employment, and in unemployment, are, respectively, $\mathbf{W}_{L,t}$, $\mathbf{W}_{SB,t}$, $\mathbf{W}_{SE,t}$, and $\mathbf{W}_{U,t}$. $\chi_L \in (0,1)$ and $\chi_{SB} \in (0,1)$ are, respectively, the bargaining power of workers negotiating with large firms and of workers negotiating with small firms. $\chi_{SE} \in (0,1)$ and $\chi_O \in (0,1)$ are, respectively, the bargaining power of self-employed and small business owners. For the sake of brevity, the value functions used in the determination of Nash prices as well as the statements of all Nash problems are relegated to the Appendix.

The implicit expressions for the Nash wages are given by

$$\frac{\chi_L}{(1-\chi_L)\,\tau_t^{w_L}}\mathbf{J}_{L,t} = (\mathbf{W}_{L,t} - \mathbf{W}_{U,t})\,,$$

for $w_{L,t}$ and

$$\frac{\chi_{SB}}{(1-\chi_{SB})\,\tau_t^{w_{SB}}}\mathbf{J}_{OSB,t} = (\mathbf{W}_{SB,t} - \mathbf{W}_{U,t})$$

for $w_{SB,t}$. The implicit expressions for the Nash rental rates are given by

$$\frac{\chi_K}{(1-\chi_K)}\left(\mathbf{J}_{SE,t} - (1-\delta)\right) = (\mathbf{W}_{SE,t} - \mathbf{W}_{U,t})\,,$$

for $r_{SE,t}$ and

$$\frac{\chi_O}{(1-\chi_O)}\left(\mathbf{J}_{SB,t}-(1-\delta)\right)=\left(\mathbf{W}_{OSB,t}-\mathbf{W}_{U,t}\right)$$

for $r_{SB,t}$. Note that the outside option of large firms is the value of a unit of capital net of depreciation, $(1-\delta)$ (Finkelstein Shapiro, 2014).

To understand the intuition behind the main results, it is useful to briefly consider the explicit expressions for wages in large and in small firms, respectively, without the policies in place, which, as detailed in the Appendix, are given by[19]

$$\begin{aligned}w_{L,t}=(1-\chi_L)\,b+\chi_L\theta_{L,t}\psi_L+\frac{(1-\chi_L)\,s^u_{K,t}f_{K,t}\nu_K}{(1-\chi_K)}\left(\frac{p_{L,t}z_{L,t}\mathbb{F}_{\omega k_L}(n_{L,t},\omega_t k_{L,t})}{q_{K,t}}\right)\\-\frac{(1-\chi_L)\,\chi_K s^u_{K,t}f_{K,t}}{(1-\chi_K)}(1-\rho^K_t)\left[1-\mathbb{E}_t\Xi_{t+1|t}(1-\delta)\right]\\+\frac{(1-\chi_L)\,\chi_{SB}}{(1-\chi_{SB})}\theta_{SB,t}\psi_{SB}+\frac{(1-\chi_L)\,\chi_{SB}f_{SE,t}}{(1-\chi_{SB})}\frac{\psi_{SB}}{q_{SB,t}},\end{aligned}$$

and

$$\begin{aligned}w_{SB,t}=(1-\chi_{SB})\,b+\chi_{SB}\theta_{SB,t}\psi_{SB}+\chi_{SB}\frac{f_{SE,t}\psi_{SB}}{q_{SB,t}}\\+\frac{(1-\chi_{SB})\,\chi_K s^u_{K,t}f_{K,t}}{(1-\chi_K)}\left(\frac{p_{L,t}z_{L,t}\mathbb{F}_{\omega k_L}(n_{L,t},\omega_t k_{L,t})}{q_{K,t}}\right)\\-\frac{(1-\chi_{SB})\,\chi_K s^u_{K,t}f_{K,t}}{(1-\chi_K)}(1-\rho^K_t)\left[1-\mathbb{E}_t\Xi_{t+1|t}(1-\delta)\right]+\frac{(1-\chi_{SB})\,\chi_L}{(1-\chi_L)}\theta_{L,t}\psi_L,\end{aligned}$$

where $s^u_{K,t}\equiv s_{K,t}/u_t$. Note that both large-firm and small-firm wages depend on labor market tightness for large, small, and self-employed firms, as well as capital market tightness.[20] Intuitively, higher labor market tightness (irrespective of the firm type) will put upward pressure on wages. This will also put downward pressure on capital rental rates as the outside options for both small firm owners and the self-employed are more attractive. Conversely, and similar to Finkelstein Shapiro (2014), higher capital market tightness will put downward pressure wages and upward pressure on capital rental rates. This is the case since higher

[19]We abstract from deriving the wage expressions with the different policies since the presence of time-varying wage subsidies prevent us from obtaining wage expressions that do not include the different value functions in $t+1$.

[20]Recall that $f_{K,t}$, $q_{K,t}$, $f_{SE,t}$, and $q_{SB,t}$ are functions of their respective market tightness.

capital market tightness—a lower probability of becoming self-employed—implies a less attractive self-employment outside option for salaried workers. By affecting market tightness, each of the cyclical subsidies will have an impact on wage dynamics and hence on labor demand.

2.5 Final Goods Production

Final output results as follows. We assume that composite output from the small firm and self-employment sectors is given by $y_{S,t} = y_S(y_{SB,t}, y_{SE,t})$. In turn, final output aggregates large firm output and the preceding composite output using the production function $y_t = y(y_{L,t}, y_{S_t})$.

Turning toward profit maximization, because the price of final output is normalized to one, the problem of the final goods producer is:

$$\max_{y_{L,t}, y_{SB,t}, y_{SE,t}} \{y_t - p_{L,t}y_{L,t} - p_{SB,t}y_{SB,t} - p_{SE,t}y_{SE,t}\}.$$

The solution to this problem is straightforward and yields optimal relative prices $p_{L,t}$, $p_{SB,t}$, and $p_{SE,t}$.

2.6 Government

The government uses lump-sum taxes to finance expenditures, and its budget constraint is

$$\begin{aligned} T_t = bu_t + (1-\tau_t^{v_L})\psi_L v_{L,t} + (1-\tau_t^{v_{SE}})\psi_{SE} v_{SE,t} n_{SE,t} + (1-\tau_t^{v_{SB}})\psi_{SB} v_{SB,t} o_{SB,t} \\ +(1-\tau_t^{\kappa})\kappa(s_{K,t}) + g_t + (1-\tau_t^{w_L}) w_{L,t} n_{L,t} + (1-\tau_t^{w_{SB}}) w_{SB,t} n_{SB,t} \\ +(\tau_t^{m_L} - 1) m_L(v_{L,}, u_t) + (\tau_t^{m_{SB}} - 1) m_{SB}(v_{SB,t} o_{SB,t}, u_t). \end{aligned}$$

Above, g_t is exogenous government spending and the last two terms represent the fiscal cost of job intermediation services. In what follows, we assume that g_t is constant. [21]

[21] As part of our robustness checks, we analyze the case where the cyclical policies are financed through government debt. The results remain qualitatively the same. Also, the assumption of lump-sum taxation is not as restrictive if we consider that some developing countries used commodity revenue to finance part of their stimulus packages. One example is Mexico, which used oil revenue as a source of financing for some of

2.7 Aggregate Resource Constraint

The resource constraint of the economy is given by

$$y_t = c_t + g_t + i_t + \kappa(s_{K,t}) + \psi_L v_{L,t} + \psi_{SE} v_{SE,t} n_{SE,t} + \psi_{SB} v_{SB,t} o_{SB,t}$$
$$+ \frac{\varphi_k}{2}\left(\frac{k_{L,t+1}}{k_{L,t}} - 1\right)^2 k_{L,t} + (\tau_t^{m_L} - 1) m_L\left(v_{L,}, u_t\right) + (\tau_t^{m_{SB}} - 1) m_{SB}\left(v_{SB,t} o_{SB,t}, u_t\right),$$

where the cost of posting vacancies, searching for capital, and providing job intermediation services are resource costs.

2.8 Definition of Competitive Equilibrium

Taking the stochastic processes $\{z_{L,t}, z_{SE,t}, z_{SB,t}\}$ *and the policies as given, the allocations and prices* $\{n_{L,t},\ n_{SB,t},\ o_{SB,t},\ n_{SE,t},\ u_t,\ \theta_{K,t},\ \theta_{SE,t},\ \theta_{SB,t},\ \omega_t,\ k_{L,t},\ \theta_{L,t},\ c_t,\ w_{L,t},\ w_{SB,t},\ r_{SE,t},\ r_{SB,t},\ T_t,\ y_t\}$ *and* $\{p_{L,t},\ p_{SB,t},\ p_{SE,t}\}$ *satisfy: the law of motion for salaried employment in large firms; the law of motion for small firm salaried employment; the law of motion for small business owners; the law of motion for self-employed individuals; the definition of unemployment; the household's optimal choice to search for capital suppliers; the optimal decision to transition from self-employment into small business owner status; the job creation decision by small business owners; the capital supply decision by large firms; the capital Euler equation of large firms; the job creation decision by large firms; the resource constraint; the Nash wage for salaried workers in large firms; the Nash wage for small firm salaried workers; the Nash rental rate for self-employed individuals; the Nash rental rate for small business owners; the three relative prices of sectoral output; the government budget constraint; and the definition of total output.*

3 Calibration

We assume a time period of one quarter. In addition, we choose Mexico as our benchmark since this country has high-quality data on labor flows and existing studies provide well-

the stimulus programs during the 2008-2009 financial crisis.

documented stylized facts about cyclical employment dynamics.

Functional Forms and Shocks Final output is given by a constant elasticity of substitution (CES) aggregate of large firm output and composite output from the self-employment and small firm sectors:

$$y_t = \left[\gamma_a y_{L,t}^{\phi_a} + (1-\gamma_a)\, y_{S,t}^{\phi_a}\right]^{\frac{1}{\phi_a}},$$

where $\gamma_a \in (0,1)$ and $\phi_a \leq 1$. In turn, composite output from the small firm and self-employment sectors, $y_{S,t}$, is given by the CES aggregate:

$$y_{S,t} = \left[\gamma_s y_{SB,t}^{\phi_s} + (1-\gamma_s)\, y_{SE,t}^{\phi_s}\right]^{\frac{1}{\phi_s}},$$

where $\gamma_s \in (0,1)$ and $\phi_s \leq 1$.[22] The production function for large firms is Cobb-Douglas: $\mathbb{F}(n_{L,t}, \omega_t k_{L,t}) = (n_{L,t})^{1-\alpha_L} (\omega_t k_{L,t})^{\alpha_L}$, where $\alpha_L \in (0,1)$. The production function for the aggregate small firm sector is also Cobb-Douglas: $F(n_{SB,t}, o_{SB,t}) = (n_{SB,t})^{1-\alpha_{SB}} (o_{SB,t})^{\alpha_{SB}}$, where $\alpha_L \in (0,1)$. We assume that $\alpha_L > \alpha_{SB}$ so that production in large firms is more capital intensive, in line with existing evidence. In each intermediate production sector productivity follows an AR(1) process with a common aggregate shock:

$$\ln z_{i,t} = (1-\rho_{z_i}) \ln(\overline{z}_i) + \rho_{z_i} \ln z_{L,t-1} + \varepsilon_t^z,$$

where $i \in \{L, SB, SE\}$. $\overline{z}_i$ is a constant and we assume that $\forall i\ \rho_{z_i} = \rho_z$. Also, $\varepsilon_t^z \sim N(0, \sigma_z^2)$ denotes the aggregate productivity shock (i.e., common to all sectoral productivity processes). Therefore, while all sectors are subject to a common productivity shock, each sector's steady state productivity can potentially differ.

We follow related literature and assume that all matching functions are Cobb-Douglas. In particular, $m_{L,t} = \tau_t^{m_L} M_L (u_t)^{\xi_L} (v_{L,t})^{1-\xi_L}$, $m_{SB,t} = \tau_t^{m_{SB}} M_{SB} (u_t)^{\xi_{SB}} (v_{SB,t} o_{SB,t})^{1-\xi_{SB}}$, $m_{SE,t} = M_{SE} (u_t)^{\xi_{SE}} (v_{SE,t} n_{SE,t})^{1-\xi_{SE}}$, and $m_{K,t} = M_K (s_{K,t})^{\xi_K} ((1-\omega_t) k_{L,t})^{1-\xi_K}$ where, for $j \in \{L, SB, SE, K\}$, M_j is the matching efficiency parameter and ξ_j is the matching elasticity parameter. For tractability, we assume that all separation rates are countercyclical relative

[22] The elasticities of substitution are $\frac{1}{1-\phi_a}$ and $\frac{1}{1-\phi_s}$, respectively.

to fluctuations in total output so that

$$\rho_t^j = \rho_{ss}^j \left[\exp\left(\eta_{\rho^j} \left(1 - \frac{y_t}{y_{ss}} \right) \right) \right],$$

for $j \in \{L,K,O,S\}$, where $\eta_{\rho^j} > 0$ determines the sensitivity of the separation rates to output deviations from trend and ρ_{ss}^j is the steady-state job destruction probability. This specification implies that separation rates increase (decrease) above their steady state values when output is below (above) trend. Finally, with regards to the household, we assume that the cost of searching for capital is $\kappa(s_{K,t}) = \psi_K \left(s_{K,t}\right)^{\eta_K}$, with $\psi_K > 0$ and $\eta_K \geq 1$. In addition, the household's utility function is characterized by constant relative risk aversion, $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$.

Parameters from Literature We borrow a number of parameters commonly used in business cycle models with search and matching. These parameters are summarized in Table 7. We set the capital share in large firms α_L to 0.32. Small firms tend to be less capital intensive, so we set the capital share in the small firm sector, α_{SB}, to 0.27.[23] Most Latin American countries, including Mexico, do not have a national unemployment insurance scheme, so we set b to 0. The subjective discount factor β is set to 0.985, in line with the literature (Boz, Durdu, and Li, 2012). The capital depreciation rate δ is in line with other studies on Mexico. The curvature of capital search $\eta_K = 1$.[24] We set the bargaining power for salaried workers, the self-employed, and small business owners to 0.5, and the matching elasticities to 0.5 so that the Hosios condition holds. The elasticity of substitution parameters in the CES aggregator functions are set to 0.7 so that production inputs are somewhat imperfectly substitutable.[25] The steady-state separation rates are based on Bosch and Maloney (2008).[26] The persistence parameters for each of the sectoral productivity shocks are set to 0.92. The coefficient of relative risk aversion is set to 2. Finally, based

[23] Our results remain qualitatively the same for larger differences in the capital share.

[24] Assuming that the cost of searching for capital is convex does not change the main conclusions.

[25] This yields an elasticity between self-employment and small firm output and an elasticity between large firm output and the combination of self-employed and small firm output of around 3.33. We explore whether our results change if we assume different values for the substitutability parameters, as well as asymmetries in substitutability.

[26] For the salaried separation rate in large firms, we take into account that a small share of workers in these firms is informal. This increases the separation rate in the sector.

on Busso, Fazio, and Levy (2012), we compute productivity differentials by type of firm so that the model is consistent with the large productivity differences that exist between self-employed firms, small firms, and large firms.[27]

Calibrated Parameters Calibrated parameters are summarized in Table 8. The matching efficiency parameters are chosen so that the model is in line with the allocation of employment across firms in the data. Our mapping between the data and the model is based on evidence on firm size and formality/legality status for Mexico (Busso, Fazio, and Levy, 2012), and yields the following distribution of individuals across employment states: the self-employed represent 16 percent of the labor force, small business owners represent 7 percent, salaried workers in small firms represent 30 percent, and salaried workers in large firms account for 42 percent of the labor force.[28] The cost of adjusting capital is set to replicate the volatility of the probability of entering self-employment (Bosch and Maloney, 2008). We set the cost of posting vacancies in small and large firms to 3.5 percent of sectoral wages (Levy, 2007).[29] The per-unit cost of searching for capital is set to 3 months of (small firm) wages, in line with the evidence in McKenzie and Woodruff (2006).[30] We set the job destruction rate of small firms, ρ^O_{ss}, so that the separation rate for small firm workers (a combination of ρ^K, ρ^O, and ρ^S) is 8 percent, in line with the evidence on informal separation rates (Bosch and Maloney, 2008). To establish the sensitivity of separation rates to output deviations, we fix η^K_{ss}, η^O_{ss}, and η^S_{ss} to one. In turn, we set η^L_{ss} to capture the volatility of transitions from formal salaried employment to unemployment relative to the volatility of transitions from informal salaried employment to unemployment from Bosch and Maloney (2008).[31] We calibrate the standard deviation of the aggregate productivity shock to match

[27]The Appendix describes the details on how we compute total factor productivity differentials across firm types in the data, as well as the mapping between the data and the model.

[28]Total self-employment in Mexico, which includes small business owners and own-account workers, is around 23 percent. Based on Perry et al. (2007) and others, between 65 and 70 percent of the self-employed are own-account workers (the remaining share being mostly individuals who operate small firms). This yields a self-employment (or own-account) share of 16 percent.

[29]Neither of these costs include the cost of hiring regulations. The results remain the same if we assume that the cost of posting vacancies for large firms includes the cost of hiring regulations, or if we assume a different calibration target for small firm vacancy postings.

[30]The results do not change if we assume much lower costs.

[31]Since we do not have data on separation rates by firm size, this assumes that a majority of informal salaried workers are in small firms, which is consistent with the evidence (Perry et al., 2007). The model is

the standard deviation of real GDP for years 1993 through 2007. In addition, we calibrate the steady-state government spending-output ratio to be 10.2 percent of output for Mexico.

4 Quantitative Results

4.1 Aggregate Dynamics without Policy

Table 9 shows that the model can match several stylized facts about business cycles and employment dynamics in a qualitative and quantitative way.[32] The model is successful in generating a number of stylized facts about labor market dynamics in a qualitative (and also quantitative) way. First, the model captures the countercyclicality of unemployment and self-employment. It is also able to generate a relative volatility of unemployment higher than one, which is difficult to generate in standard search models. Second, the model generates a higher volatility in the job-finding rate in large firms relative to small firms that is broadly in line with the data. The cyclical correlation of large-firm employment and output and the probability of entering self-employment from unemployment are also consistent with the evidence. Finally, the cyclical correlation between the job-finding rate in large firms is higher than the one in small firms.[33]

unable to generate the volatility of separation rates in the data without running into convergence problems. Targeting the relative volatility of transitions from self-employment to unemployment yields qualitatively similar results and generates additional unemployment volatility, but significantly reduces the countercyclicality of self-employment. However, the main conclusions do not change.

[32]We log-linearize the model around the non-stochastic steady-state and implement a first-order approximation. We simulate the model for 2100 periods, discard the first 100 periods, and apply the Hodrick-Prescott filter with smoothing parameter 1600 to the simulated series to obtain the cyclical component and compute second moments. We use Dynare for all dynamic simulations. To compare the model to the data, we use series from 1993:Q1 to 2007:Q4 for output, consumption, and unemployment to compute second moments, as well as the evidence in Bosch and Maloney (2008) and Fernández and Meza (2013) for the labor market. The output, consumption, and unemployment series are obtained from the Federal Reserve Bank of Saint Louis' FRED database.

[33]The fact that the cyclical correlation of the job-finding rates for salaried workers is particularly high is due to the presence of a single aggregate shock driving the model's dynamics. Introducing (correlated) sectoral shocks would allow us to quantitatively capture the cyclical correlation of these finding rates in the data.

4.2 Policy Experiments

We compare the response of the economy to a negative aggregate productivity shock when no labor market policies are in place—the no-policy scenario—to the response under the earlier noted set of *cyclical* labor market policies. We assume that these policies respond to fluctuations in aggregate productivity as follows. For $j \in \{v_L, v_{SB}, v_{SE}, \kappa, m_L, m_{SB}, w_L, w_{SB}\}$:

$$\tau_t^j = \exp\left[\tau_{ss}^j \left(\frac{y_t}{y_{ss}} - 1\right)\right], \tag{10}$$

where j is any of the policies above, $\tau_{ss}^j \geq 0$ captures the intensity of the policy, and y_{ss} denotes output in steady state.[34]

We calibrate each of the individual policies τ_{ss}^j to obtain a stimulus of 0.2 percent of output in the period of the negative aggregate shock. The fiscal package is in line with the size of the labor market measures introduced in Mexico as part of the stimulus package during the financial crisis (ILO, 2011).[35]

Figures 1 and 2 show the impulse responses of the economy to a one standard deviation negative aggregate productivity shock, comparing the benchmark (no-policy) scenario to two economies, one with hiring subsidies for small firms and one with hiring subsidies for large firms. Figures 3 and 4 compare the benchmark scenario to otherwise identical economies, one with hiring subsidies for self-employed firms and the other with subsidies for the creation of self-employment ventures. Figures 5 and 6 compare the benchmark economy to economies with better job intermediation for employment in large and small firms. Figures 7 and 8 compare the benchmark economy to economies with wage subsidies for large firms and wage subsidies for small firms, respectively. In what follows, the impact of each individual policy is compared to the results from the benchmark (no-policy) scenario.

[34]For a related approach, see Canzoneri et al. (2011). The specification in equation (10) has several attractive features. First, given how we introduced the policies in the model, τ_t^j is purely cyclical in nature and has no impact on steady-state allocations. Second, for $\tau_{ss}^j \geq 0$, the policy acts as a subsidy when output is below trend and as a tax when output is above trend, so that the government budget is always balanced. Third, in the case of a negative aggregate shock, the policy becomes active on impact and exhibits endogenous persistence, where the latter is determined by the severity and length of the recession.

[35]Recall that policy is endogenously determined once the shock takes place, which implies that the total fiscal cost in present value terms (once the economy returns back to steady state) is around 2.5 percent of GDP. Assuming a smaller fiscal package of 0.1 percent of output on impact, which naturally also delivers a lower total fiscal cost in present value terms of around 1.45 percent of GDP, yields very similar results.

Hiring Subsidies for Larger Firms The policy generate a rise in large-firm vacancy postings, which puts upward pressure on large-firm labor market tightness and limits the contraction in capital usage, which results in a smaller contraction in investment and also puts upward pressure on large- and small-firm wages, with the latter recovering earlier and the former initially falling by less. The fall in consumption is now limited by upward pressure on wages, the fall in expenditures on capital search, and the expansion in large-firm salaried employment (the contraction in self-employment earnings (not shown) is also subdued). Thus, some income protection ensues.[36] The behavior of salaried employment and the decision to allocate relatively more capital within large firms generates a much faster recovery in large-firm output, and in turn, total output. The expansion in large-firm employment counteracts the larger contraction in small-firm employment and self-employment, which explains unemployment rising by less and returning back to steady state faster.[37]

Hiring Subsidies for Existing Small Firms Higher vacancy postings by existing small firms puts upward pressure on small-firm salaried labor market tightness and leads to a reduction in both large-firm market tightness (which induces a larger initial fall in wages) and capital market tightness. The reduction in expenditures on capital search and the sharp expansion in small-firm salaried employment initially reduce the size of the contraction in consumption. The fall in large-firm vacancies reduces large-firm capital usage and increases capital supply to the self-employed in the period of the shock, ultimately resulting in a smaller initial fall in investment. Higher small firm vacancy posting puts upward pressure on large-firm wages, leading to a more sluggish recovery in large-firm vacancies and to a medium term slower recovery in investment. The sharp rise in small-firm employment induces a faster rebound in small-firm output, generating a faster recovery in total output (although only initially), and is large enough to offset the fall in both large-firm employment and self-

[36]Note that this may not be the case for *total labor income* from each employment type since the smaller contraction in wages in a given sector can be offset by the larger contraction in employment in that same sector.

[37]These results are consistent with the positive effects of hiring subsidies documented in Campolmi, Faia, and Winkler (2011) and Faia, Lechthaler, and Merkl (2013) in a different context, mainly in models with a single firm and employment type.

employment (so unemployment in fact decreases).[38] Ultimately, the more sluggish recovery in large-firm output slows down the aggregate recovery process in the medium term.

Hiring Subsidies for Self-Employed Individuals Small firm owners expand, generating a larger reallocation of capital from the large-firm sector into the self-employment sector (the increase in vacancy postings by the self-employed puts upward pressure on large-firm wages via labor market tightness, reducing the incentive of large firms to hire workers and hence in-house capital usage). Capital supply to the self-employed surges, and the Nash rental rate for self-employed workers falls by more so that self-employment earnings contract by less (not shown). Wages for both small and large-firm workers are less volatile, but consumption initially falls by slightly more given higher capital-search expenditures. Vacancy postings by existing small firms and the self-employed bounce back after the shock putting upward pressure on wages via higher market tightness. The rise in capital search expenditures is short-lived, and expenditures begin their return back to steady state faster, which explains why consumption does not fall by more after the shock. As capital usage among large firms initially contracts by more, investment exhibits a sharper contraction as well. The rise in both small-firm salaried employment and the measure of small business owners leads to a sharp recovery in small-firm output, generating a faster recovery in total output despite the more persistent contraction in large-firm output. The behavior of employment outside the large firm sector is responsible for limiting the rise in unemployment. However, unemployment remains persistently above steady-state due partly to the persistent fall in both large-firm employment and self-employment, where the latter is explained by the outflow out of self-employment and into small business owner status.

A caveat regarding hiring subsidies for self-employed individuals: the model assumes that self-employed individuals are homogeneous, and the only requirement to become a small firm is to successfully match with salaried workers via vacancy posting. However, in the data, the

[38]Note that the large response of small-firm salaried employment is mainly due to the fact that the subsidy rate is much larger compared to the hiring subsidy rate for larger firms. This is partly explained by the fact that we are assuming a fiscal package of the same size (0.2 percent of output), but the cost of posting vacancies for small firms is slightly lower than the cost for large firms. Furthermore, the subsidy rate adjusts to the size of the fiscal package. If we were to assume the same subsidy rate for all policies (and let the cost of the fiscal package adjust endogenously), unemployment would rise by less, but remain fairly persistent as the recovery takes hold.

self-employed that expand tend to be those who are more successful and have higher ability and skills, which would translate into higher productivity. The model in its current form implicitly assumes that those who expand automatically inherit higher productivity. If we were to differentiate between high and low-ability entrepreneurs and use targeted subsidies for those with high ability (or the most potential), the quantitative impact of hiring subsidies for the self-employed would likely be smaller since only a very small fraction of the self-employed would benefit from the subsidy (those with higher ability). As such, our results represent an upper bound of the impact of these subsidies on aggregate dynamics.

Subsidies for the Creation of Self-Employed Firms The demand for capital supplied by large firms rises and a reallocation of capital towards the self-employment sector takes place. The surge in capital demand from potential self-employed individuals reduces capital usage among large firms by more, leading to a larger contraction in large-firm vacancies and employment, large-firm output, and investment. A similar but milder effect is observed for small-firm salaried employment (small firms are not directly impacted by capital reallocation, but they are affected by the change in the self-employment outside option).[39] Salaried labor market tightness across firm types contracts and puts downward pressure on wages, but it becomes easier to move into self-employment, which puts upward pressure on wages. Ultimately wages do not adjust much relative to the benchmark economy, but salaried employment contracts by more, which along with the sharp rise in expenditures on capital search explains the larger contraction in consumption. The rise in self-employment is large enough to generate a smaller increase in unemployment (this merely slows down the rise in unemployment, and the policy has no discernible impact on unemployment dynamics in the medium term). Finally, note that, while the contraction in small-firm output worsens only slightly, the more sluggish recovery in large-firm output explains the slower recovery in total output.[40]

[39] If each small firm were using more than one unit of matched capital, the reallocation process would likely have a larger impact on salaried employment among large and small firms. Thus, our results can be seen as a lower bound for the response of small-firm salaried employment.

[40] Since it may be difficult to raise revenue for this policy by increasing the cost of hiring during an expansion, we can alternatively assume that this subsidy is financed via a temporary increase in payroll taxes for large firms. This enhances the adverse quantitative effects of the policy by generating an even larger contraction in large-firm employment and output.

Cyclical Improvements in Job Intermediation for Employment in Large Firms Results are analogous to hiring subsidies for large firms. The expected cost of posting vacancies for large firms decreases (mainly due to an increase in the job-filling rate instead of a fall in the cost per vacancy as was the case with the hiring subsidy), bolstering vacancies in the period of the shock and increasing capital usage among large firms. This leads to a smaller fall in investment and in turn a faster recovery in total output. The reduction in capital search expenditures by households implies that consumption contracts by less. The behavior of large-firm vacancies affects labor market tightness and implies that the policy provides partial income protection by preventing wages across all firm types from falling as much. Finally, the policy induces an expansion in employment among large firms that more than offsets the fall in employment among small firms and ultimately generates a reduction in unemployment. While the qualitative impact of the policy is similar to the hiring subsidy, the quantitative effects are larger.

Cyclical Improvements in Job Intermediation for Employment in Existing Small Firms There is a subdued effect on aggregate dynamics, and a similar effect to hiring subsidies for existing small firms. Interestingly, this job intermediation policy is less effective in reducing unemployment in the aftermath of the shock for this group of firms relative to a similar policy for large firms (despite the importance of small firms for aggregate employment). One key difference relative to hiring subsidies is that capital supply to the self-employed rises my more, which explains the behavior of investment. Moreover, since the downward adjustment in small firm wages is less stark, the recovery in small firm output is weaker, which in turn explains the short-term positive effects on unemployment.

Wage Subsidies for Large Firms The reduction in large-firm capital usage is subdued, which implies that salaried vacancies among large firms fall by less, generating a smaller contraction in investment and employment among large firms and a faster recovery in investment. In turn, large-firm output recovers at a faster rate. The supply of capital to the self-employed is smaller, so self-employment initially expands by less. The smaller decrease in large-firm vacancies implies that large-firm labor market tightness falls by less, putting

upward pressure on wages among all firm types. Since the adjustment in wages is smaller, both small firms and the self-employed cut vacancies by more, generating a larger contraction in small-firm salaried employment in subsequent periods and small-firm output exhibits a more sluggish recovery. The smaller fall in large-firm employment and wages, combined with the fall in household resources spent on capital search, leads to a smaller fall in aggregate consumption. Importantly, the contraction in small-firm salaried employment more than offsets the smaller fall in large-firm employment, which leads to a sharper increase in unemployment.

Wage Subsidies for Small Firms The value of having salaried workers in small firms rises and limits the fall in vacancy postings among small and self-employed firms, putting upward pressure on labor market tightness and wages in both salaried firm types, and pushing larger firms to reduce vacancy postings by more. Surprisingly, capital usage exhibits a similar fall to the one observed under the wage subsidies for large firms (despite having a larger fall in large-firm vacancies), which also implies a smaller fall in investment.[41] Despite the larger contraction in salaried employment among large firms in the aftermath of the shock, the rise in small-firm salaried employment and wages for both firm types, along with the initial reduction in the resources devoted to capital search (not shown), leads to a marginally smaller fall in consumption. Even though small-firm output recovers faster, the fall in large-firm output implies that the policy has a negligible effect on the recovery path of total output. Finally, given the large expansion in small-firm salaried employment, the rise of unemployment after the shock is smaller.

From a practical standpoint, wage subsidies for small firms may be hard to implement, and obtaining revenue for these subsidies from these same firms during expansions is not trivial. Assuming that only a fraction of small firms benefit from the subsidy (those that are formal), or assuming that the subsidy is financed via an increase in the payroll tax for large firms (see the Appendix), yields qualitatively similar results, with the benefits from small-firm wage subsidies under these two scenarios being more subdued.

[41] Existing small firms already have a capital relationship and the behavior of labor market tightness puts downward pressure on the Nash rental rates, which decreases the incentive to supply capital to the self-employed.

5 Fiscal Implications

Fiscal Cost of Policies and Subsidy Rates Recall that the assumed size of the fiscal package in the period of the shock is 0.2 percent of output. However, to the extent that policies affect the recovery speed of output, and the policy remains in place as long as output is below trend, the total fiscal cost can vary across policies. While the fiscal cost in the first few periods after the shock is lower when hiring subsidies for small firms are in place, the fiscal cost of the policy falls back more rapidly with hiring subsidies for larger firms, job intermediation for large firms, and hiring subsidies for the self-employed. This is driven by the behavior of output.

In addition, note that for the same size of the fiscal package, small and self-employed firms seem to require larger hiring subsidy rates relative to those for large firms. Indeed, the subsidy rates for large firms do not have to be high to have an important quantitative impact on aggregate dynamics. Similarly, wage subsidy rates for both small and large firms end up being very small, below 1 percent on impact, which may explain their limited impact on the recovery process.

Cumulative Fiscal Multipliers During Downturns To provide a better metric of the effectiveness of each of the policies considered, we follow the literature and compute cumulative multipliers after a negative aggregate shock in the following way:[42]

$$M_d(p) = \frac{\sum_{i=0}^{d} \beta^i (x_{t+i}(p) - x_{t+i})}{\sum_{i=0}^{d} \beta^i (fc_{t+i}(p))},$$

where d is the number of quarters after the shock and p is the policy for which the fiscal multiplier is computed. $x(p)$ is either total output, y, or total employment, n, under policy p, whereas x represents either of these two variables under the no-policy scenario. $fc(p)$ is the fiscal cost of policy p, where the fiscal cost under the no-policy scenario is zero.

As shown in Table 10, small firm creation and self-employed firm creation policies are not particularly effective in bolstering output or employment. Hiring subsidies for existing

[42] See, for example, Faia, Lechthaler, and Merkl (2013).

small firms yield output and employment multipliers higher than one in the first year, which subsequently fall below one after three years. Focusing on employment, the fiscal multiplier is fairly stable in the medium run. Conversely, the output multiplier falls below one after three years.

While hiring subsidies for large firms appear to be less effective in the first year, the output multiplier rises above one in the medium run because the policy fosters a faster recovery in total output, but the latter takes some time to materialize. However, the employment multiplier remains below one. Finally, the output multiplier generated by job intermediation services among large firms suggests that this policy is particularly effective in fostering a faster output recovery, even at short horizons. If we consider the medium term, this policy delivers an employment multiplier above one. These last two results suggest that job intermediation for employment among large firms is the most effective policy tool, followed by hiring subsidies for large firms. Wage subsidies yield very small (or even negative) output and employment multipliers and appear to be the least effective out of the set of policies we consider.

5.1 Robustness and Additional Experiments

The Appendix includes a brief summary of the results from a series of robustness checks, including different financing alternatives for the subsidies and different parameterizations of the model. The main results of the paper are robust to these and other alternative experiments and calibrations (included in the Appendix).

6 Conclusions

The 2008-2009 financial crisis rekindled considerable interest in labor market policies that can lessen the employment and output costs of recessions. A number of recent studies have analyzed the impact of various employment subsidies for the U.S. However, little work has focused on developing economies, where the breadth of self-employment and the large allocation of salaried employment in small firms implies non-trivial differences in the labor market and firm structure. Emerging-economy data limitations make it difficult to asses

empirically the effectiveness of any one policy. In order to get around these limitations, and using evidence on Latin America, we build a business cycle model with frictional labor markets consistent with the salient features of the firm and employment structure of Latin American and other developing countries. In turn, we use the model to analyze the aggregate impact of different cyclical labor market policies during downturns.

Our findings suggest that cyclical hiring subsidies and improved intermediation services for larger firms can yield positive results across the board and, importantly, provide partial income protection by reducing the contraction in individual wages and self-employment income. Cyclical policies aimed at protecting employment during downturns—especially among small firms, which employ a sizeable share of the labor force in developing countries—may have some success in reducing the initial rise in unemployment. However, some of these same policies reallocate resources towards less productive firms, which ultimately puts a dent on the recovery pace. In turn, wage subsidies are the least effective policy tools in fostering a faster recovery, even though they provide partial income protection.

Our model provides the basis for several interesting extensions. These include accounting for the implications of entry dynamics among larger firms, worker and consumption heterogeneity, as well as the difference between low and high-ability self-employed, and job-to-job transitions. Moreover, while the model is consistent with existing evidence on the continued use of informal input credit among small firms, studying the implications of financial development is important as it may lead to a change in the availability of financing sources and the productivity profile of firms. This, in turn, could modify the effectiveness of cyclical policies aimed at protecting employment and fostering faster recoveries in these countries. We plan to explore these extensions in future work.

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7 Appendix

7.1 Household Optimization

Assigning the multipliers Λ_t, $\Lambda_{K,t}$, $\Lambda_{OSB,t}$ and $\Lambda_{NSB,t}$ to the household's budget constraint, the household's perceived law of motion for self-employment, its perceived law of motion for small business owners, and its perceived (from the labor demand perspective) law of motion for employment in small firms, respectively, the first order conditions are as follows. For consumption c_t,

$$u'(c_t) - \Lambda_t = 0.$$

For self-employed individuals, $n_{SE,t+1}$,

$$\begin{aligned}
&\beta \mathbb{E}_t \Lambda_{t+1} \left[(p_{SE,t+1} z_{SE,t+1} - r_{SE,t+1}) - \tau_{t+1}^{v_{SE}} \psi_{SE} v_{SE,t+1} \right] \\
&+\beta(1-\rho_{t+1}^K)\mathbb{E}_t[\left(1-(1-\rho_{t+1}^O)(1-\rho_{t+1}^S)v_{SE,t+1}q(\theta_{SE,t+1})\right)\Lambda_{K,t+1} \\
&+(1-\rho_{t+1}^O)(1-\rho_{t+1}^S)v_{SE,t+1}q(\theta_{SE,t+1})\Lambda_{OSB,t+1}] \\
&+\beta(1-\rho_{t+1}^K)(1-\rho_{t+1}^O)(1-\rho_{t+1}^S)\mathbb{E}_t v_{SE,t+1}q(\theta_{SE,t+1})\Lambda_{NSB,t+1}^d - \Lambda_{K,t} = 0.
\end{aligned}$$

For self-employment projects, $s_{K,t}$:

$$-\Lambda_t \tau_t^\kappa \kappa'(s_{K,t}) + (1-\rho_t^K) p(\theta_{K,t}) \Lambda_{K,t} = 0.$$

For small business owners, $o_{SB,t+1}$:

$$\begin{aligned}
&\beta \mathbb{E}_t \Lambda_{t+1} [p_{SB,t+1} z_{SB,t+1} F_{o_{SB,t+1}^h}(n_{SB,t+1}, o_{SB,t+1}) - r_{SB,t+1} - \tau_{t+1}^{v_{SB}} \psi_{SB} v_{SB,t+1}] \\
&+\beta(1-\rho_{t+1}^K)\rho_{t+1\mathbb{E}t}^O \Lambda_{K,t+1} \\
&+\beta(1-\rho_{t+1}^K)(1-\rho_{t+1}^O)\mathbb{E}_t \Lambda_{OSB,t+1} \\
&+\beta(1-\rho_{t+1}^K)(1-\rho_{t+1}^O)(1-\rho_{t+1}^S)\mathbb{E}_t \Lambda_{NSB,t+1} v_{SB,t+1} q(\theta_{SB,t+1}) - \Lambda_{OSB,t} = 0.
\end{aligned}$$

The demand for individuals employed by small businesses, $n^d_{SB,t+1}$:

$$\beta \mathbb{E}_t \Lambda_{t+1}[p_{SB,t+1} z_{SB,t+1} F_{n^d_{SB,t+1}}(n_{SB,t+1}, o_{SB,t+1}) - \tau^{w_{SB}}_{t+1} w_{SB,t+1}]$$
$$+\beta(1-\rho^K_{t+1})\left(1-\rho^O_{t+1}\right)\left(1-\rho^S_{t+1}\right)\mathbb{E}_t \Lambda_{NSB,t+1} - \Lambda_{NSB,t} = 0.$$

Vacancies posted per self-employed individual, $v_{SE,t}$:

$$-\Lambda_t \tau^{v_{SE}}_t \psi_{SE} - \Lambda_{K,t}(1-\rho^K_t)(1-\rho^O_t)(1-\rho^S_t) q(\theta_{SE,t})$$
$$+\Lambda_{OSB,t}(1-\rho^K_t)(1-\rho^O_t)(1-\rho^S_t) q(\theta_{SE,t})$$
$$+(1-\rho^K_t)(1-\rho^O_t)(1-\rho^S_t) q(\theta_{SE,t}) \Lambda_{NSB,t} = 0.$$

And vacancies posted per small business, $v_{SB,t}$:

$$-\Lambda_t \tau^{v_{SB}}_t \psi_{SB} + (1-\rho^K_t)(1-\rho^O_t)(1-\rho^S_t) q(\theta_{SB,t}) \Lambda_{NSB,t} = 0.$$

From the first-order condition for consumption, we know that

$$\Lambda_t = u'(c_t).$$

Then, we can write

$$\frac{\Lambda_{K,t}}{u'(c_t)} = \frac{\tau^\kappa_t \kappa'(s_{K,t})}{(1-\rho^K_t) p(\theta_{K,t})},$$

and obtain

$$\frac{\Lambda_{NSB,t}}{u'(c_t)} = \frac{\tau^{v_{SB}}_t \psi_{SB}}{(1-\rho^K_t)(1-\rho^O_t)(1-\rho^S_t) q(\theta_{SB,t})},$$

Also, we have

$$(1-\rho^K_t)(1-\rho^O_t)(1-\rho^S_t)\frac{\Lambda_{OSB,t}}{u'(c_t)} = \frac{\tau^{v_{SE}}_t \psi_{SE}}{q(\theta_{SE,t})} - \frac{\tau^{v_{SB}}_t \psi_{SB}}{q(\theta_{SB,t})} + \frac{\tau^\kappa_t \kappa'(s_{K,t})}{p(\theta_{K,t})}$$

To determine the optimal decision to become a small firm, we write:

$$\begin{aligned}\frac{\Lambda_{OSB,t}}{\Lambda_t} = \mathbb{E}_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\Bigg\{&-\frac{h'_{t+1}}{\Lambda_{t+1}} + p_{SB,t+1}z_{SB,t+1}F_{o_{SB}}(n_{SB,t+1}, o_{SB,t+1})\\ &-r_{SB,t+1} - \tau_t^{v_{SB}}\psi_{SB}v_{SB,t+1} + (1-\rho^K)\rho^O\frac{\Lambda_{K,t+1}}{\Lambda_{t+1}}\\ &+(1-\rho^K_{t+1})(1-\rho^O_{t+1})(1-\rho^S_{t+1})\frac{\Lambda_{NSB,t+1}}{\Lambda_{t+1}}v_{SB,t+1}q(\theta_{SB,t+1})\\ &+(1-\rho^K_{t+1})(1-\rho^O_{t+1})(1-\rho^S_{t+1})\frac{\Lambda_{OSB,t+1}}{\Lambda_{t+1}}\Bigg\}.\end{aligned}$$

Then, use the expressions above as well as the optimal decision to move into self-employment to yield (iterating forward when necessary):

$$\begin{aligned}\left[\frac{\psi_{SE}\tau_t^{v_{SE}}}{q(\theta_{SE,t})} - \frac{\psi_{SB}\tau_t^{v_{SB}}}{q(\theta_{SB,t})}\right] = (1-\rho^K_t)(1-\rho^O_t)(1-\rho^S_t)\mathbb{E}_t\Xi_{t+1|t}\Big\{&p_{SB,t+1}z_{SB,t+1}F_{o_{SB}}(n_{SB,t+1}, o_{SB,t+1})\\ &-r_{SB,t+1} - (p_{SE,t+1}z_{SE,t+1} - r_{SE,t+1}) + \left[\frac{\psi_{SE}\tau_{t+1}^{v_{SE}}}{q(\theta_{SE,t+1})} - \frac{\psi_{SB}\tau_{t+1}^{v_{SB}}}{q(\theta_{SB,t+1})}\right]\Big\}.\end{aligned}$$

where $\Xi_{t+1|t} = \beta u'_{t+1}/u'_t$ is the stochastic discount factor. This equation implicitly defines $v_{SE,t}$.

To derive the decision to enter self-employment, first write:

$$\begin{aligned}\frac{\Lambda_{K,t}}{\Lambda_t} = \mathbb{E}_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\Big\{&(p_{SE,t+1}z_{SE,t+1} - r_{SE,t+1}) - \tau_{t+1}^{v_{SE}}\psi_{SE}v_{SE,t+1}\\ &+(1-\rho^K_{t+1})\left(1-(1-\rho^O_{t+1})(1-\rho^S_{t+1})v_{SE,t+1}q(\theta_{SE,t+1})\right)\frac{\Lambda_{K,t+1}}{\Lambda_{t+1}}\\ &+(1-\rho^K_{t+1})(1-\rho^O_{t+1})(1-\rho^S_{t+1})v_{SE,t+1}q(\theta_{SE,t+1})\frac{\Lambda_{OSB,t+1}}{\Lambda_{t+1}}\\ &+(1-\rho^K_{t+1})(1-\rho^O_{t+1})(1-\rho^S_{t+1})v_{SE,t+1}q(\theta_{SE,t+1})\frac{\Lambda_{NSB,t+1}}{\Lambda_{t+1}}\Bigg\}.\end{aligned}$$

Using the equations above, we can rewrite the above expression to have:

$$\frac{\tau_t^\kappa\kappa'(s_{K,t})}{p(\theta_{K,t})} = (1-\rho^K_t)\mathbb{E}_t\Xi_{t+1|t}\left\{p_{SE,t+1}z_{SE,t+1} - r_{SE,t+1} + \frac{\tau_{t+1}^\kappa\kappa'(s_{K,t})}{p(\theta_{K,t+1})}\right\}.$$

This equation implicitly defines $s_{K,t}$.

7.2 Value Equations and Nash Price Determination

7.2.1 Value Equations

In what follows, we define the household and firm value functions to determine the Nash wage and rental rates. The value of having an additional salaried worker in a large firm, $\mathbf{J}_{L,t}$, is given by the difference between the firm's marginal revenue product of labor and the subsidy-adjusted wage, plus the continuation value in the event that the match survives into the following period (in equilibrium the value of any vacancy is zero—this reflects that vacancies are posted until the value of doing so is exhausted):

$$\mathbf{J}_{L,t} = p_{L,t} z_{L,t} \mathbb{F}_{n_L}(n_{L,t}, \omega_t k_{L,t}) - \tau_t^{w_L} w_{L,t} + (1 - \rho_t^L) \mathbb{E}_t \Xi_{t+1|t} \mathbf{J}_{L,t+1}.$$

Similar intuition lies behind the value of an additional salaried worker for a small business, $\mathbf{J}_{SB,t}$:

$$\mathbf{J}_{OSB,t} = p_{SB,t} z_{SB,t} F_{n_{SB}}(n_{SB,t}, o_{SB,t}) - \tau_t^{w_{SB}} w_{SB,t} + \mathbb{E}_t \Xi_{t+1|t} \left\{ (1 - \rho_t^K)(1 - \rho_t^O)(1 - \rho_t^S) \mathbf{J}_{OSB,t+1} \right\}.$$

The household's value of having an additional household member in salaried employment in a large firm, $\mathbf{W}_{L,t}$, is given by corresponding wage payment plus the expected continuation value:

$$\mathbf{W}_{L,t} = w_{L,t} + \mathbb{E}_t \Xi_{t+1|t} \left\{ (1 - \rho_t^L) \mathbf{W}_{L,t+1} + \rho_t^L \mathbf{W}_{U,t+1} \right\},$$

where: $\mathbf{W}_U$ is the household's value of unemployment. Similar intuition lies behind the value of having an additional household member employed by a small business:

$$\mathbf{W}_{SB,t} = w_{SB,t} + E_t \Xi_{t+1|t} \left\{ \begin{array}{c} (1 - \rho_t^K)(1 - \rho_t^O)(1 - \rho_t^S) \mathbf{W}_{SB,t+1} \\ + \left[1 - (1 - \rho_t^K)(1 - \rho_t^O)(1 - \rho_t^S) \right] \mathbf{W}_{U,t+1} \end{array} \right\}.$$

In turn, the household's value of unemployment is:

$$\mathbf{W}_{U,t} = b + \mathbb{E}_t \Xi_{t+1|t} \left\{ \begin{array}{c} (1-\rho_t^L) p(\theta_{L,t}) \mathbf{W}_{L,t+1} + (1-\rho_t^K) s_{K,t}^u f_{K,t} \mathbf{W}_{SE,t+1} \\ +(1-\rho_t^K)(1-\rho_t^O)(1-\rho_t^S)\left[f_{SB,t} + f_{SE,t}\right] \mathbf{W}_{SB,t+1} \\ + \left[\begin{array}{c} 1-(1-\rho_t^L) f_{L,t} - (1-\rho_t^K) s_{K,t}^u f_{K,t} \\ -(1-\rho_t^K)(1-\rho_t^O)(1-\rho_t^S)\left[f_{SB,t} + f_{SE,t}\right] \end{array} \right] \mathbf{W}_{U,t+1} \end{array} \right\},$$

where: $\mathbf{W}_{SE,t}$ is the value to the household of an additional self-employed individual (defined below) and $s_{K,t}^u$ is the amount of resources spent on searching for a capital supplier per unemployed individual, $s_{K,t}^u \equiv s_{K,t}/u_t$.

The value to a large firm of having a capital relationship with a self-employed individual, $\mathbf{J}_{SE,t}$, is given by the corresponding rental rate adjusted for the probability that the capital relationship is destroyed net of the capital depreciation rate plus the expected continuation value:

$$\mathbf{J}_{SE,t} = r_{SE,t} + (\rho_t^K - \delta) + E_t \Xi_{t+1|t} \left\{ \begin{array}{c} (1-\rho_t^K)(1-(1-\rho_t^O)(1-\rho_t^S) v_{SE,t} q_{SE,t}) \mathbf{J}_{SE,t+1} \\ +(1-\rho_t^K)(1-\rho_t^O)(1-\rho_t^S) v_{SE,t} q_{SE,t} \mathbf{J}_{SB,t+1} \end{array} \right\}.$$

Similar intuition lies behind the value to a large firm of having a capital relationship with a small business, $\mathbf{J}_{SB,t}$:

$$\mathbf{J}_{SB,t} = r_{SB,t} + \left[(1-(1-\rho_t^O)(1-\rho_t^S) - \delta\right] + E_t \Xi_{t+1|t} \left\{ \begin{array}{c} (1-\rho_t^K)(1-\rho_t^O) \mathbf{J}_{SB,t+1} \\ +(1-\rho_t^K) \rho_t^O \mathbf{J}_{SE,t+1} \end{array} \right\}.$$

In turn, the value to a self-employed individual of having a capital relationship with a large firm is, $\mathbf{W}_{SE,t}$, is given by the difference between the marginal revenue product of capital and the rental rate, plus the expected continuation value:

$$\mathbf{W}_{SE,t} = p_{SE,t} z_{SE,t} - r_{SE,t} + \mathbb{E}_t \Xi_{t+1|t} \left\{ \begin{array}{c} (1-\rho_t^K)\left[1-(1-\rho_t^O)(1-\rho_t^S) v_{SE,t} q_{SE,t}\right] \mathbf{W}_{SE,t+1} \\ +(1-\rho_t^K)(1-\rho_t^O)(1-\rho_t^S) v_{SE,t} q_{SE,t} \mathbf{J}_{OSB,t+1} + \rho_t^K \mathbf{W}_{U,t+1} \end{array} \right\}.$$

Finally, similar intuition as before applies to the value to a small business from having a

capital relationship with a large firm, $\mathbf{W}_{OSB,t}$, which is given by:

$$\mathbf{W}_{OSB,t} = p_{SB,t} z_{SB,t} F_{o_{SB}}\left(n^{d}_{SB,t}, o_{SB,t}\right) - r_{SB,t} + \mathbb{E}_t \Xi_{t+1|t} \left\{ \begin{array}{c} (1-\rho^K_t)(1-\rho^O_t)\mathbf{W}_{OSB,t+1} \\ +(1-\rho^K_t)\rho^O_t \mathbf{W}_{SE,t+1} + \rho^K_t \mathbf{W}_{U,t+1} \end{array} \right\}.$$

7.2.2 Nash Wage Rate Determination

Large firms and salaried workers choose a wage $w_{L,t}$ to

$$\max_{w_{L,t}} \left\{ \left(\mathbf{W}_{L,t} - \mathbf{W}_{U,t}\right)^{\chi_L} \left(\mathbf{J}_{L,t}\right)^{1-\chi_L} \right\}.$$

Small business owners and small firm salaried workers choose a wage $w_{SB,t}$ to

$$\max_{w_{SB,t}} \left\{ \left(\mathbf{W}_{SB,t} - \mathbf{W}_{U,t}\right)^{\chi_{SB}} \left(\mathbf{J}_{OSB,t}\right)^{1-\chi_{SB}} \right\}.$$

Above $\chi_L \in (0,1)$ and $\chi_{SB} \in (0,1)$ are, respectively, the bargaining power of workers negotiating with large firms and the bargaining power of workers negotiating with small businesses. $\mathbf{W}_{L,t} - \mathbf{W}_{U,t}$ and $\mathbf{J}_{L,t}$ are the household's and large firm's respective capital gains from large firm salaried employment. Similarly, $\mathbf{W}_{SB,t} - \mathbf{W}_{U,t}$ and $\mathbf{J}_{OSB,t}$ are the household's and small business owner's capital gains from salaried employment in small firms. $\mathbf{W}_{U,t}$ is the household's value of having a household member in unemployment. The implicit expressions for the Nash wages are given by

$$\frac{\chi_L}{\left(1-\chi_L\right)\tau^{w_L}_t} \mathbf{J}_{L,t} = \left(\mathbf{W}_{L,t} - \mathbf{W}_{U,t}\right)$$

for $w_{L,t}$ and

$$\frac{\chi_{SB}}{\left(1-\chi_{SB}\right)\tau^{w_{SB}}_t} \mathbf{J}_{OSB,t} = \left(\mathbf{W}_{SB,t} - \mathbf{W}_{U,t}\right)$$

for $w_{SB,t}$.

7.2.3 Nash Rental Rate Determination

Large firms and the self-employed choose a rental rate $r_{SE,t}$ to

$$\max_{r_{SE,t}} \left\{ (\mathbf{W}_{SE,t} - \mathbf{W}_{U,t})^{\chi_K} \left(\mathbf{J}_{SE,t} - (1-\delta) \right)^{1-\chi_K} \right\}.$$

Large firms and small business owners choose a rental rate $r_{SB,t}$ to

$$\max_{r_{SB,t}} \left\{ (\mathbf{W}_{OSB,t} - \mathbf{W}_{U,t})^{\chi_O} \left(\mathbf{J}_{SB,t} - (1-\delta) \right)^{1-\chi_O} \right\}.$$

Above, the outside option of large firms is the value of a unit of capital net of depreciation, $(1-\delta)$. Also, $\chi_{SE} \in (0,1)$ and $\chi_O \in (0,1)$ are, respectively, the bargaining power of self-employed and small business owners. $\mathbf{W}_{SE,t} - \mathbf{W}_{U,t}$ and $\mathbf{J}_{SE,t} - (1-\delta)$ are the household's and large firm's respective capital gains from self-employment capital relationships. Similarly, $\mathbf{W}_{OSB,t} - \mathbf{W}_{U,t}$ and $\mathbf{J}_{SB,t} - (1-\delta)$ are the household's and the large firm's respective capital gains from small firm capital relationships. The implicit expressions for the Nash rental rates are given by

$$\frac{\chi_K}{(1-\chi_K)} \left(\mathbf{J}_{SE,t} - (1-\delta) \right) = (\mathbf{W}_{SE,t} - \mathbf{W}_{U,t})$$

for $r_{SE,t}$ and

$$\frac{\chi_O}{(1-\chi_O)} \left(\mathbf{J}_{SB,t} - (1-\delta) \right) = (\mathbf{W}_{OSB,t} - \mathbf{W}_{U,t})$$

for $r_{SB,t}$.

7.2.4 Explicit Wage Expressions Without Policy

To obtain explicit expressions for wages without the policies in place, first note that

$$\frac{\psi_L}{q_{L,t}} = (1-\rho_t^L) \mathbb{E}_t \Xi_{t+1|t} \mathbf{J}_{L,t+1},$$

$$\frac{p_{L,t} z_{L,t} \mathbb{F}_{\omega k_L}(n_{L,t}, \omega_t k_{L,t}) + (1-\rho_t^K) q_{K,t}}{q_{K,t}} = (1-\rho_t^K) \mathbb{E}_t \Xi_{t+1|t} \mathbf{J}_{SE,t+1},$$

and

$$\frac{\psi_{SB}}{q_{SB,t}} = (1-\rho_t^O)(1-\rho_t^S)(1-\rho_t^K)\mathbb{E}_t\Xi_{t+1|t}\mathbf{J}_{OSB,t+1}.$$

Then, write

$$\begin{aligned}\mathbf{W}_{L,t} - \mathbf{W}_{U,t} &= w_{L,t} + \mathbb{E}_t\Xi_{t+1|t}\left\{(1-\rho_t^L)\mathbf{W}_{L,t+1} + \rho_t^L\mathbf{W}_{U,t+1}\right\} - b \\ &+\mathbb{E}_t\Xi_{t+1|t}\left\{\begin{array}{c} (1-\rho_t^L)f_{L,t}\mathbf{W}_{L,t+1} + (1-\rho_t^K)s_{K,t}^u f_{K,t}\mathbf{W}_{SE,t+1} \\ +(1-\rho_t^K)(1-\rho_t^O)(1-\rho_t^S)(f_{SB,t}+f_{SE,t})\mathbf{W}_{SB,t+1} \\ +\left[\begin{array}{c} 1-(1-\rho_t^L)p_{Lt} - (1-\rho_t^K)s_{K,t}^u f_{K,t} \\ -(1-\rho_t^K)(1-\rho_t^O)(1-\rho_t^S)(f_{SB,t}+f_{SE,t}) \end{array}\right]\mathbf{W}_{U,t+1} \end{array}\right\}.\end{aligned}$$

After some algebra, we have

$$\begin{aligned}\mathbf{W}_{L,t} - \mathbf{W}_{U,t} &= w_{L,t} - b + \mathbb{E}_t\Xi_{t+1|t}\left\{(1-\rho_t^L)(1-f_{L,t})(\mathbf{W}_{L,t+1} - \mathbf{W}_{U,t+1})\right\} \\ &+\mathbb{E}_t\Xi_{t+1|t}\left\{\begin{array}{c} (1-\rho_t^K)s_{K,t}^u f_{K,t}(\mathbf{W}_{SE,t+1} - \mathbf{W}_{U,t+1}) \\ +(1-\rho_t^K)(1-\rho_t^O)(1-\rho_t^S)(f_{SB,t}+f_{SE,t})(\mathbf{W}_{SB,t+1} - \mathbf{W}_{U,t+1}) \end{array}\right\}.\end{aligned}$$

Using the first-order conditions that determine the implicit functions for $w_{L,t}, w_{SB,t}$, and $r_{SE,t}$, we can write

$$\begin{aligned}\frac{\chi_L}{(1-\chi_L)}\mathbf{J}_{L,t} &= w_{L,t} - b + \mathbb{E}_t\Xi_{t+1|t}\left\{(1-\rho_t^L)(1-f_{L,t})\frac{\chi_L}{(1-\chi_L)}\mathbf{J}_{L,t+1}\right\} \\ &+\mathbb{E}_t\Xi_{t+1|t}\left\{\begin{array}{c} (1-\rho_t^K)s_{K,t}^u f_{K,t}\frac{\chi_K}{(1-\chi_K)}\left(\mathbf{J}_{SE,t+1} - (1-\delta)\right) \\ +(1-\rho_t^K)(1-\rho_t^O)(1-\rho_t^S)(f_{SB,t}+f_{SE,t})\frac{\chi_{SB}}{(1-\chi_{SB})}\mathbf{J}_{OSB,t+1} \end{array}\right\}.\end{aligned}$$

Now, based on as the job creation and capital supply conditions above when the policies are suppressed, we have

$$\begin{aligned}\frac{\chi_L}{(1-\chi_L)}\left(p_{L,t}\mathbb{F}_{n_{L,t}}(n_{L,t},\omega_t k_{L,t}) - w_{L,t} + \frac{\psi_L}{q_{L,t}}\right) &= w_{L,t} - b + \frac{(1-f_{L,t})\chi_L}{(1-\chi_L)}\frac{\psi_L}{q_{L,t}} \\ -\frac{\chi_K s_{K,t}^u f_{K,t}}{(1-\chi_K)}&\left(\frac{p_{L,t}z_{L,t}\mathbb{F}_{\omega k_L}(n_{L,t},\omega_t k_{L,t}) + (1-\rho_t^K)q_{K,t}}{q_{K,t}}\right) \\ +\frac{\chi_K s_{K,t}^u f_{K,t}}{(1-\chi_K)}&\mathbb{E}_t\Xi_{t+1|t}(1-\rho_t^K)(1-\delta) - \frac{\chi_{SB}(f_{SB,t}+f_{SE,t})}{(1-\chi_{SB})}\frac{\psi_{SB}}{q_{SB,t}}.\end{aligned}$$

Rearranging terms, we have

$$w_{L,t} = (1-\chi_L)\, b + \chi_L \theta_{L,t}\psi_L + \frac{(1-\chi_L)\, s^u_{K,t} f_{K,t}\nu_K}{(1-\chi_K)} \left(\frac{p_{L,t} z_{L,t} \mathbb{F}_{\omega k_L}(n_{L,t}, \omega_t k_{L,t})}{q_{K,t}} \right) - \frac{(1-\chi_L)\, \chi_K s^u_{K,t} f_{K,t}}{(1-\chi_K)} (1-\rho^K_t) \left[1 - \mathbb{E}_t \Xi_{t+1|t} (1-\delta) \right] + \frac{(1-\chi_L)\, \chi_{SB} (f_{SB,t} + f_{SE,t})}{(1-\chi_{SB})} \frac{\psi_{SB}}{q_{SB,t}},$$

where we make use of fact that $f_{L,t}/q_{L,t} = \theta_{L,t}$. Finally, we can write

$$\begin{aligned} w_{L,t} &= (1-\chi_L)\, b + \chi_L \theta_{L,t} \tau^{v_L}_t \psi_L + \frac{(1-\chi_L)\, s_{K,t} f_{K,t} \chi_K}{(1-\chi_K)} \left(\frac{p_{L,t} z_{L,t} \mathbb{F}_{\omega k_L}(n_{L,t}, \omega_t k_{L,t})}{q_{K,t}} \right) \\ &- \frac{(1-\chi_L)\, \chi_K s_{K,t} f_{K,t}}{(1-\chi_K)} (1-\rho^K_t) \left[1 - \mathbb{E}_t \Xi_{t+1|t} (1-\delta) \right] \\ &+ \frac{(1-\chi_L)\, \chi_{SB}}{(1-\chi_{SB})} \theta_{SB,t} \psi_{SB} + \frac{(1-\chi_L)\, \chi_{SB} f_{SE,t}}{(1-\chi_{SB})} \frac{\psi_{SB}}{q_{SB,t}}, \end{aligned}$$

where we make use of the fact that $f_{SB,t}/q_{SB,t} = \theta_{SB,t}$. Following similar steps by constructing $(\mathbf{W}_{SB,t} - \mathbf{W}_{U,t})$, the wage for small-firm workers is

$$w_{SB,t} = (1-\chi_{SB})\, b + \chi_{SB} \theta_{SB,t} \psi_{SB} + \nu_{SB} \frac{f_{SE,t} \psi_{SB}}{q_{SB,t}} + \frac{(1-\chi_{SB})\, \chi_K s^u_{K,t} f_{K,t}}{(1-\chi_K)} \left(\frac{p_{L,t} z_{L,t} \mathbb{F}_{\omega k_L}(n_{L,t}, \omega_t k_{L,t})}{q_{K,t}} \right) - \frac{(1-\chi_{SB})\, \chi_K s^u_{K,t} f_{K,t}}{(1-\chi_K)} (1-\rho^K_t) \left[1 - \mathbb{E}_t \Xi_{t+1|t} (1-\delta) \right] + \frac{(1-\chi_{SB})\, \chi_L}{(1-\chi_L)} \theta_{L,t} \psi_L.$$

7.3 Calibration of Productivity Differentials Across Firm Types

Busso, Fazio, and Levy (2012) use Mexican Census data to characterize the distribution of employment (as well as capital) based on firm size and the formality and legality status of the firm. Fernández and Meza (2013) use their evidence to determine the productivity gap between formal and informal firms (focusing on legal firms alone since they consider that informal firms in their model are household firms that do not hire workers (i.e., household firms are self-employed firms)).[43] We follow their approach (and notation) in calibrating the productivity parameters across firm types.

7.3.1 Distribution of Employment by Employment Status and Definition of Firm Size in the Model

Given the different types of firms in our model, our mapping between productivity in the data and productivity in the model is more detailed as it considers both firm size and legal/formality status. We assume that the self-employed are those workers in firms of size [0-5] that are legal and informal. These represent self-employed (own-account) firms in the model. Workers in small firms are those working in firms of size [0-5] excluding informal and legal firms, as well as firms of sizes [6-10] and [11-50] excluding legal and formal firms. Large firms include workers that are formal and legal (regardless of firm size), as well as all workers in firms of size [+50].[44] Combining evidence on the share of self-employment for Mexico and the share of workers that are informal but legal (Busso, Fazio, and Levy, 2012), we assume that the self-employed account for 16 percent of the labor force, and small business owners 7 percent of the labor force. In turn, salaried workers in small and large firms account for 30 and 42 percent of the labor force, respectively.

[43] As defined in Busso, Fazio, and Levy (2012), legal firms are those that comply with mandated social security contributions for the all their salaried workers. Semi-legal firms have salaried workers but pay social security for only a fraction of them.

[44] We include all legal and formal firms within the same category – regardless of firm size – since their legality implies that policymakers can readily introduce cyclical wage subsidies by using firms' tax statements and social security records. This is not the case for firms that hire informal salaried workers since these workers are often not reported to the government.

7.3.2 Productivity Differentials Based on Firm Size and Formality/Legality Status

Based on the mapping between firms in the data and firm types in the model, we can use evidence on productivity differences by firm size to compute the steady-state sectoral productivity differentials in the model. Using Table 10 in Busso, Fazio, and Levy (2012) and following Fernández and Meza (2013), we have

$$\log\left(\frac{TFP_{LSF}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -0.959 \quad \Rightarrow \quad \frac{TFP_{LSF}}{TFP_{LF}} = 0.3833,$$

$$\log\left(\frac{TFP_{LI}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -1.759 \quad \Rightarrow \quad \frac{TFP_{LI}}{TFP_{LF}} = 0.1722,$$

$$\log\left(\frac{TFP_{SLSF}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -0.214 \quad \Rightarrow \quad \frac{TFP_{SLSF}}{TFP_{LF}} = 0.8073,$$

$$\log\left(\frac{TFP_{II}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -0.881 \quad \Rightarrow \quad \frac{TFP_{II}}{TFP_{LF}} = 0.4144,$$

for firms of size [0-5],

$$\log\left(\frac{TFP_{LSF}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -0.910 \quad \Rightarrow \quad \frac{TFP_{LSF}}{TFP_{LF}} = 0.4025,$$

$$\log\left(\frac{TFP_{LI}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -0.688 \quad \Rightarrow \quad \frac{TFP_{LI}}{TFP_{LF}} = 0.5026,$$

$$\log\left(\frac{TFP_{SLSF}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -0.085 \quad \Rightarrow \quad \frac{TFP_{SLSF}}{TFP_{LF}} = 0.9185,$$

$$\log\left(\frac{TFP_{II}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -0.632 \quad \Rightarrow \quad \frac{TFP_{II}}{TFP_{LF}} = 0.5315,$$

for firms of size [6-10], and

$$\log\left(\frac{TFP_{LSF}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -0.746 \quad \Rightarrow \quad \frac{TFP_{LSF}}{TFP_{LF}} = 0.4743,$$

$$\log\left(\frac{TFP_{LI}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -0.395 \quad \Rightarrow \quad \frac{TFP_{LI}}{TFP_{LF}} = 0.6737,$$

$$\log\left(\frac{TFP_{SLSF}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -0.092 \quad \Rightarrow \quad \frac{TFP_{SLSF}}{TFP_{LF}} = 0.9121,$$

$$\log\left(\frac{TFP_{II}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -0.701 \quad \Rightarrow \quad \frac{TFP_{II}}{TFP_{LF}} = 0.4961,$$

for firms of size [11-50]. Finally, for firms of size [+50], we have

$$\log\left(\frac{TFP_{LSF}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -0.574 \quad \Rightarrow \quad \frac{TFP_{LSF}}{TFP_{LF}} = 0.5633,$$

$$\log\left(\frac{TFP_{LI}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -0.491 \quad \Rightarrow \quad \frac{TFP_{LI}}{TFP_{LF}} = 0.6120,$$

$$\log\left(\frac{TFP_{SLSF}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -0.159 \quad \Rightarrow \quad \frac{TFP_{SLSF}}{TFP_{LF}} = 0.8530,$$

$$\log\left(\frac{TFP_{II}}{TFP}\right) - \log\left(\frac{TFP_{LF}}{TFP}\right) = -1.039 \quad \Rightarrow \quad \frac{TFP_{II}}{TFP_{LF}} = 0.3538.$$

Using Tables 6 and 10 in Busso, Fazio, and Levy (2012), we can construct employment-weighted measures of TFP for each type of firm (self-employed, TFP_{SE}, small, TFP_{SB}, and large, TFP_L). Then,

$$TFP_{SE} = TFP_{[0-5],LI}$$

$$\begin{aligned} TFP_{SB} = &\left(\frac{0.79}{32.17}\right) TFP_{[0-5],LSF} + \left(\frac{1.69}{32.17}\right) TFP_{[0-5],SLSF} + \left(\frac{10.94}{32.17}\right) TFP_{[0-5],II} \\ &+ \left(\frac{1.63}{32.17}\right) TFP_{[6-10],LI} + \left(\frac{0.50}{32.17}\right) TFP_{[6-10],LSF} + \left(\frac{1.92}{32.17}\right) TFP_{[6-10],SLSF} \\ &+ \left(\frac{3.66}{32.17}\right) TFP_{[6-10],II} + \left(\frac{2.39}{32.17}\right) TFP_{[11-50],LI} + \left(\frac{0.57}{32.17}\right) TFP_{[11-50],LSF} \\ &+ \left(\frac{4.72}{32.17}\right) TFP_{[11-50],SLSF} + \left(\frac{3.36}{32.17}\right) TFP_{[11-50],II} \end{aligned}$$

$$\begin{aligned} TFP_{L} = &\left(\frac{0.77}{44.02}\right) TFP_{[0-5],LF} + \left(\frac{1.09}{44.02}\right) TFP_{[6-10],LF} + \left(\frac{3.84}{44.02}\right) TFP_{[11-50],LF} \\ &+ \left(\frac{14.67}{44.02}\right) TFP_{[+50],LF} + \left(\frac{7.74}{44.02}\right) TFP_{[+50],LI} + \left(\frac{1.19}{44.02}\right) TFP_{[+50],LSF} \\ &+ \left(\frac{13.75}{44.02}\right) TFP_{[+50],SLSF} + \left(\frac{1.18}{44.02}\right) TFP_{[+50],II} \end{aligned}$$

Using the values for TFP *relative to* TFP_{LF} by firm size and legality/formality status above, we have

$$TFP_{SE} = 0.1722$$

$$\begin{aligned} TFP_{SB} = &\left(\frac{0.79}{30.54}\right)(0.3833) + \left(\frac{1.69}{30.54}\right)(0.8073) + \left(\frac{10.94}{30.54}\right)(0.4144) \\ &+ \left(\frac{1.63}{32.17}\right)(0.5026) + \left(\frac{0.50}{30.54}\right)(0.4025) + \left(\frac{1.92}{30.54}\right)(0.9185) \\ &+ \left(\frac{3.66}{30.54}\right)(0.5315) + \left(\frac{2.39}{30.54}\right)(0.6737) + \left(\frac{0.57}{30.54}\right)(0.4743) \\ &+ \left(\frac{4.72}{30.54}\right)(0.9121) + \left(\frac{3.36}{30.54}\right)(0.4961) \end{aligned}$$

$$\begin{aligned} TFP_{L} = &\left(\frac{0.77}{44.02}\right)(1) + \left(\frac{1.09}{44.02}\right)(1) + \left(\frac{3.84}{44.02}\right)(1) \\ &+ \left(\frac{14.67}{44.02}\right)(1) + \left(\frac{7.74}{44.02}\right)(0.6737) + \left(\frac{1.19}{44.02}\right)(0.4743) \\ &+ \left(\frac{13.75}{44.02}\right)(0.9121) + \left(\frac{1.18}{44.02}\right)(0.4161) \end{aligned}$$

This yields $TFP_{SE} = 0.1722, TFP_{SB} = 0.6150$, and $TFP_{L} = 0.8901$. Then, normalizing TFP_{SE} to one, and using the mapping $TFP_{L} = z_{L}, TFP_{SB} = z_{SB}$, and $TFP_{SE} = z_{SE}$ between the data and the model, we have

$$z_{L} = 5.1690, \quad z_{SB} = 3.5714, \quad z_{SE} = 1.$$

7.4 Robustness and Additional Experiments

7.4.1 Asymmetry in Elasticities of Substitution between Types of Output

Assuming that $\phi_a = 0.8$ and $\phi_s = 0.6$ makes the main conclusions under the benchmark calibration starker. The main differences are as follows. With hiring subsidies for self-employed firms (i.e., small firm creation subsidies), consumption decreases by more (as capital search expenditures are more responsive) and there is no difference in the recovery speed of total output relative to the benchmark economy. A similar claim can be made regarding the expansion of intermediation services for small firms, where consumption falls by more and the output recovery is more sluggish relative to the results in the main text. Similarly, with hiring subsidies for existing small firms, the gains in terms of a faster initial recovery in consumption, investment, and output are more short-lived relative to the benchmark results. Finally, the gains from hiring subsidies for large firms and the improvement in matching for employment in large firms are larger. Thus, the main conclusions do not change. Conversely, assuming that $\phi_a = 0.6$ and $\phi_s = 0.8$ entails the following changes. Introducing hiring subsidies for self-employed firms make consumption less volatile, and investment and output recover faster. Thus, under this alternative calibration, the gains from these subsidies are somewhat larger relative to the benchmark calibration under the same policy. With hiring subsidies for small firms, the initial faster recovery in total output is longer (even though it still fades away in the medium term). Despite these changes, the main message of the paper does not change. Similarly, the recovery of output under better matching for employment in small firms is less sluggish, but the same policy for large firms still dominates. Importantly, regardless of the calibration of ϕ_a and ϕ_s, the adverse aggregate impact of introducing self-employment search (startup) subsidies does not change.

7.4.2 Higher Volatility of Self-Employment Separation Rates

Calibrating the model to capture the relative volatility of self-employment separation rates (as opposed to the relative volatility of salaried separation rates as in the benchmark model)

in the data does not change the main conclusions.

7.4.3 Financing Cyclical Subsidies Through Government Debt

We follow Campolmi, Faia, and Winkler (2011) and allow for subsidies to firms to be financed partly through government debt. If the government uses lump-sum taxes and government debt to finance expenditures, the government budget constraint is

$$\begin{aligned} T_t + b_t^* = bu_t + (1-\tau_t^{v_L})\psi_L v_{L,t} + (1-\tau_t^{v_{SE}})\psi_{SE} v_{SE,t} n_{SE,t} + (1-\tau_t^{v_{SB}})\psi_{SB} v_{SB,t} o_{SB,t} \\ +(1-\tau_t^{\kappa})\kappa(s_{K,t}) + R_{t-1} b_{t-1}^* + g_t + (1-\tau_t^{w_L}) w_{L,t} n_{L,t} + (1-\tau_t^{w_{SB}}) w_{SB,t} n_{SB,t} \\ +(\tau_t^{m_L}-1) m_L\left(v_{L,}, u_t\right) + (\tau_t^{m_{SB}}-1) m_{SB}\left(v_{SB,t} o_{SB,t}, u_t\right). \end{aligned}$$

where g_t is government spending. Following Totzek and Winkler (2010) and Campolmi, Faia and Winkler (2011), we assume that a fraction γ_b of government expenditures is financed through debt, so that

$$b_t^* = \gamma_b \left(\begin{array}{c} bu_t + (1-\tau_t^{v_L})\psi_L v_{L,t} + (1-\tau_t^{\kappa})\kappa(s_{K,t}) + R_{t-1} b_{t-1}^* + g_t - T_t \\ +(1-\tau_t^{v_{SE}})\psi_{SE} v_{SE,t} n_{SE,t} + (1-\tau_t^{v_{SB}})\psi_{SB} v_{SB,t} o_{SB,t} \\ +(1-\tau_t^{w_L}) w_{L,t} n_{L,t} + (1-\tau_t^{w_{SB}}) w_{SB,t} n_{SB,t} \\ +(\tau_t^{m_L}-1) m_L\left(v_{L,}, u_t\right) + (\tau_t^{m_{SB}}-1) m_{SB}\left(v_{SB,t} o_{SB,t}, u_t\right) \end{array} \right).$$

The impact of the policies is qualitatively similar to the one where the cyclical subsidies are fully financed using lump-sum taxation.

7.4.4 Financing Hiring Subsidies for Small Firms with Payroll Taxes on Large Firms

While it may be feasible to introduce a hiring subsidy to small firms during a downturn, raising the revenue for that subsidy through distortionary taxes on small firms may be more difficult given the high rates of informality among these firms. Assuming that the

hiring subsidy for small firms is financed through an increase in payroll taxes for large firms naturally reduces the beneficial impact from the subsidy relative to the case with lump-sum taxation or government debt. In particular, relative to the no-policy case, the recovery of consumption, investment, and output become more sluggish in the medium term, but unemployment still falls due to the surge in small-firm employment.

7.4.5 Policies with Identical Subsidy Rates

The benchmark model assumed the same fiscal cost when analyzing each of the policies and allowed the cyclical subsidies to adjust endogenously. This generated different subsidy rates depending on the type of policy. If we assume the same (impact) subsidy rate for all policies and let the fiscal cost adjust endogenously, the results can change in important ways for some of the policies. If we assume a subsidy rate of 5 percent on impact for all policies (see Banerji et al., 2014), hiring subsidies for small and self-employed firms have a negligible impact on consumption and output dynamics, though they still limit the initial rise in unemployment after the shock.[45] This takes place because the hiring subsidies need to be substantially higher to have a quantitative impact. In contrast, the effect of hiring subsidies for large firms is quantitatively similar to the main results, mainly because the subsidy rate in the latter is not much higher than 5 percent. The results for improved matching for employment in either large or small firms do not change much either, for similar reasons. Conversely, a subsidy of 5 percent implies that wage subsidies do have a larger quantitative impact (even though the fiscal cost also increases substantially), but the qualitative results do not change: wage subsidies for large firms increase unemployment by even more but also boost the recovery in output, whereas the same subsidies for small firms limit the rise in unemployment in the short term.

[45] In contrast to the benchmark results, a hiring subsidy for existing small firms only limits the rise in unemployment and does not lead to a drastic reduction in unemployment after the shock.

7.4.6 Combining Individual Policies

As should be the case, combining hiring subsidies with self-employment startup subsidies offsets the benefits from the hiring subsidies, so that the policy mix is much less effective in fostering a faster recovery and reducing aggregate volatility, even though it does limit the rise in unemployment. Similarly, combining the self-employment startup subsidies with improvements in job intermediation (i.e., matching) implies that the gains obtained when intermediation is focused on large firms are smaller.

7.4.7 Exogenous Separation Rates

The same policy experiments in a model with exogenous separation rates yield qualitatively similar results for the fiscal output and employment multipliers. Also, without the endogenous response in separations, all policies either limit the rise in unemployment or reduce unemployment by more relative to the benchmark model. The results for all other variables under exogenous separations remain qualitatively similar to those in the benchmark impulse response functions.

7.5 Tables and Figures: Motivation, Calibration, Business Cycle Statistics, Impulse Response Functions, and Additional Results

Table 1: Percent of Low- and Middle-Income Countries that Implemented Labor Demand and Job Matching Policies

Type of Policy	Percent of Countries
Direct Job Creation and Empl. Incentives	80
Credit Facilities, Access to Credit	65
Lower Non-Wage Labor Costs, Other Taxes	58
Public Employment Services	53
Special Measures for SMEs	49
Subsidies for Job Creation	44

Source: Figures 10 and 14, ILO and World Bank (2012). Notes: The sample of low- and middle-income economies includes 55 countries. 58 percent of the country sample introduced training for the unemployed. We abstract from analyzing training policies since the model would become substantially more complex.

Table 2: Allocation of Expenditures, Labor Demand and Job Matching Policies: Low- and Middle-Income Countries

Labor Demand Policies	Budget Allocation (Percent)
Direct Job Creation and Empl. Incentives	66.95
Subsidies to Employers Maintaining Existing Jobs	14.54
Credit Facilities, Access to Credit	12.97
Special Measures for SMEs	3.04

Programs that Improve Job Matching and Skills	Budget Allocation (Percent)
Public Employment Services	43
Training for Unemployed	31
Training for Employed	20

Source: Figures 11 and 15, ILO and World Bank (2012). Notes: The sample of low- and middle-income economies includes 55 countries. Subsidies to employers maintaining jobs include wage subsidies.

Table 3: Size Distribution for Selected Countries: Salaried Employment and Establishments

Firm	U.S.		Mexico		Argentina		Bolivia	
Size	Empl.	Estab.	Empl.	Estab.	Empl.	Estab.	Empl.	Estab.
$1-9$	4.2	54.5	22.7	90.5	22	84.0	43.6	91.7
$10-19$	4.8	14.5	5.5	4.2	25	12.9	10.0	4.2
$20-49$	11	14.7	8	2.7	19	2.5	13.6	2.6
$50-99$	12.3	7.4	7.5	1.1	35	0.8	9.8	0.8
100+	67.7	8.9	56.3	1.6	18	0.2	23.0	0.6

Source: Taken directly from Busso, Madrigal, and Pagés (2012). Note: Empl. and Establ. stand for Employment and Establishments, respectively. The sample year varies by country. The data in the Table is for manufacturing firms only. Similar patterns hold for other sectors.

Table 4: Consequences from Lack of External Financing for Latin American Small Firms

Consequences from Restricted Access to Financing	Percent of Entrepreneurs
Reduced Scale	56.0
Search for Partners	11.0
Search for Support from Suppliers or Customers	51.0
Delay in Launching Enterprise	32.0

Source: Taken directly from Table 6.5, IDB (2005b).

Table 5: Percent of Small Entrepreneurs Using Alternative Sources of Financing: Latin America

Financing Sources	Startup	Early Years
Suppliers	32.0	36.6
Purchase of Second-Hand Machinery and Equipment	27.5	20.6
Customers	18.0	19.1
Delaying Payment of Services	8	3.5

Source: Taken directly from Table 6.2, IDB (2005b).

Table 6: Allocation of Resources by Firm Size: Mexico

Firm Size	Capital	Workers	Value Added
$0-5$	13.2	37.8	10.3
$6-10$	4.5	8.8	4.6
$11-50$	10.2	14.9	12.5
$+50$	72.1	38.5	72.5

Source: Busso, Fazio, and Levy (2012).

Table 7: Parameterization for Benchmark Economy

Parameter	Value	Parameter Description	Parameter Source
α_L	0.32	Capital Share, Large Firms	DSGE Literature
α_{SB}	0.27	Capital Share, Small Firms	-
b	0	Unempl. Insurance	No Unempl. Benefits
β	0.985	Discount Factor	DSGE Literature
δ	0.025	Capital Depreciation	DSGE Literature
η_K	1	Curvature Search Cost	Search Literature
χ	0.50	Bargaining Power	Search Literature
ξ	0.50	Matching Elasticity	Search Literature
ϕ_a	0.7	Elasticity Param.	Assumption
ϕ_s	0.7	Elasticity Param.	Assumption
ρ_{ss}^L	0.05	Sep. Rate, Large Firms	Bosch, Maloney (2008)
ρ_{ss}^K	0.03	Sep. Rate, SE	Bosch, Maloney (2008)
ρ_{ss}^S	0.03	Sep. Rate, Small Firms	Assumption
ρ_{z_L}	0.92	Autocorrelation of z_L	DSGE Literature
$\rho_{z_{SB}}$	0.92	Autocorrelation of z_{SB}	DSGE Literature
$\rho_{z_{SE}}$	0.92	Autocorrelation of z_{SE}	DSGE Literature
σ	2	CRRA Utility Parameter	DSGE Literature
$\overline{z}_L$	5.17	Large Firm Productivity	Busso et al. (2012)
$\overline{z}_{SB}$	3.57	Small Firm Productivity	Busso et al. (2012)
$\overline{z}_{SE}$	1	SE Productivity	Normalization

Table 8: Calibrated Parameters and Targets: Benchmark Economy

Parameter	Value	Parameter Description	Target
g	0.0967	Steady State Gov. Spending	10.2 percent of output
M_L	0.111	Large Firm Match. Eff.	$n_L = 0.42$
M_K	0.045	SE Match. Eff.	$n_{SE} = 0.16$
M_{SB}	0.213	Old Small Firm Match. Eff.	$o_{SB} = 0.07$
M_{SE}	0.027	New Small Firm Match. Eff.	$n_{SB} = 0.30$
φ_k	0.496	Capital Adj. Cost Param.	$\sigma_{p^u_{K,t}} = 9.4$
ψ_L	0.027	Large Firm Vacancy Cost	3.5 percent of w_L
ψ_K	0.735	Capital Search Cost	3 months of w_{SB}
ψ_{SB}	0.026	Small Firm Vacancy Cost	3.5 percent of w_{SB}
ψ_{SE}	0.026	SE Vacancy Cost	3.5 percent of w_{SB}
ρ^O_{ss}	0.022	Destruction Rate, Small Firms	$(1 - \rho^{SB}) = 0.92$
η^L_{ss}	1.27	Sensitivity, Sep. Rate ρ^L_t	$\frac{\sigma_{E\to U}}{\sigma_{I\to U}} = 1.22$
η^K_{ss}	1.00	Sensitivity, Sep. Rate ρ^K_t	Assumption
η^O_{ss}	1.00	Sensitivity, Sep. Rate ρ^O_t	Assumption
η^S_{ss}	1.00	Sensitivity, Sep. Rate ρ^S_t	Assumption
σ_z	0.0173	SD Productivity Shock	$\sigma_y = 2.39$
γ_a	0.192	CES Parameter	$\frac{p_L y_L}{y} = 0.55$
γ_s	0.453	CES Parameter	$\frac{p_{SB} y_{SB}}{y} = 0.33$

Table 9: Business Cycle Statistics: Data vs. Model

Targeted Second Moments	Data	Benchmark Model
σ_{y_t}	2.39	2.39
$\sigma_{f^u_{K,t}}$	9.40	9.40
$\sigma_{\rho^{SB}_t}/\sigma_{\rho^{L}_t}$	1.22	1.23
Non-Targeted Moments		
$\sigma_{c_t}/\sigma_{y_t}$	1.13	0.56
$\sigma_{i_t}/\sigma_{y_t}$	2.78	7.54
$\sigma_{u_t}/\sigma_{y_t}$	6.28	1.31
$\sigma_{n_{L,t}}/\sigma_{n_{SB,t}}$	[1.00,1.05]	0.92
$\sigma_{f_{L,t}}/\sigma_{(f_{SB,t}+f_{SE,t})}$	2.18	2.29
$\rho(n_{L,t}, y_t)$	[0.740,0.840]	0.778
$\rho(n_{SB,t}, y_t)$	[-0.470,0.740]	0.767
$\rho(n_{SE,t}, y_t)$	-0.450	-0.886
$\rho(u_t, y_t)$	-0.889	-0.706
$\rho(f_{L,t}, y_t)$	0.798	0.984
$\rho(f_{SB,t} + f_{SE,t}, y_t)$	0.366	0.524
$\rho(f^u_{K,t}, y_t)$	-0.433	-0.538
$\rho(y_t, y_{t-1})$	0.846	0.734
$\rho(u_t, u_{t-1})$	0.878	0.748

Note: One caveat regarding the comparison between the model and the data. We assume that informal salaried workers in the data represent in broad terms the salaried workers in small firms in our model, even though informal salaried workers are present mostly in micro, and small firms, and to a much lesser extent in large firms. Also, a (relatively small) fraction of formal workers are present in small firms in both the data and in the calibration of the model (Busso, Fazio, and Levy, 2012). Finally, for the cyclical correlation of output with employment in large and small firms, we present lower and upper bounds for the moments in the data (based on different definitions of formal and informal employment in Fernández and Meza, 2013).

Figure 1: Response to a Negative Productivity Shock: Hiring Subsidies for Large Firms and Existing Small Firms

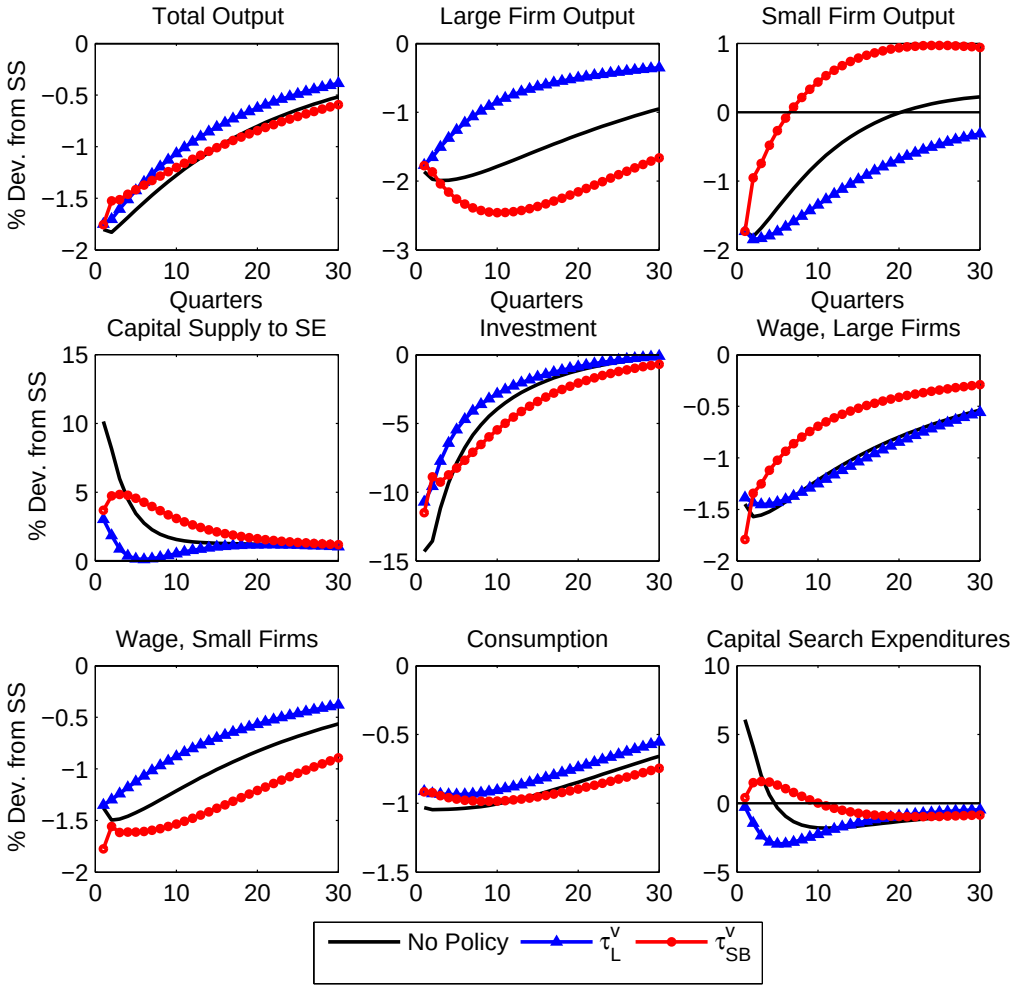

Figure 2: Response to a Negative Productivity Shock: Hiring Subsidies for Large Firms and Existing Small Firms (Cont.)

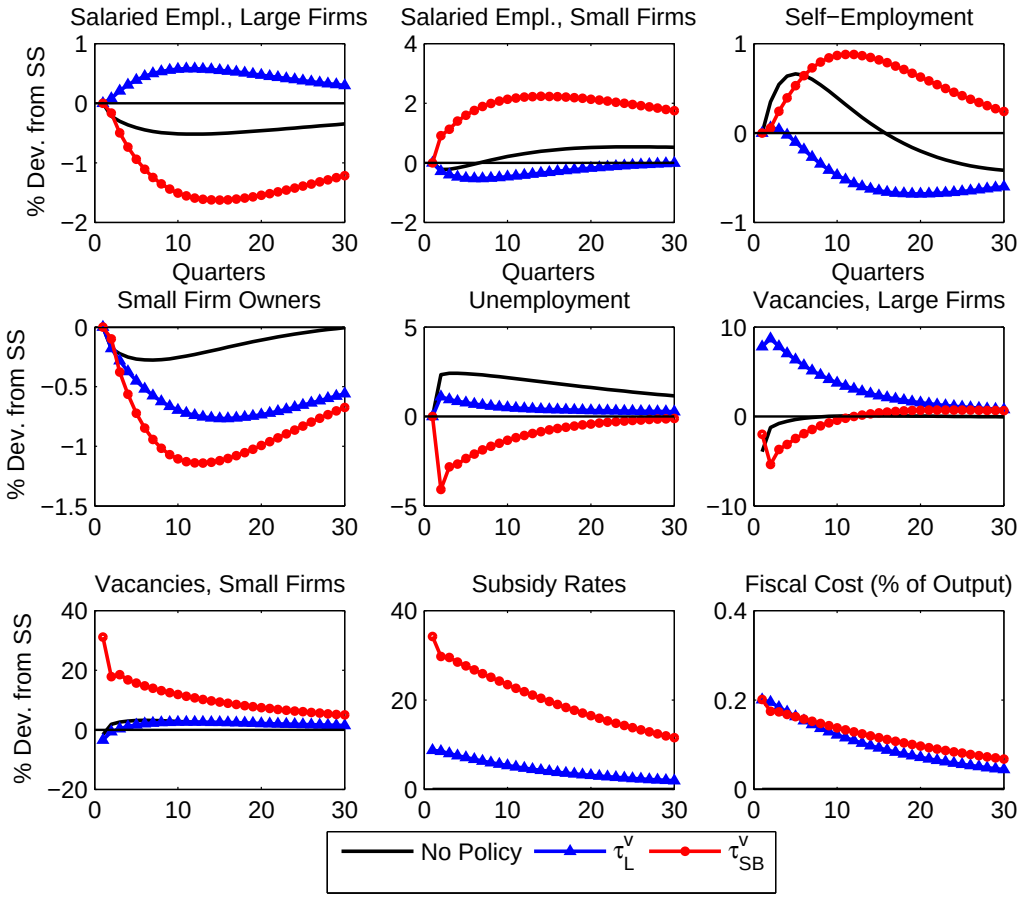

Figure 3: Response to a Negative Productivity Shock: Hiring Subsidies for Self-Employed Firms and Subsidies for Self-Employed Firm Creation

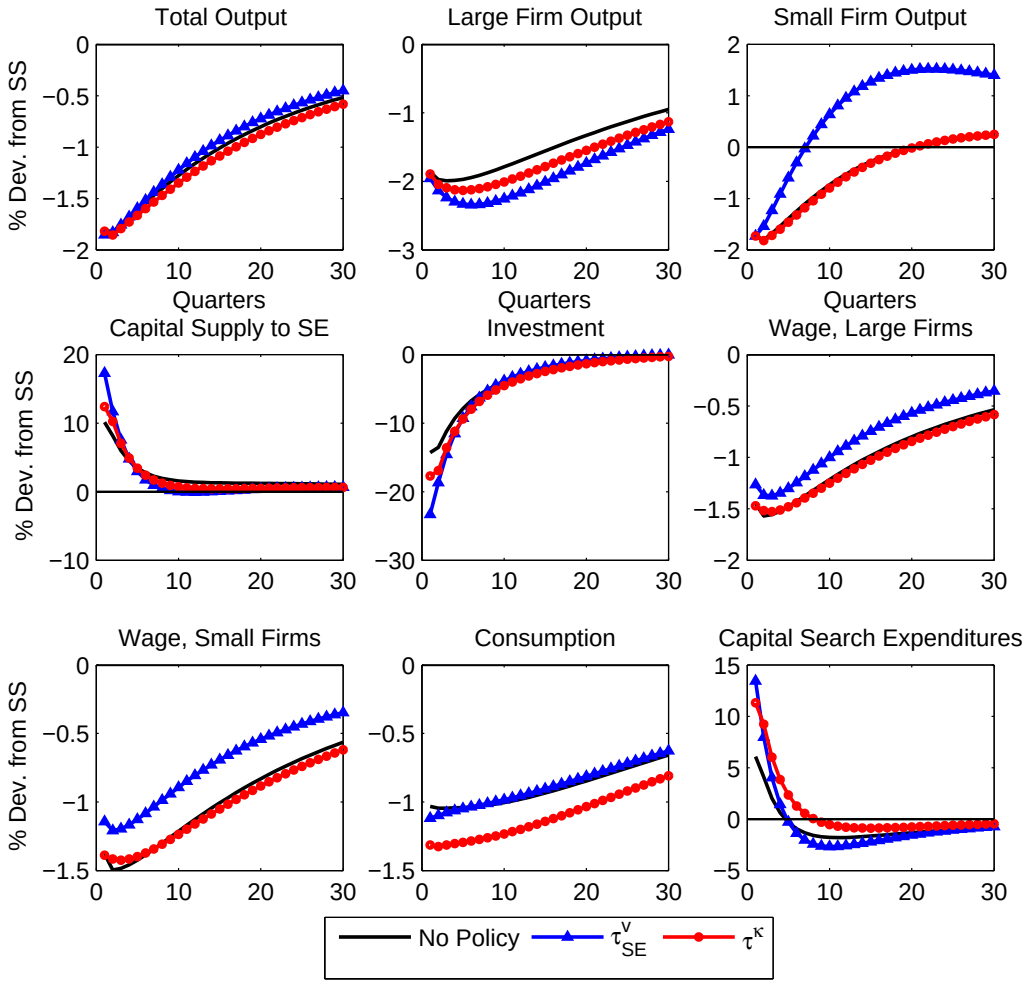

Figure 4: Response to a Negative Productivity Shock: Hiring Subsidies for Self-Employed Firms and Subsidies for Self-Employed Firm Creation (Cont.)

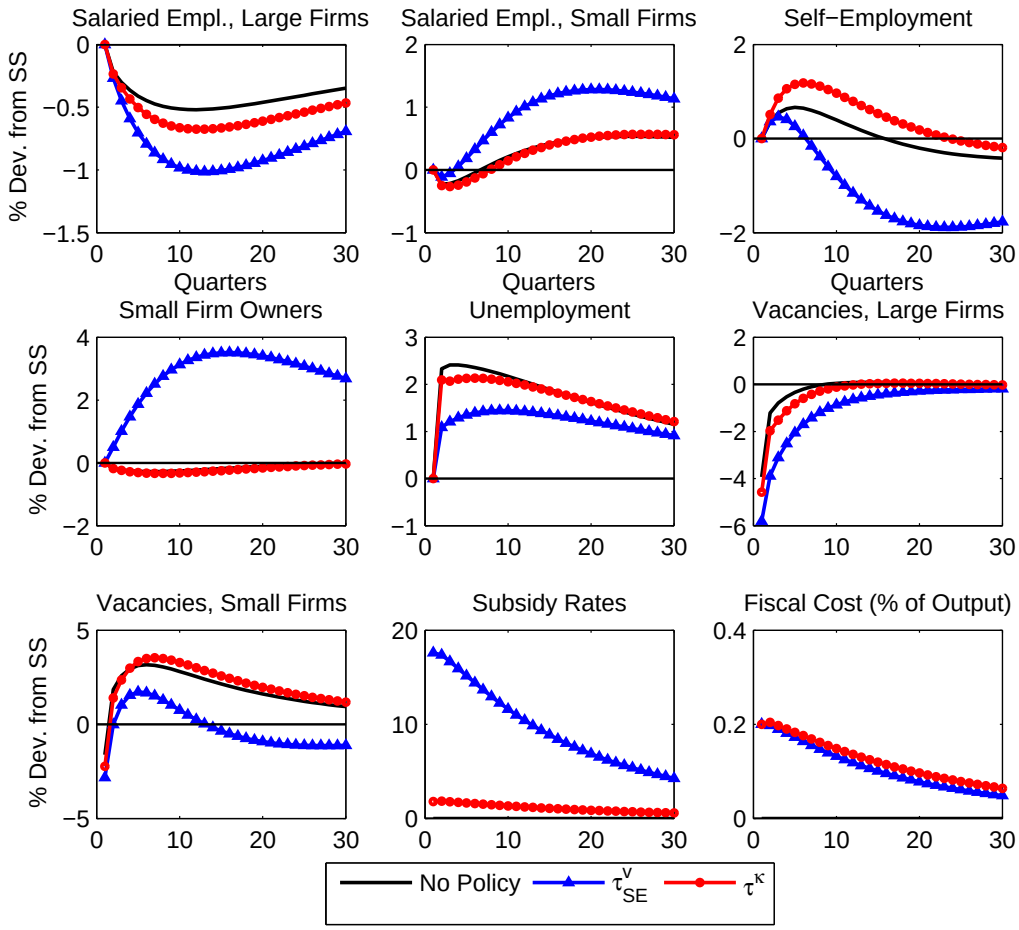

Figure 5: Response to a Negative Productivity Shock: Improvement in Matching Efficiency for Salaried Firms

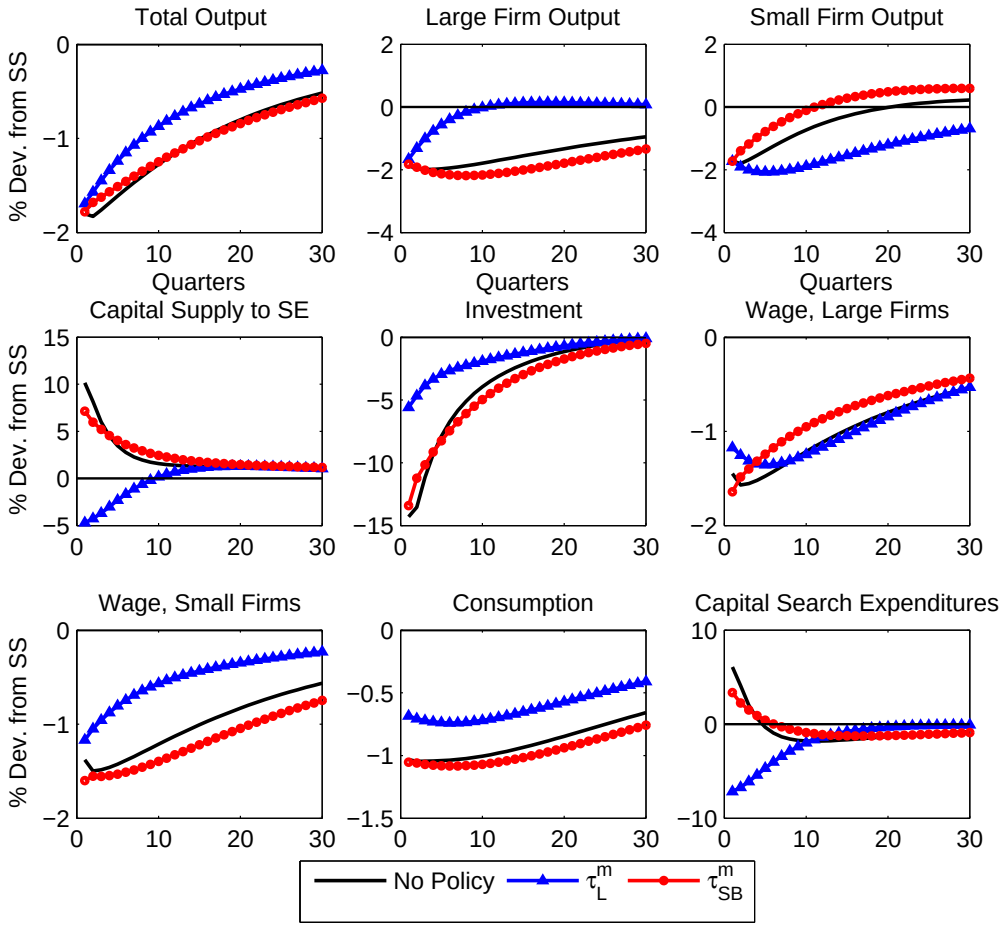

Figure 6: Response to a Negative Productivity Shock: Improvement in Matching Efficiency for Salaried Firms (Cont.)

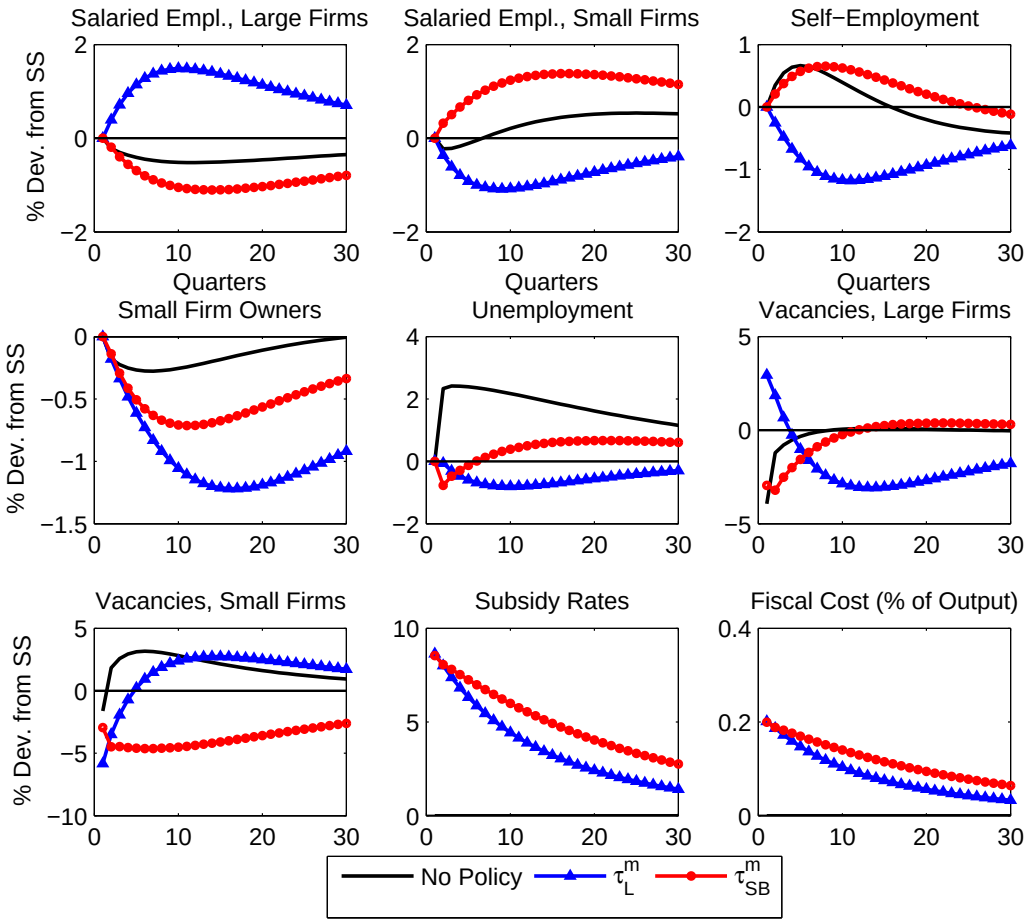

Figure 7: Response to a Negative Productivity Shock: Wage Subsidies for Large and Small Firms

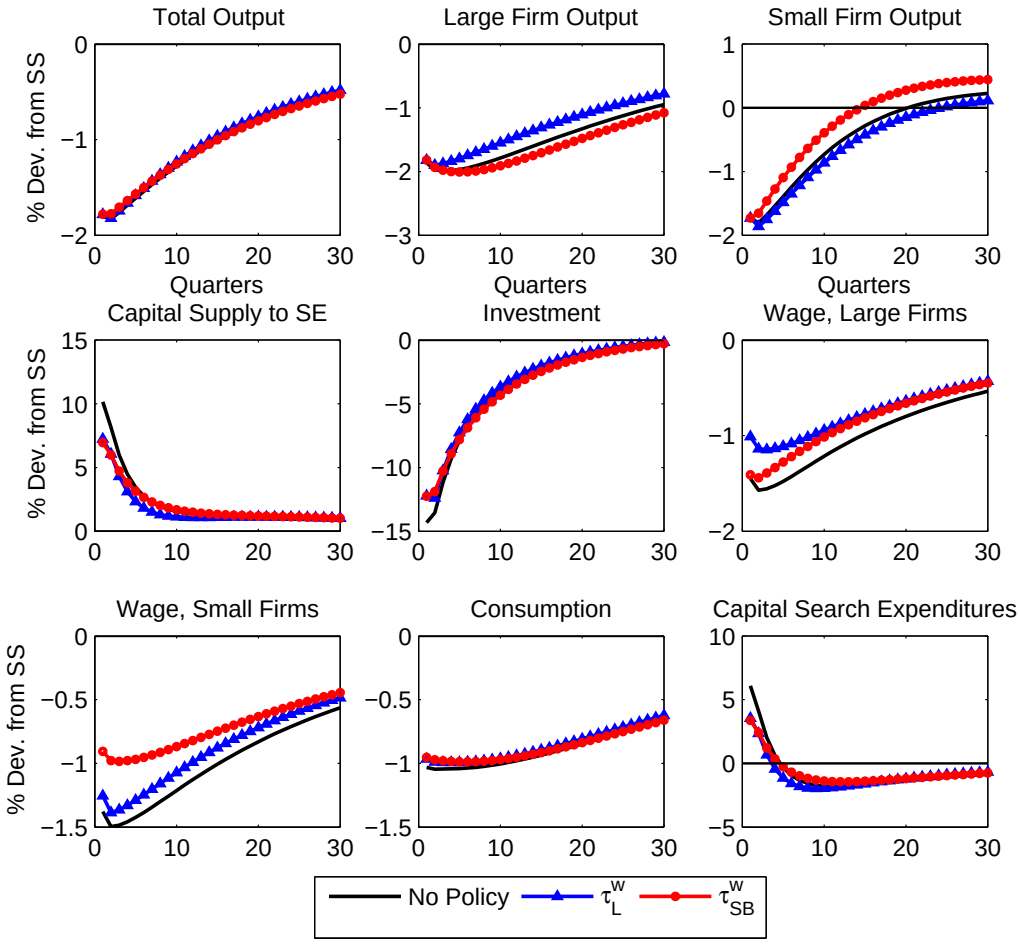

Figure 8: Response to a Negative Productivity Shock: Wage Subsidies for Large and Small Firms (Cont.)

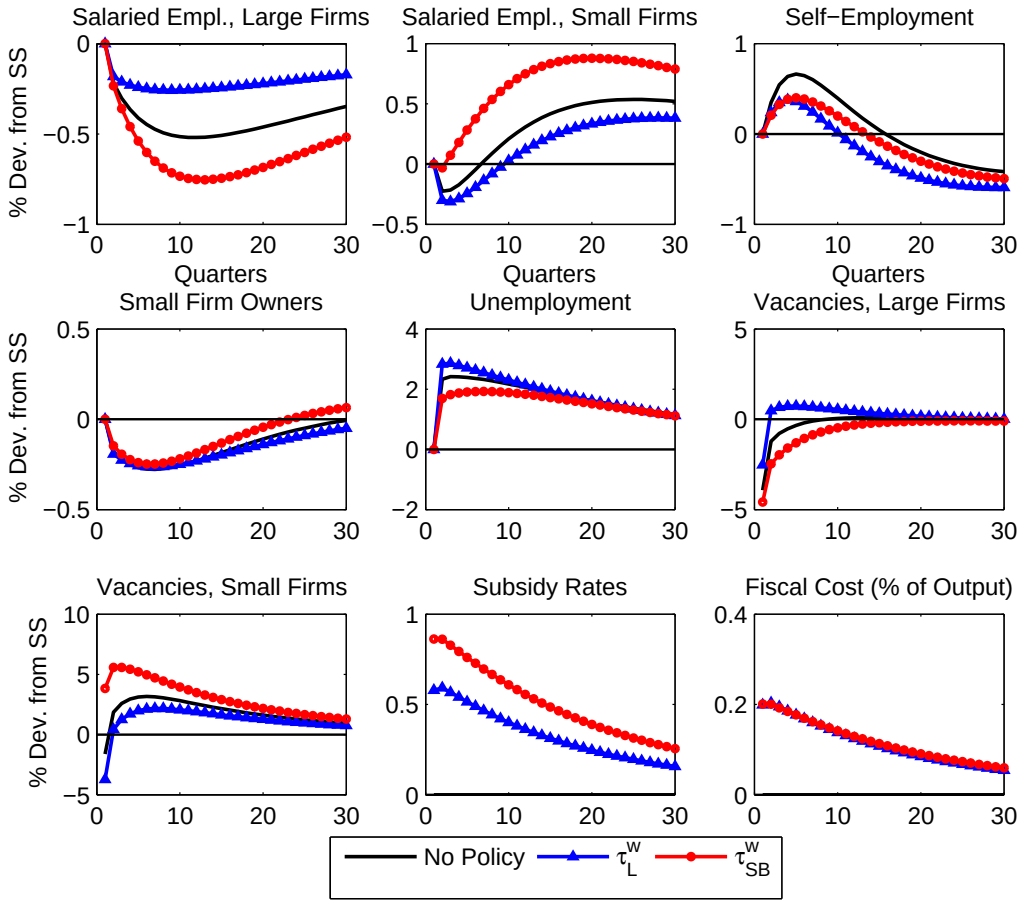

Table 10: Cumulative Multipliers During Recessions

	Output Multipliers		
Policy p	Quarters $d = 4$	Quarters $d = 12$	Quarters $d = 30$
τ^{v_L}	0.710	1.214	1.684
$\tau^{v_{SB}}$	1.204	0.976	0.383
$\tau^{v_{SE}}$	−0.056	0.148	0.465
τ^{κ}	−0.155	−0.325	−0.520
τ^{m_L}	1.494	2.628	3.727
$\tau^{m_{SB}}$	0.594	0.455	0.096
τ^{w_L}	0.091	0.198	0.320
$\tau^{w_{SB}}$	0.242	0.218	0.129

	Employment Multipliers		
Policy p	Quarters $d = 4$	Quarters $d = 12$	Quarters $d = 30$
τ^{v_L}	0.310	0.521	0.681
$\tau^{v_{SB}}$	1.284	1.403	1.308
$\tau^{v_{SE}}$	0.256	0.291	0.288
τ^{κ}	0.062	0.058	0.032
τ^{m_L}	0.611	1.048	1.388
$\tau^{m_{SB}}$	0.642	0.705	0.648
τ^{w_L}	−0.093	−0.080	−0.051
$\tau^{w_{SB}}$	0.126	0.127	0.103

Figure 9: Response to a Negative Productivity Shock: Hiring Subsidies for Large Firms and Existing Small Firms (Cont.)

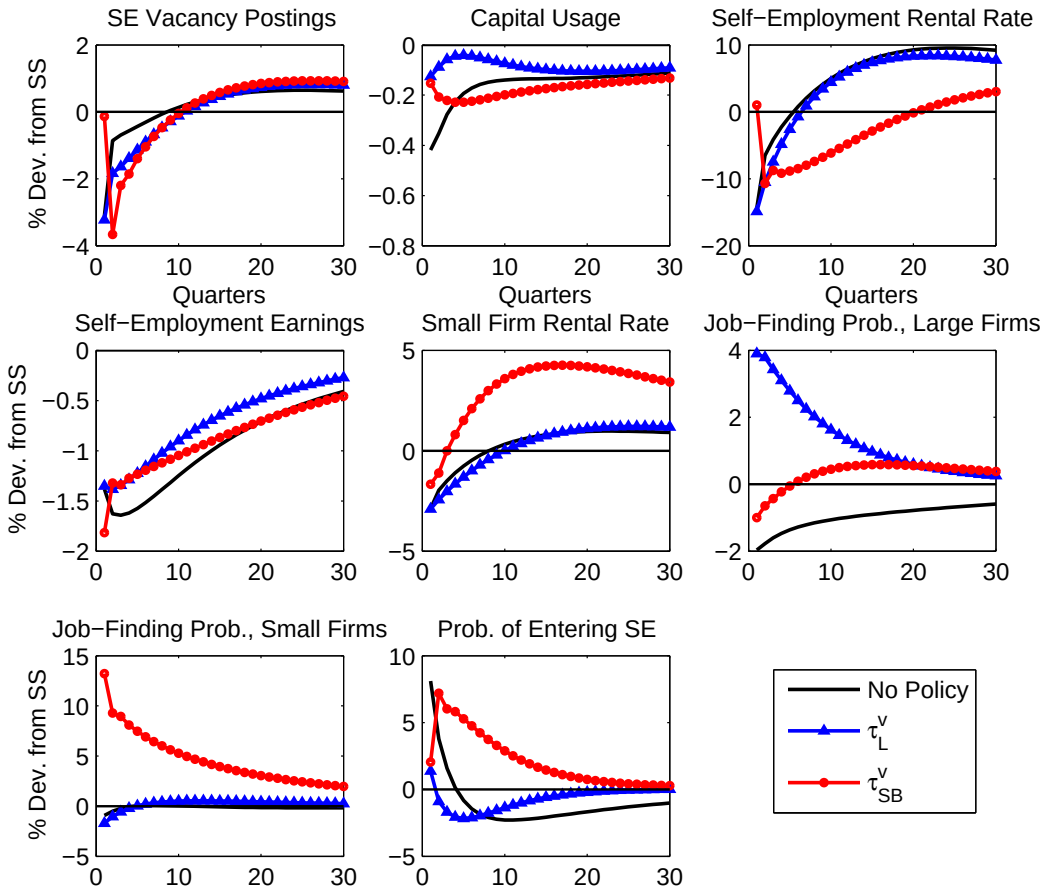

Figure 10: Response to a Negative Productivity Shock: Hiring Subsidies for Self-Employed Firms and Subsidies for Self-Employed Firm Creation (Cont.)

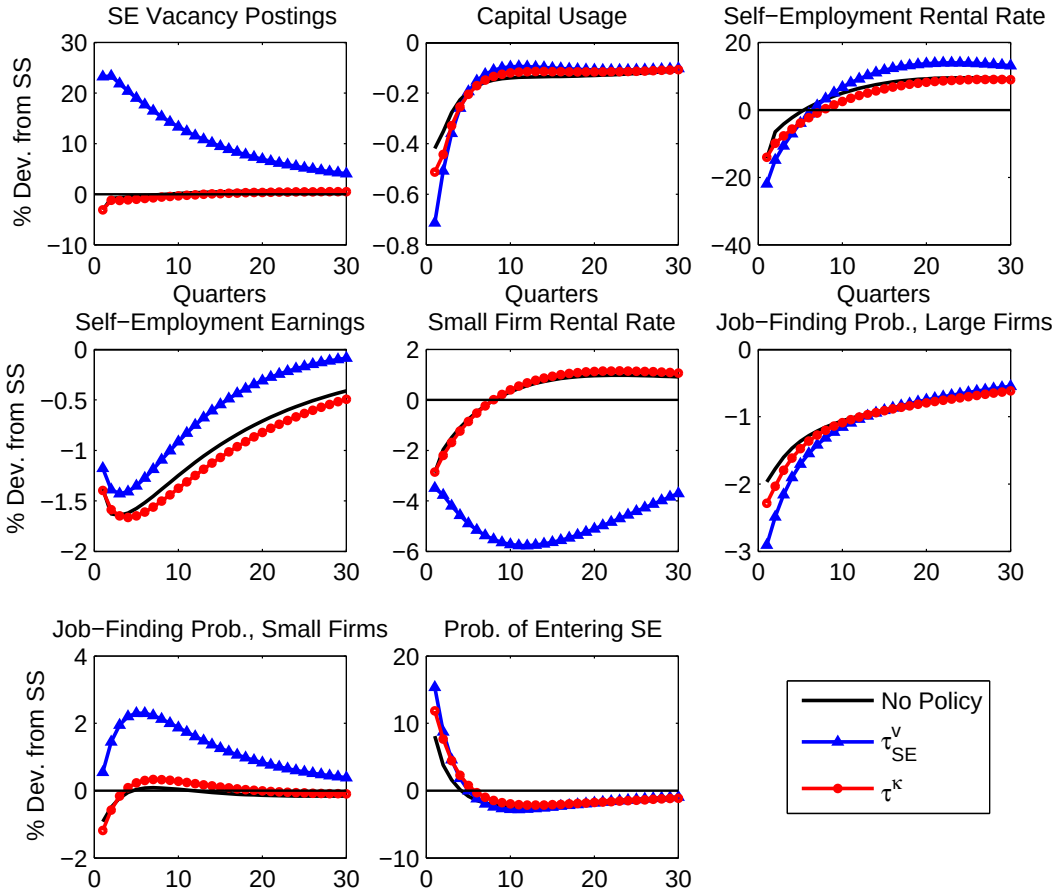

Figure 11: Response to a Negative Productivity Shock: Improvement in Matching Efficiency for Salaried Firms (Cont.)

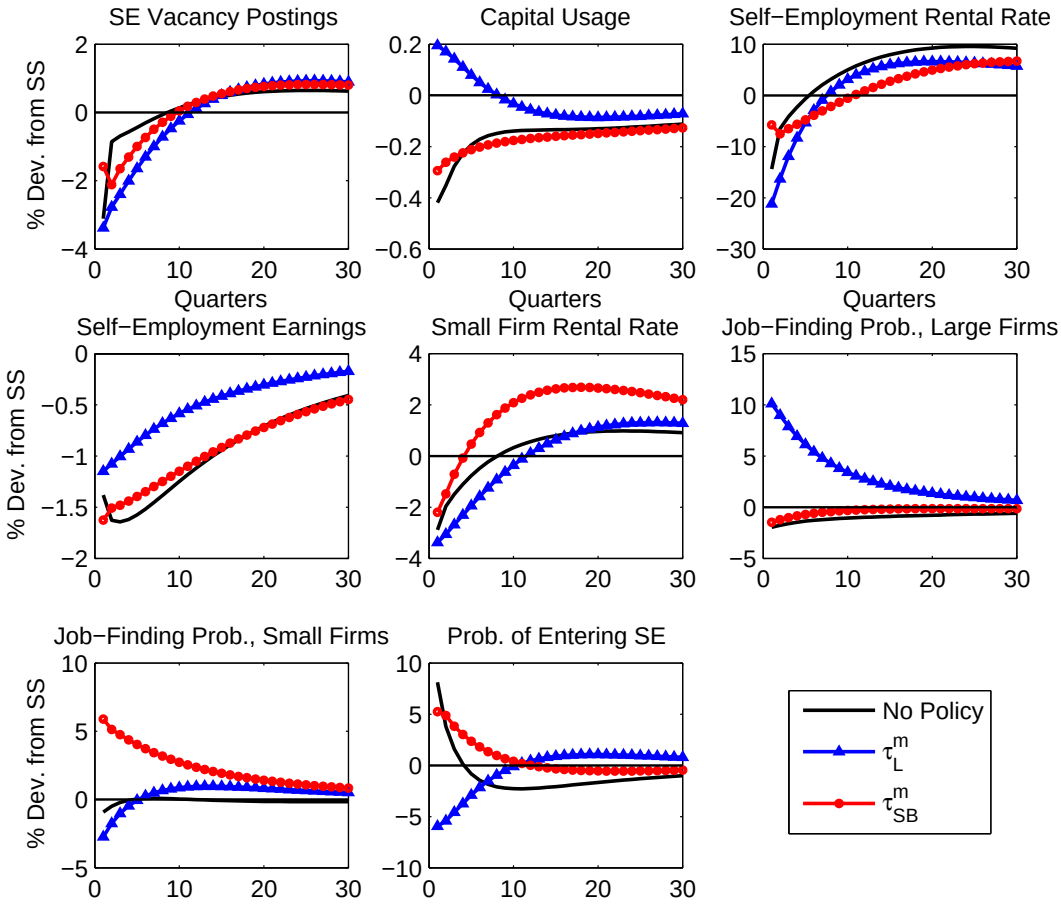

Figure 12: Response to a Negative Productivity Shock: Wage Subsidies for Large and Small Firms (Cont.)