Commodity Prices and Growth*

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Abstract

In this paper we propose an endogenous growth model of small commodity-rich economies in which: (i) long-run (steady-state) growth is endogenous and yet independent of commodity prices; (ii) commodity prices affect short-run growth through transitional dynamics; and (iii) the status of net commodity importer/exporter is endogenous. We argue that these predictions are consistent with historical evidence from the 19th to the 21st century.

J.E.L. Codes: O3; O4; Q4
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1 Introduction

Historical evidence from the 19th to the 21st century provides three stylized facts for commodity-rich countries: (1) commodity prices are generally un-correlated with long-run growth and (2) commodity prices are instead correlated with growth in the short-run. The two facts together, which we discuss in Section 2, raise an important question: what is the economic mechanism that drives the short-run comovement between commodity prices and growth to vanish in the long-run? Moreover, (3) the status of commodity importer/exporter changes over time. In fact, commodity-rich economies switch from being net importers to net commodity exporters and viceversa. For instance, Canada became a net oil exporter in the mid 1980's (see Issa et al., 2008) and China was a net oil exporter until the early 1990's and according to U.S. Energy Information Administration (EIA) it became the world's second-largest net importer of crude oil in 2009.

In this paper we study the link between commodity prices and growth within a model of endogenous growth.[1] The insight of the analysis is that accounting for the first two facts jointly amounts to sterilizing market-size effects induced by commodity price movements, which is a theoretical challenge analogous to the well-known problem of the "scale effect."[2]

The paper is naturally linked to the literature on the "curse of natural resources" which hints at large and persistent short- and long-run effects of natural resource abundance.[3] However, as exemplified by the title of the survey article by Van der Ploeg (2011)—"Natural Resources: Curse or Blessing?"—the outcome of this research effort is far from conclusive with mixed empirical evidence and relatively little theoretical work.

In Section 3 we propose a small open economy (SOE) model of endogenous growth that reconciles the first two empirical observations and provides conditions on the country's commodity price/endowment which determine whether an economy is a net importer or exporter of the commodity. Specifically, we assume that the endowment of the commodity is exogenous and constant. Moreover, in the spirit of the SOE tradition, we assume that commodity prices are taken parametrically by the agents inside the model.[4] As such we

[1] We recognize that variants of the neoclassical growth model would be consistent with the first two facts. In that type of models, long-run (steady-state) growth is determined by the pace of exogenous technical progress as such by construction independent of commodity prices.

[2] The so-called "scale effect" refers to the unappealing feature of first-generation models of endogenous growth à la Romer (1990) that steady-state growth depends on the scale of the economy, e.g., population.

[3] See Gelb (1988), Auty (1990), Sachs and Warner (1995, 1999, 2001), Deaton and Miller (1995), Deaton (1999), Gylfason et al. (1999), Sala-i-Martin and Subramanian (2003), Raddatz (2007), Brunnschweiler and Bulte (2008), Collier and Goderis (2009), Alexeev and Conrad (2009), and Smith (2013).

[4] We view price-taking as a convenient working assumption since it affords analytical tractability that is

also abstract from the determination of world commodity prices and focus on their effects on aggregate variables as consumer expenditures on home and foreign goods, manufacturing production, and total factor productivity (TFP).[5]

To explain the economic mechanism that drives the results it is useful to describe the structure of the model. As in Peretto (1998) the model combines horizontal (expanding variety) and vertical (quality-upgrading and/or cost-reducing) innovation. Manufacturing is the engine of long-run growth. In this sector, incumbent firms engage in two activities. (1) They use labor services and materials to produce intermediate goods supplied to the downstream consumption sector. Materials are purchased from an upstream sector which uses labor and the commodity as inputs. (2) They allocate labor to reduce unit production costs (i.e., vertical innovation). Market structure is endogenous in that both firm size and the mass of firms in the manufacturing sector are jointly determined in the free-entry equilibrium. In fact, firm size, which is proportional to the rate of gross profitability, is the key variable regulating the incentives to innovate.

Movements in commodity prices affect the economy via two channels: (1) they change the value of the endowment as such inducing income/wealth effects—"commodity wealth channel"—and (2) they affect the demand for the commodity in the materials sector and, through the demand of materials in manufacturing and inter-sectoral labor reallocation, have cascade effects through all the vertical cost structure of production—"cost channel."

In Section 4 we derive a "*long-run commodity price super-neutrality*" result: the steady-state growth rate of the economy is independent of commodity prices. The mechanism that drives this result is a market-size sterilization effect through firm entry: given the number of firms, movements in commodity prices change the size of the manufacturing sector, firm size, and so incentives to vertical innovation. Ceteris paribus, this would have steady-state growth effects. However, as the size and so the profitability of incumbent firms change, the mass of firms endogenously adjusts to bring the economy back to the initial steady-state value of firm size, thereby sterilizing the long-run growth effects of the price change.[6]

valuable for the understanding of the economic mechanism at work in our model. We acknowledge that, for certain commodities and time periods, countries may have some degree of market power (e.g., New Zealand supplies close to half of the total world exports of lamb and mutton).

[5]Kilian (2008b, 2009) argues for the need to account for the endogeneity of energy prices when studying their effects on the economy. We acknowledge that studying the joint dynamics of commodity prices and growth, and their interdependence, is of first-order importance but it goes beyond the scope of this paper. See Peretto and Valente (2011) and Peretto (2012) for papers that endogenize the price of the commodity within the same class of models we use in this paper.

[6]The forces that yield sterilization of commodity price changes in the long run are also responsible for the sterilization of the so-called scale effect. See Peretto (1998) and Peretto and Connolly (2007) for a detailed

We argue that the neutrality of commodity prices for long-run growth is critical for the model to be consistent with two basic time-series observations: commodity prices exhibit large and persistent long-run movements (see Jacks, 2013) whereas trend growth in several commodity-rich economies (e.g., Western offshoots) has been approximately constant since the 19th century (see Section 2). This argument, which parallels that in Jones (1995), draws an analogy between the effects of commodity price on growth and the literature on the (lack of) growth effects of taxation (see Easterly and Rebelo, 1993; Easterly et al., 1993; Stokey and Rebelo, 1995; Mendoza et al., 1997; Peretto, 2003; Jaimovich and Rebelo, 2012).

In Section 5 we derive conditions for which a commodity price boom increases, decreases, or leaves unchanged manufacturing production and so short-run (transitional) growth. The sign of the effect depends upon the substitution possibilities between labor and materials in manufacturing and between labor and the commodity in the materials sector. Specifically, we identify four cases: after a commodity price boom (1) the value of manufacturing production raises if the demand for the commodity is overall inelastic—"global complementarity"—(2) it falls if the demand for the commodity is overall elastic—"global substitution"—(3) it does not change if manufacturing and materials sectors have Cobb-Douglas production functions—"Cobb-Douglas-like economy"—and (4) it raises or falls depending on the initial level of the price if materials and manufacturing sectors display opposite substitution/complementarity properties—"complementarity/substitution switch."

These predictions are inherently related to the literature on the "Dutch Disease."[7] In fact, how a commodity price boom affects manufacturing production is ultimately an empirical matter. Yet the empirical literature provides a spectrum of findings ranging from (i) little/no effect (see Gelb, 1988; Sala-i-Martin and Subramanian, 2003; Black et al., 2005; Caselli and Michaels, 2013) (ii) positive (see Allcott and Keniston, 2014; Smith, 2014) to (iii) negative effects (see Ismail, 2010; Rajan and Subramanian, 2011; Harding and Venables, 2013; Charnavoki and Dolado, 2014).

As a final note on the relation to the literature, we point out that the model we propose is also well suited to study the growth effects of the Prebisch-Singer hypothesis.[8] In this paper we make no attempt to explain why commodity prices would fall relative to the prices of imported goods. However, in Section 6 we show that a downward trend in the

analysis of the mechanism driving the sterilization of the scale effect in this class of models.

[7]The "Dutch Disease" hypothesis posits that a boom in the natural resource sector shrinks manufacturing production through crowding out and an appreciation of the real exchange rate.

[8]The Prebisch-Singer hypothesis (see Prebisch, 1959; Singer, 1950) posits that in the long-run commodity prices fall relative to the prices of the manufactures that the exporting country imports. See Harvey et al. (2010) for recent empirical evidence.

commodity/imports relative price has no steady-state growth effect.

In Section 7 we implement a numerical exercise that further illustrates the dynamic properties of the model. We offer some concluding remarks in Section 8.

2 Facts

Empirical work on long-run trends in commodity prices and growth has been for long time hindered by the shortness of the time period for which reliable data are available. However, Angus Maddison (see Bolt and van Zanden, 2013) for real GDP per capita and Jacks (2013) for commodity prices have provided data that span the 19th, 20th, and 21st century.[9] The increased time span, approximately 150 years of data, allows us to relate the long-run trend components in commodity prices and growth for several commodity-rich countries. This is especially important for the scope of the paper since we aim to make a marked distinction between the *steady-state* (long-run) and *transitional dynamics* (short-run) relationship between commodity prices and growth.

Consistently with the literature on commodity price super-cycles (see Cuddington and Jerrett, 2008; Jerrett and Cuddington, 2008; Erten and Ocampo, 2013; Jacks, 2013), we adopt the following definition of Long-Run (LR).

Definition 1 (**Long-run trend**). *Given a time series x_t, the Long-Run trend (LR) component, x_t^{LR}, corresponds to the component of x_t with periodicity larger than 70 years.*[10]

Our first fact follows directly from this definition.

Fact 1. *Commodity prices exhibit large and persistent long-run movements whereas growth rates of real GDP per capita exhibit no such large persistent changes.*

Fact 1, which we take as the key empirical observation of our analysis, posits an important disconnect between the long-run properties of commodity prices and growth. As a result, we argue it is a litmus test for endogenous growth models along the lines of Jones (1995): if

[9]Data on real GDP per capita are from the Maddison Project database which is publicly available at http://www.ggdc.net/maddison/maddison-project/home.htm. Commodity price data are from Jacks (2013) and downloaded from the author's website at http://www.sfu.ca/~djacks/.

[10]We use a band-pass filter, as implemented by Christiano and Fitzgerald (2003), to isolate the Short-Run component (SR), x_t^{SR}, which corresponds to the component of x_t with periodicity between 2 and 70 years. The Long-Run trend (LR) component is then $x_t^{LR} = x_t - x_t^{SR}$.

long-run (steady-state) growth dependend on commodity prices then we would observe large and persistent swings in growth rates which is at odd with the data. This type of argument is analogous to the one made by Stokey and Rebelo (1995) in the context of taxation and growth.

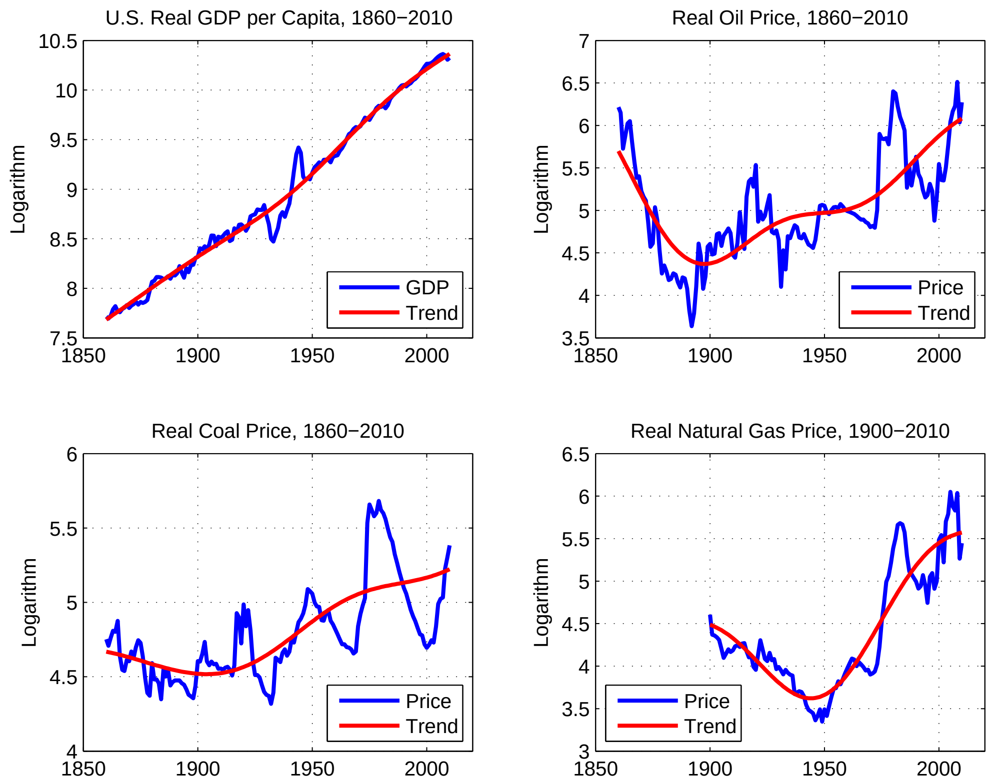

Figure 1: U.S. Real GDP per Capita and Energy Prices

Notes: Data for the U.S. real GDP per capita are from the Angus Maddison's dataset which is publicly available at http://www.ggdc.net/maddison/maddison-project/home.htm. Real energy prices are available from David Jacks's website at http://www.ggdc.net/maddison/maddison-project/home.htm. Trend (red line) is the long-run trend (LR) component of the series as in Definition 1.

Figure 1 illustrates Fact 1 for the U.S. and energy prices. The LR component in real GDP per capita is almost a straight line implying that trend growth has been approximately constant for the last 150 years. Figure 2 shows that the same pattern emerges for all Western offshoots. On the other hand, commodity prices exhibit large and persistent movements in the LR component. This observation is not specific to energy prices but it is a robust finding across several commodities (e.g., animal products, grains, metals, minerals, precious metals, softs).[11]

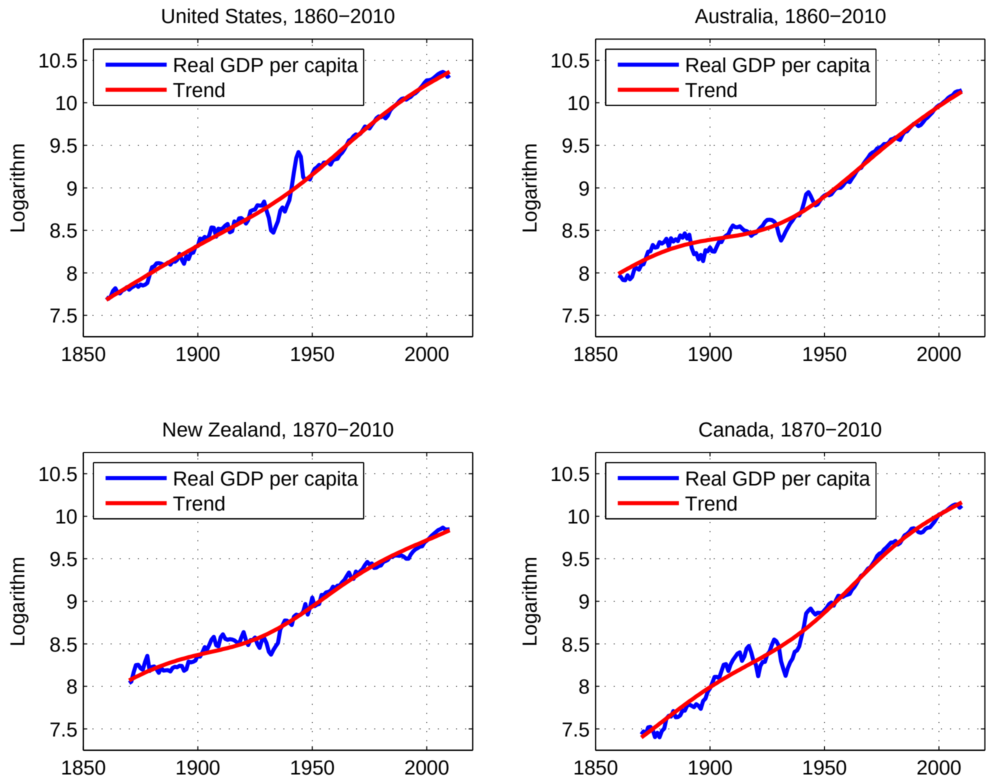

Figure 2: Real GDP per Capita in the Western Offshoots

Notes: Data for real GDP per capita are from the Angus Maddison's dataset which is publicly available at `http://www.ggdc.net/maddison/maddison-project/home.htm`. Trend (red line) is the long-run trend (LR) component of the series as in Definition 1.

The second fact that we highlight is the following.

Fact 2. *Commodity prices and growth rates of real GDP per capita co-move in the short-run.*

Evidence for Fact 2 comes from a variety of sources. Despite mixed evidence on the sign of the relationship, overall the empirical literature strongly supports the view that commodity prices are generally correlated with growth in the short-run.

The first source of evidence is the empirical literature on the "curse of natural resources." Sachs and Warner (1995, 1999, 2001) find a statistically significant negative relationship beween natural resource intensity (e.g., exports of natural resources in percent of GDP) and

[11]See Jacks (2013) for an extensive treatment of long-run trends, medium-term cycles, and short-run boom/bust episodes in commodity prices.

average growth over a twenty-year period. However, the existence of a resource curse has been called into question by several papers (see Deaton and Miller, 1995; Brunnschweiler and Bulte, 2008; Alexeev and Conrad, 2009; Smith, 2013). The common theme of these papers is that a resource boom is indeed associated with positive instead of negative growth effects as the resource curse hypothesis would predict.

We share with the resource curse papers their focus on the low-frequency relationship between commodity prices and growth. However, we differ from them to the extent in which we make a sharp distinction between what we consider to be a long-run (steady-state) as opposed to a short-run (transitional dynamics) commodity price/growth relationship.

The second source of evidence is the literature on oil prices and the business cycle (see Hamilton, 1988, 1996, 2003, 2009; Kilian, 2008a,c, 2009). This strand of literature focuses instead on the high-frequency relationship between oil prices and growth. Specifically, it posits exogenous movements in oil prices as a source of cyclical fluctuations in real GDP per capita. However, it abstracts from the possibility of growth effects of oil prices in the long-run.

3 A Model Economy

3.1 Overview

We consider a small open economy (SOE) populated by a representative household that supplies labor services inelastically in a competitive labor market. The household faces a standard expenditure/saving decision problem: it chooses the path of expenditures (home and foreign goods) and savings by freely borrowing and lending in a competitive market for financial assets at the prevailing interest rate.[12] Household's income consists of returns on asset holdings, labor income, profits, and commodity income which is the (constant) commodity endowment valued at the world commodity price.

The production side of the economy consists of three sectors: (1) consumption goods, (2) intermediate goods or manufacturing, and (3) materials. The consumption sector consists of a representative competitive firm which combines differentiated intermediate goods to produce an homogeneous final good. Upon entry (horizontal innovation), manufacturing

[12] It is possible to think of our model economy as taking the world interest rate parametrically. Since the model has the property that the domestic interest rate jumps to its steady-state level, given by the domestic discount rate, as long the SOE has the same discount rate as the rest of the world, the equilibrium discussed in the paper displays the same properties as an equilibrium with free financial flows.

firms combine labor services and materials to produce differentiated intermediate goods. They also engage in activities aimed to reduce unit production costs (vertical innovation). Entry requires the payment of a sunk cost. Finally, materials are supplied by an upstream competitive sector which uses labor services and the commodity as inputs.

Manufacturing is the engine of endogenous growth. Specifically, the economy starts out with a given range of intermediate goods, each supplied by one firm. Entrepreneurs compare the present value of profits from introducing a new good to the entry cost. They only target new product lines because entering an existing product line in Bertrand competition with the existing supplier leads to losses. Once in the market, firms devote labor to cost-reducing (or, equivalently, productivity enhancing) projects. As each firm strives to figure out how to improve efficiency, it contributes to the pool of public knowledge that benefits the future cost reduction activity of all firms. This allows the economy to grow at a constant rate in steady state, which is reached when entry stops and the economy settles into a stable industrial structure.

Throughout, we omit time subscripts unless needed for clarity.

3.2 Households

The representative household solves the following maximization problem:

$$\max_{\{Y_H, Y_F\}} U(t) = \int_t^{\infty} e^{-\rho(s-t)} \log u(s)\, ds, \quad \rho > 0 \tag{1}$$

where

$$\log u = \varphi \log\left(\frac{Y_H}{P_H L}\right) + (1-\varphi)\log\left(\frac{Y_F}{P_F L}\right), \quad 0 < \varphi < 1 \tag{2}$$

subject to the budget constraint,

$$\dot{A} = rA + WL + \Pi_H + \Pi_M + p\Omega - Y_H - Y_F, \quad \Omega > 0 \tag{3}$$

where ρ is the discount rate, φ controls the degree of home bias in preferences, A is assets holding, r is the rate of return on financial assets, W is the wage, L is population size which equals labor supply since there is no preference for leisure, Y_H is expenditure on home consumption good whose price is P_H, and Y_F is expenditure on foreign consumption good whose price is P_F. In addition to asset and labor income, the household receives the dividends paid out by the producers of the home consumption good, Π_H, the dividends paid out by firms in the material sector, Π_M, and the revenues from sales of the domestic commodity endowment, Ω, at the price p. The solution to this problem consists of the optimal consumption/expenditure allocation rule,

$$\varphi Y_F = (1 - \varphi) Y_H, \tag{4}$$

and the Euler equation governing saving behavior,

$$r = r_A \equiv \rho + \frac{\dot{Y}_H}{Y_H} = \rho + \frac{\dot{Y}_F}{Y_F}. \tag{5}$$

3.3 Trade Structure

The economy can be either an importer or exporter of the commodity. In the first case (commodity importer), it sells the home consumption good to buy the commodity in the world market. As in the SOE tradition, we assume the world commodity market is willing to accomodate any demand at the exogenous constant price p. In the second case (commodity exporter), it accepts the foreign consumption good as payment for its commodity exports. The foreign good is imported at the constant exogenous price P_F. Only final goods and the commodity are tradable. The balanced trade condition, which is also the market clearing condition for the consumption good market, is $Y_H + Y_F + p(O - \Omega) = Y$, where Y is the aggregate value of production of the home consumption good. Using the consumption expenditure allocation rule (4), we can rewrite the balance trade condition as

$$\frac{1}{\varphi} Y_H + p(O - \Omega) = Y, \tag{6}$$

where O denotes the home use of the commodity. From (6) it is easily established that: (1) $O > \Omega$ (commodity importer) implies $Y > Y_H$, i.e., the model economy exchanges home consumption goods for the commodity. And conversely (2) $O < \Omega$ (commodity exporter) implies $Y < Y_H$, i.e., it exchanges the commodity for foreign consumption goods.

3.4 Consumption Goods

The home (homogeneoeus) consumption good is produced by a representative competitive firm with the following technology:

$$C_H = N^{\chi} \left[\frac{1}{N} \int_0^N X_i^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \chi > 0,\ \epsilon > 1 \tag{7}$$

where ϵ is the elasticity of product substitution, X_i is the quantity of the non-durable intermediate good i, and N is the mass of goods. Based on Ethier (1982) we separate the elasticity of substitution between intermediate goods from the degree of increasing returns to variety, χ. The final good producer solves the following maximization problem:

$$\max_{\{X_i\}} \Pi_H = P_H C_H - \int_0^N P_i X_i di$$

subject to (7). This structure yields the demand curve for each intermediate good,

$$X_i = Y \cdot \frac{P_i^{-\epsilon}}{\int_0^N P_i^{1-\epsilon} di}, \tag{8}$$

where $Y = P_H C_H$. Because the sector is perfectly competitive, $\Pi_H = 0$.

3.5 Manufacturing

In this section we abstract from firms' behavior, which we discuss in Section 4.1, and detail the technologies available to the incumbent firm.

The typical firm produces one differentiated good with the following technology:

$$X_i = Z_i^{\theta} \cdot F\left(L_{X_i} - \phi, M_i\right), \quad 0 < \theta < 1,\ \phi > 0 \tag{9}$$

where X_i is output, L_{X_i} is production employment, ϕ is a fixed labor cost, M_i is use of materials, and Z_i^{θ} is the firm's total factor productivity (TFP) which is a function of the stock of firm-specific knowledge, Z_i. $F(\cdot)$ is a standard production function, which is homogeneous of degree one in its arguments. Total production costs are

$$W\phi + C_X(W, P_M) Z_i^{-\theta} \cdot X_i, \tag{10}$$

where $C_X(\cdot)$ is the associated unit-cost function which is homogeneous of degree one in its arguments. Hicks-neutral technological change internal to the firm shifts this function downward. The firm accumulates knowledge according to the technology

$$\dot{Z}_i = \alpha K L_{Z_i}, \quad \alpha > 0 \tag{11}$$

where $\dot{Z}_i$ is the flow of firm-specific knowledge generated by a project employing L_{Z_i} units of labor (for an interval of time dt) and αK is the productivity of labor in such a project as determined by the parameter α and by the stock of public knowledge, K. Public knowledge accumulates as a result of spillovers:

$$K = \frac{1}{N} \int_0^N Z_i di,$$

which posits that the knowledge frontier is determined by the average knowledge of all firms.[13]

When a firm generates a new idea to improve the efficiency/productivity of the production process, it also generates general-purpose knowledge which is not excludable. Specifically, firms appropriate the economic returns from firm-specific knowledge but they cannot prevent others from using the general-purpose knowledge that spills over into the public domain. Formally, a project that produces $\dot{Z}_i$ units of firm-specific knowledge also generates $\dot{Z}_i$ units of public knowledge.

3.6 Materials

In this section we abstract from firm's behavior, which we discuss in Section 4.2, and detail the production technology available to the representative firm.

A competitive firm uses labor services, L_M, and the commodity, O, as inputs to produce materials, M, which are purchased by the manufacturing sector at the price P_M. The production technology is $M = G(L_M, O)$, where $G(\cdot)$ is a standard production function, which is homogeneous of degree one in its arguments. Total production costs are

$$C_M(W, p) M, \tag{12}$$

where $C_M(\cdot)$ is the associated unit-cost function which is homogeneous of degree one in the wage, W, and the commodity price, p.

3.7 Taking Stock: Vertical Cost Structure

Let us assess what we have so far. Given the vertical structure of production, a commodity price change has cascade effects: (1) it directly affects production costs and so pricing, i.e., P_M, in the upstream materials sector through the unit-cost function $C_M(W, p)$; (2)

[13] See Peretto and Smulders (2002) for a detailed discussion of a spillovers function of this class.

the change in P_M in turn affects production costs and so pricing, i.e., P_i, in manufacturing through the unit-cost function $C_X(W, P_M)$; (3) the change in P_i finally affects production costs and so pricing, i.e., P_H, in the consumption goods sector through the demand for intermediate goods. Hence the initial change in the price of the commodity affects the home Consumer Price Index (CPI).

Note also that the materials sector competes for labor services with the manufacturing sector. This captures the inter-sectoral allocation problem faced by the economy.

4 Firms' Behavior and General Equilibrium

In this section we first construct the equilibrium in manufacturing and materials sector, then we impose general equilibrium conditions to study the aggregate dynamics of the economy.

4.1 Firms' Behavior in Manufacturing

The typical intermediate firm maximizes the present discounted value of net cash flows:

$$\max_{\{L_{X_i},\, L_{Z_i},\, M_i\}} V_i(t) = \int_t^{\infty} e^{-\int_t^s [r(v)+\delta]dv} \Pi_i(s) ds, \quad \delta > 0$$

where δ is a "death shock."[14] Using the cost function (10), instantaneous profits are

$$\Pi_i = \left[P_i - C_X(W, P_M) Z_i^{-\theta} \right] X_i - W\phi - W L_{Z_i},$$

where L_{Z_i} are labor sevices allocated to cost reduction.[15] Each firm i maximizes $V_i(t)$, which is the value of the firm, subject to the cost-reduction technology (11), the demand schedule (8), taking as given $Z_i(t) > 0$ (initial stock of knowledge), $Z_j(t')$ for $t' \geq t$ and $j \neq i$ (rivals' knowledge accumulation paths), and $Z_j(t') \geq 0$ for $t' \geq t$ (knowledge irreversibility constraint). The solution of this problem yields the (maximized) value of the firm given the time path of the number of firms, $N(t)$.

To characterize entry, we assume that upon payment of a sunk cost, $W \cdot (\beta Y/N)$, an entrepreneur can create a new firm that starts out its activity with productivity equal to

[14] $\delta > 0$ is required for the model to have symmetric dynamics in the neighborhood of the steady-state.

[15] If $\phi = 0$ then horizontal innovation becomes a source of steady-state growth as in first-generation models of endogenous growth à la Romer (1990). In this case, the model economy displays scale effects. In the rest of the paper, we focus on the case $\phi > 0$ for which vertical innovation is the only source of steady-state growth.

the industry average.[16] Once in the market, the new firm solves a problem identical to the one outlined above for the incumbent firm. Therefore, a free-entry equilibrium requires $V_i(t) = W(t) \cdot (\beta Y(t)/N(t))$ for all t.

Appendix A.1 shows that the equilibrium thus defined is symmetric and it is characterized by the following factor demands:

$$WL_X = Y\frac{\epsilon - 1}{\epsilon}S_X^L + W\phi N, \tag{13}$$

and

$$P_M M = Y\frac{\epsilon - 1}{\epsilon}S_X^M, \tag{14}$$

where the shares of the firm's variable costs due to labor and materials are respectively,

$$S_X^L \equiv \frac{WL_{X_i}}{C_X(W, P_M)Z_i^{-\theta}X_i} = \frac{\partial \log C_X(W, P_M)}{\partial \log W},$$

and

$$S_X^M \equiv \frac{P_M M_i}{C_X(W, P_M)Z_i^{-\theta}X_i} = \frac{\partial \log C_X(W, P_M)}{\partial \log P_M}.$$

Note that $S_X^L + S_X^M = 1$. Associated to these factor demands are the rates of return to cost reduction, r_Z, and entry, r_N:

$$r = r_Z \equiv \frac{\alpha}{W}\left[\frac{Y}{\epsilon N}\theta(\epsilon - 1) - W\frac{L_Z}{N}\right] + \frac{\dot{W}}{W} - \delta, \tag{15}$$

and

$$r = r_N \equiv \frac{N}{W\beta Y}\left[\frac{Y}{\epsilon N} - W\phi - W\frac{L_Z}{N}\right] + \frac{\dot{W}}{W} - \delta + \frac{\dot{Y}}{Y} - \frac{\dot{N}}{N}. \tag{16}$$

Neither the return to cost reduction in (15) nor the return to entry in (16) directly depend on factors related to the commodity market. Why is this the case? The technology (9) yields a unit-cost function that depends only on input prices and it is independent of the quantity produced and thus of inputs use. Since the optimal pricing rule features a constant markup over unit cost, the firm's gross-profit flow (revenues minus variable costs), $Y/\epsilon N$, is independent of input prices. Equations (15) and (16), then, capture the idea that investment

[16] See Peretto and Connolly (2007) for an interpretation of this assumption.

decisions by incumbents and entrants do not directly respond to conditions in the commodity market because they are guided by the gross-profit flow. Conditions in the commodity market have instead an indirect effect through aggregate spending on intermediate goods, Y, which are nonetheless sterilized by net entry/exit of firms.

4.2 Firms' Behavior in Materials Sector

Given the unit-cost function (12), competitive producers of materials operate along the infinitely elastic supply curve:

$$P_M = C_M\left(W, p\right). \tag{17}$$

In equilibrium then materials production is given by (14) evaluated at the price P_M. Defining the commodity share in material costs as

$$S_M^O \equiv \frac{pO}{C_M\left(W, p\right) M} = \frac{\partial \log C_M(W, p)}{\partial \log p},$$

we can write the associated demand for labor and the commodity:

$$WL_M = M\frac{\partial C_M(W, p)}{\partial W} = Y\frac{\epsilon - 1}{\epsilon} S_X^M \left(1 - S_M^O\right), \tag{18}$$

and

$$pO = M\frac{\partial C_M(W, p)}{\partial p} = Y\frac{\epsilon - 1}{\epsilon} S_X^M S_M^O. \tag{19}$$

4.3 General Equilibrium

The main equilibrium conditions of the model are the rate of return to saving (5), to cost reduction (15), and to entry (16), then labor demand in manufacturing (13) and materials sector (18), and the household's budget constraint (3).[17] Asset market equilibrium requires return equalization, i.e., $r = r_A = r_Z = r_N$, and that the value of the household's portfolio equal the total value of the securities issued by firms, i.e., $A = NV = \beta WY$.[18] We choose

[17] The household's budget constraint (3) and balanced trade (6) imply the labor-market clearing condition: $L = L_N + L_X + L_Z + L_M$, where L_N is aggregate employment in entrepreneurial activity (i.e., labor services used to enter manufacturing), $L_X + L_Z$ is aggregate employment in production and cost-reduction of existing manufacturing firms, and L_M is aggregate employment in materials sector.

[18] The first equality derives from the symmetry of the equilibrium, i.e., $A = \int_0^N V_i di = NV_i = NV$. The second equality derives instead from imposing the free-entry condition: $V_i = W \cdot (\beta Y/N)$.

labor as the numeraire, i.e., $W \equiv 1$, which is a convenient normalization since it implies that all expenditures are constant.

The following proposition characterizes the equilibrium value of home manufacturing production, balanced trade, and expenditures on home and foreign consumption goods.

Proposition 1. *At any point in time, the value of home manufacturing production and the balanced trade condition are, respectively:*

$$Y(p) = \frac{L}{1 - \xi(p) - \rho\beta} \qquad \text{with} \quad \xi(p) \equiv \frac{\epsilon - 1}{\epsilon} S_X^M(p)\, S_M^O(p)\,, \tag{20}$$

and

$$\frac{1}{\varphi} Y_H(p) - p\Omega = Y(p)\,(1 - \xi(p))\,. \tag{21}$$

The associated expenditures on home and foreign consumption goods are, respectively:

$$Y_H(p) = \varphi \left[\frac{L(1 - \xi(p))}{1 - \xi(p) - \rho\beta} + p\Omega \right], \tag{22}$$

and

$$Y_F(p) = (1 - \varphi) \left[\frac{L(1 - \xi(p))}{1 - \xi(p) - \rho\beta} + p\Omega \right]. \tag{23}$$

Because $Y_H(p)$ and $Y_F(p)$ are constant, the interest rate is $r = \rho$ at all times.

Proof. See Appendix A.2.

The following proposition characterizes the equilibrium dynamics of the model.

Proposition 2. *Let $x \equiv Y/\epsilon N$ denote the gross profit rate. The general equilibrium of the model reduces to the following system of piece-wise linear differential equations in the gross profit flow, x:*

$$\dot{x} = \begin{cases} \frac{\delta L \epsilon / N_0}{(1 - \xi(p)) - \frac{1}{\epsilon}} & \text{if} \quad \phi \le x \le x_N \\[2ex] \frac{\phi}{\beta\epsilon} - \left[\frac{1}{\beta\epsilon} - (\rho + \delta) \right] x & \text{if} \quad x_N < x \le x_Z \\[2ex] \frac{\phi - \frac{\rho+\delta}{\alpha}}{\beta\epsilon} - \left[\frac{1 - \theta(\epsilon - 1)}{\beta\epsilon} - (\rho + \delta) \right] x & \text{if} \quad x > x_Z. \end{cases}$$

Assuming that

$$\frac{\phi - (\rho + \delta)/\alpha}{1 - \theta(\epsilon - 1) - \beta\epsilon(\rho + \delta)} > \frac{\rho + \delta}{\alpha\theta(\epsilon - 1)},$$

the economy asymptotically converges to the steady-state value of x,

$$x^* = \frac{\phi - (\rho + \delta)/\alpha}{1 - \theta(\epsilon - 1) - \beta\epsilon(\rho + \delta)} > x_Z. \tag{24}$$

The associated steady-state growth rate of cost-reduction is

$$\hat{Z}^* = \frac{(\phi\alpha - \rho - \delta)\,\theta(\epsilon - 1)}{1 - \theta(\epsilon - 1) - \beta\epsilon(\rho + \delta)} - (\rho + \delta) > 0. \tag{25}$$

Proof. See Appendix A.3.

Figure 3 illustrates the equilibrium dynamics of the model by means of a phase diagram.[19]

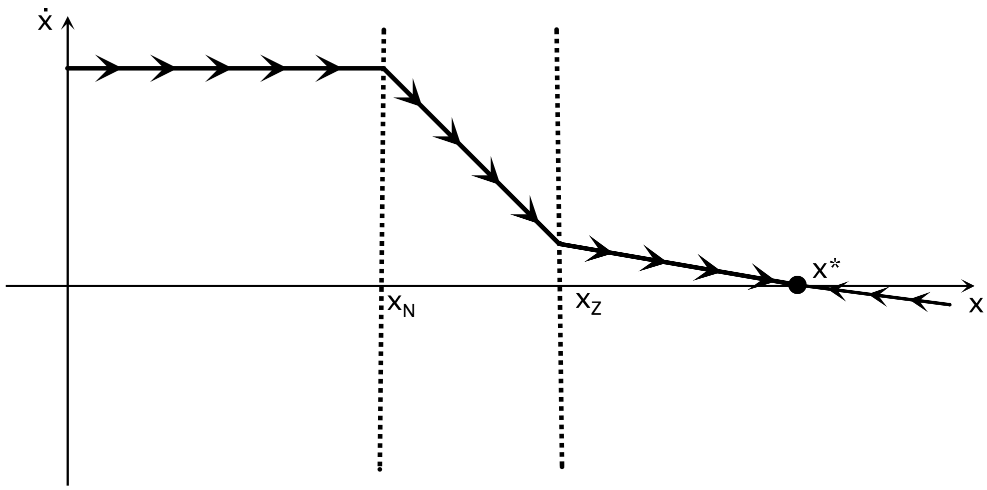

Figure 3: Global Equilibrium Dynamics

Proposition 2 states a **"long-run commodity price super-neutrality"** result: the steady-state growth rate of cost reduction, $\hat{Z}^*$, which is the only source of steady-state growth, is independent of the commodity price, p.

[19]The phase diagram in Figure 3 depicts the global model dynamics implied by Proposition 2, which refers to the region of the parameter space in which $x_N < x_Z$ and $x^* > x_Z$ (Case A). In Case A, horizontal innovation comes first and vertical innovation follows guaranteeing positive steady-state growth. The model is also consistent with the case $x_N < x_Z$ and $x_N < x^* < x_Z$ (Case B). In Case B, vertical innovation never arises in equilibrium and the economy features no steady-steady growth. The global dynamics are well defined also in the case in which the ranking of the thresholds is inverted, i.e., $x_N > x_Z$ (Case C). To streamline the presentation of the results, for the rest of the paper we focus on Case A.

The mechanism that drives this super-neutrality result is a market-size sterilization effect: (1) fix the number of firms at $\overline{N}$, then a change in the commodity price affects the size of the manufacturing sector, $Y(p)$ (see Proposition 1), firm's gross profitability, $x \equiv Y(p)/\left(\epsilon\overline{N}\right)$ (see Proposition 2), and so incentives to vertical innovation. Ceteris paribus, this would have steady-state growth effects. (2) Now let the mass of firms vary as in the free-entry equilibrium. As the profitability of incumbent firms varies, the mass of firms endogenously adjusts (net entry/exit) to bring the economy back to the initial steady-state value of firm size. As a result, the entry process fully sterilizes the long-run growth effects of the initial price change.[20]

4.3.1 Total Factor Productivity and Welfare

In this section we derive the closed-form solution for aggregate total factor productivity (TFP) and welfare in the region $x(t) > x_Z$ of the phase diagram in Figure 3.

In this model economy, aggregate (TFP) is

$$T = N^{\chi} Z^{\theta}. \tag{26}$$

As a result, $\hat{T}(t) = \chi\hat{N}(t) + \theta\hat{Z}(t)$, where $\hat{T}(t) \equiv \dot{T}(t)/T(t)$. Using (25), $\hat{T}(t)$ in steady state is

$$\hat{T}^{*} = \theta\hat{Z}^{*} = \theta\left[\frac{(\phi\alpha - \rho - \delta)\,\theta\,(\epsilon - 1)}{1 - \theta\,(\epsilon - 1) - \beta\epsilon\,(\rho + \delta)} - (\rho + \delta)\right] \equiv g. \tag{27}$$

In the neighborhood of the steady-state $x^{*} > x_Z$, the dynamics of the gross-profit rate, x, are governed by the following differential equation:

$$\dot{x} = \nu\,(x^{*} - x)\,,$$

where

$$\nu \equiv \frac{1 - \theta\,(\epsilon - 1) - \beta\epsilon\,(\rho + \delta)}{\beta\epsilon} \quad \text{and} \quad x^{*} \equiv \frac{\phi - (\rho + \delta)/\alpha}{1 - \theta\,(\epsilon - 1) - \beta\epsilon\,(\rho + \delta)}.$$

We thus work with the solution

[20] The mechanism that yield sterilization of commodity price changes in the long run is also the one responsible for the sterilization of the so-called scale effect, i.e., the steady-state growth rate of cost reduction is also independent of population size, L.

$$x(t) = x_0 e^{-\nu t} + x^* \left(1 - e^{-\nu t}\right), \tag{28}$$

where $x_0 \equiv x(0)$ is the initial condition on $x(t)$.

The following proposition characterizes the time path for aggregate TFP and welfare.

Proposition 3. *Consider an economy starting at time $t = 0$ with initial condition x_0. At any time $t > 0$ the log of TFP is*

$$\log T(t) = \log\left(Z_0^{\theta} N_0^{\chi}\right) + gt + \left(\frac{\gamma}{\nu} + \chi\right) \Delta \left(1 - e^{-\nu t}\right), \tag{29}$$

where

$$\Delta \equiv \frac{x_0}{x^*} - 1.$$

The instantaneous utility flow is

$$\log u(t) = \log \varphi \left(\frac{1 - \xi(p)}{1 - \xi(p) - \rho\beta} + \frac{p\Omega}{L}\right) - \varphi \log c(p) + \varphi g t + \varphi \left(\frac{\gamma}{\nu} + \chi\right) \Delta \left(1 - e^{-\nu t}\right). \tag{30}$$

The resulting level of welfare is

$$U(0) = \frac{1}{\rho}\left[\log \varphi \left(\frac{1 - \xi(p)}{1 - \xi(p) - \rho\beta} + \frac{p\Omega}{L}\right) - \varphi \log c(p) + \frac{\varphi g}{\rho} + \frac{\varphi\left(\frac{\gamma}{\nu} + \chi\right)}{\rho + \nu}\Delta\right]. \tag{31}$$

Proof. See Appendix A.4.

Equation (29) shows that commodity prices affect the time path for aggregate TFP only through the displacement term, Δ. Steady-state growth, g, and the speed of reversion to the steady state, ν, are both independent of the commodity price, p.

Equation (31) identifies three channels through which commodity prices affect welfare: (1) *"windfall effect"* through the term $p\Omega$; (2) *"cost of living/CPI effect"* through the term $c(p) \equiv C_X\big(1, C_M(1, p)\big)$; and (3) *"curse/blessing effect "* through transitional dynamics associated with the term Δ (i.e., initial displacement from the steady state) and/or steady-state growth, g.

The first two effects capture (static) forces that the literature on the curse of natural resources has discussed at length. An economy with a commodity endowment experiences a windfall when the price of the commodity raises. In our model, this would be analogous to a lump-sum transfer from abroad. The cost of living effect is instead due to the fact that the economy uses the commodity for home production: an increase in the commodity price works its way through the home vertical structure of production—from upstream materials production to downstream manufacturing—and it manifests itself as a higher price of the home consumption good (i.e., higher CPI).

The third effect captures (dynamic) forces that are critical for our analysis. The steady-state growth rate of aggregate TFP is independent of the commodity price, p. This is due to the sterilization of market-size effects (see Section 4.3). However, there are effects due to transitional dynamics of TFP: (1) cumulated gain/loss from the acceleration/deceleration of the rate of cost reduction and (2) cumulated gain/loss from the acceleration/deceleration of product variety expansion. These two transitional components amplify the change in manufacturing expenditure induced by the change in the commodity price.

5 Manufacturing Production and Commodity Prices

In this section we study how a permanent change in the commodity price affects the value of manufacturing production. The following lemma derives a set of elasticities which are key determinants of the comparative statics.[21]

Lemma 1. *Let:*

$$\epsilon_X^M \equiv -\frac{\partial \log M}{\partial \log P_M} = 1 - \frac{\partial \log S_X^M}{\partial \log P_M} = 1 - \frac{\partial S_X^M}{\partial P_M} \cdot \frac{P_M}{S_X^M};$$

$$\epsilon_M^O \equiv -\frac{\partial \log O}{\partial \log p} = 1 - \frac{\partial \log S_M^O}{\partial \log p} = 1 - \frac{\partial S_M^O}{\partial p} \cdot \frac{p}{S_M^O}.$$

Then,

$$\xi'(p) = \left(\frac{\epsilon - 1}{\epsilon}\right) \cdot \frac{\partial\left(S_M^O(p)\, S_X^M(p)\right)}{\partial p} = \frac{\xi(p)}{p} \cdot \Gamma(p),$$

where

$$\Gamma(p) \equiv \left(1 - \epsilon_X^M(p)\right) S_M^O(p) + 1 - \epsilon_M^O(p). \tag{32}$$

[21] The following results of comparative statics are inherently related to the literature on the "Dutch Disease" hypothesis, which posits that a boom in the natural resource sector shrinks manufacturing production through a crowding out effect and an appreciation of the real exchange rate.

Proof. See Appendix A.5.

The key object in Lemma 1 is $\Gamma(p)$, which is the elasticity of $\xi(p) \equiv \left(\frac{\epsilon-1}{\epsilon}\right) S_M^O(p) S_X^M(p)$ with respect to the commodity price, p.[22] Differentiating (20), rearranging terms, and using (32) yields

$$\frac{d\log Y(p)}{dp} = \frac{\xi'(p)}{1-\xi(p)-\beta\rho} = \frac{\xi(p)}{p[1-\xi(p)-\beta\rho]} \cdot \Gamma(p),$$

which says that the effect of commodity price changes on (the value of) manufacturing production depends on the overall pattern of substitution/complementarity that is reflected in the price elasticities of materials, ϵ_X^M, and commodity demand, ϵ_M^O, and in the commodity share of materials production costs, S_M^O.

The following proposition states the results formally.

Proposition 4. *Depending on the properties of the function $\Gamma(p)$, there are four cases:*

1. ***Global complementarity**. Suppose that $\Gamma(p) > 0$ for all p. Then, manufacturing expenditure $Y(p)$ in (20) is a monotonically increasing function of p.*

2. ***Cobb-Douglas-like economy**. Suppose that $\Gamma(p) = 0$ for all p. This occurs when S_M^O and S_X^M are exogenous constants. Then, manufacturing expenditure $Y(p)$ in (20) is independent of p.*

3. ***Global substitution**. Suppose that $\Gamma(p) < 0$ for all p. Then, manufacturing expenditure $Y(p)$ in (20) is a monotonically decreasing function of p.*

4. ***Endogenous switching from complementarity to substitution.** Suppose that there exists a price p^v where $\Gamma(p)$ changes sign, from positive to negative. Then, the value of manufacturing production $Y(p)$ in (20) is a hump-shaped function of p with a maximum at p^v.*

Proof. See Appendix A.6.

The Cobb-Douglas-like case in Proposition 4 occurs when the production technologies in the materials and manufacturing sector are both Cobb-Douglas, i.e., $\epsilon_X^M = \epsilon_M^O = 1$. We do

[22] According to (19), $\Gamma(p)$ is the elasticity of the home demand for the commodity with respect to the commodity price, holding constant manufacturing expenditure. It thus captures the partial equilibrium effects of price changes in the commodity and materials markets for given market size.

not discuss this case further since it is a knife-edge specification in which commodity price changes have no effect on manufacturing production. The main insight of Proposition 4 is that the sign of the comparative statics depends on the substitution possibilities between labor and materials in the manufacturing sector and between labor and the commodity in the materials sector. Arguably, the most interesting case is when the function $\Gamma(p)$ switches sign as the model generates an endogenous switch from overall complementarity to substitution. This happens if production in manufacturing and materials sectors displays opposite substitution/complementarity properties: e.g., materials production exhibits labor-commodity complementarity while manufacturing exhibits labor-materials substitution. As a result, there exists a threshold price p^v such that $\Gamma(p) < 0$ for $p < p^v$ and $\Gamma(p) > 0$ for $p > p^v$: when p is low, the cost share $S_M^O(p)$ is small and the function $\Gamma(p)$ is then dominated by the term $1-\epsilon_M^O(p)$, which is positive since complementarity implies $\epsilon_M^O(p) < 1$ (i.e., *inelastic commodity demand*); conversely, when p is high, the cost share $S_M^O(p)$ is large and $\Gamma(p)$ is dominated by the term $1-\epsilon_X^M(p)$, which is negative since substitution implies $\epsilon_X^M(p) > 1$ (i.e., *elastic materials demand*).

The analysis suggests that a commodity price boom induces a decline in manufacturing activity (i.e., "Dutch Disease") when the economy exhibits overall substituion. It instead raises manufacturing activity when the economy exhibits overall complementarity between labor and the commodity.

Net commodity importer/exporter status.—An important building block of our model is that the commodity is used as input into production of materials. Therefore, the demand of the commodity is endogenous as such it responds to variations in the (exogenous) commodity price, p. As a result, the status of commodity importer/exporter is determined within the model as a function of the endowment, Ω, price, p, technological properties subsumed in the term $\xi(p)$, and other relevant parameters.

The following proposition characterizes the commodity exporting/importing region.

Proposition 5. *The economy is an exporter of the commodity when*

$$\frac{\Omega}{L} > \frac{\xi(p)}{p\left[1-\xi(p)-\beta\rho\right]}.$$

Proof. See Appendix A.7.

Proposition 5 provides a formal notion of **"commodity supply dependence."** For a given commodity price, p, there exists a threshold for the commodity/population endowment ratio Ω/L such that: (i) if Ω/L lies below the threshold, the economy is a *commodity importer*, i.e., $O > \Omega$, and conversely (ii) if Ω/L is above the threshold, the economy is a *commodity exporter*, i.e., $O < \Omega$. An extreme case of dependence is when $\Omega = 0$ such that by construction the country must import the commodity. Another way to interpret the link between the commodity price and the importer/exporter status is to note that, for a given relative endowment Ω/L, there exists a commodity price threshold p^d such that for $p < p^d$ the economy is a commodity importer whereas for $p > p^d$ the economy is a commodity exporter.

Figure 4 illustrates the determination of the commodity exporting/importing region.

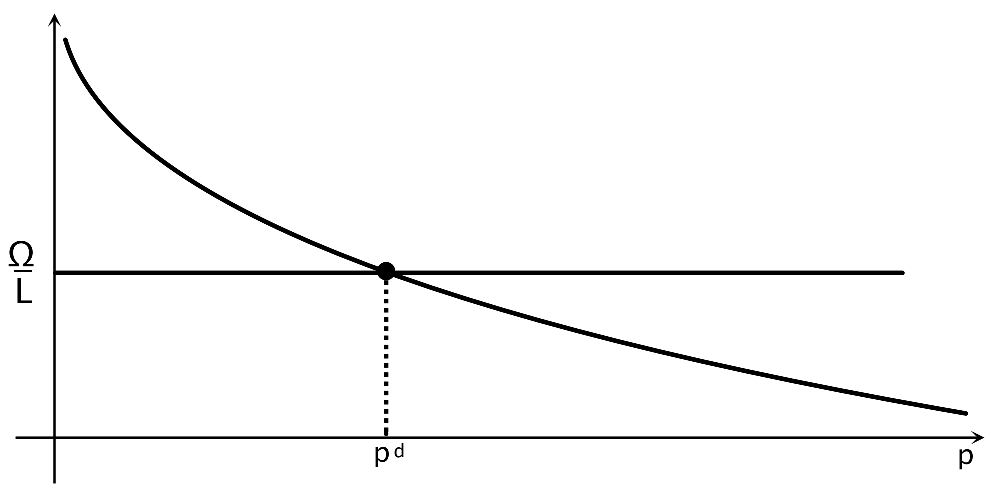

Figure 4: Commodity Supply Dependence

Figure 4 shows that the threshold price p^d is decreasing in the ratio Ω/L: economies with relatively high commodity endowments are commodity exporters for a larger range of prices, i.e., they are more likely to be net commodity exporters. On the other hand, economies with no commodity endowment, $\Omega = 0$, depend by construction on the foreign supply of the commodity for all p.

Commodity supply dependence, commodity price boom, and welfare.—Overall, the analysis suggests that an economy with a positive commodity endowment can gain (in terms of welfare) from a commodity price boom even if it is a commodity importer. Why is this the case? The reason is that revenues from sales of the endowment, Ω, go up one-for-one with p while import costs, pO, go up less than linearly since commodity consumption, O, responds negatively to an increase in p. Intuitively, this *specialization effect* is stronger if home commodity demand is elastic, i.e., under global substitution (see Proposition 4).

What matters for welfare is not the commodity trade balance, but how manufacturing activity reacts to commodity price changes. Under global substitution, the contraction of the commodity demand after a price boom mirrors the contraction of manufacturing activity associated with the specialization effect. The Schumpeterian mechanism at the heart of the model amplifies such a contraction—the instantaneous fall in $Y(p)$—into a deceleration of the rate of TFP growth. The economy eventually reverts to the initial steady-state growth rate g, but the temporary deceleration has negative welfare effects.

With these considerations in mind, let us consider a permanent change in the commodity price: for $p' > p$ we write

$$\Delta \equiv \frac{x_0}{x^*} - 1 = \frac{Y(p')/\epsilon N(p)}{Y(p')/\epsilon N(p')} - 1.$$

The term Δ is the percentage displacement of the state variable x from its steady state that occurs at time $t = t_0$, when the commodity price jumps up from p to p'. The numerator is the value of profitability holding constant the mass of firms; the denominator is instead the value of profitability at the end of the transition, that is, when the mass of firms has fully adjusted to the new market size.

Let us consider a commodity exporting economy under overall substitution, i.e., $\Gamma(p) < 0$. Figure 5 illustrates three possible paths of $\log u(t; p')$ as the economy transits to the new steady state with a permanently higher commodity price, $p' > p$.

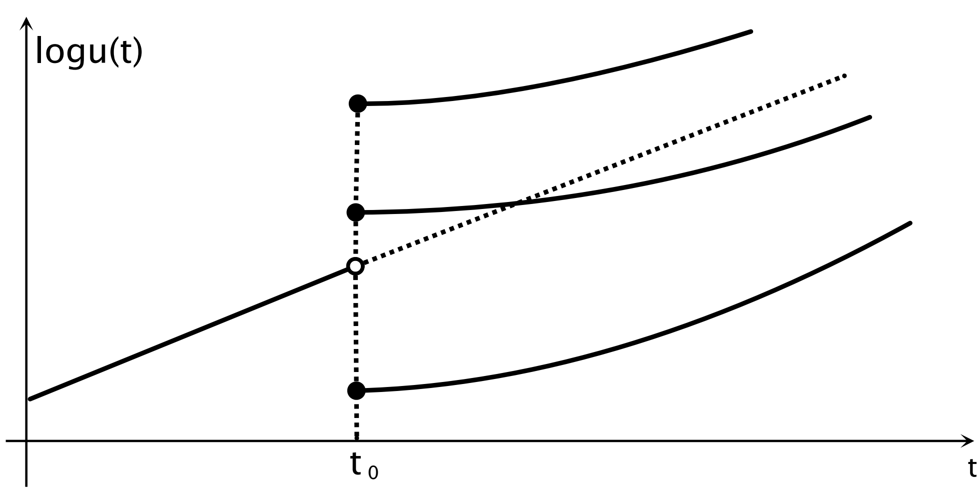

Figure 5: Utility Transition Path After a Commodity Price Boom

Since aggregate TFP is a pre-determined variable at $t = t_0$, the impact response in

$\log u(t_0)$ is exclusively driven by the jump in the home CPI index and the windfall effect. However, these two forces work in opposite directions such that the initial jump in utility has an ambiguous sign. After the initial impact response, the transition path of $\log u(t)$, for $t > t_0$, is governed by the transitional dynamics of aggregate TFP: the permanent fall in manufacturing activity—from $Y(p)$ to $Y(p') < Y(p)$—produces a slowdown in TFP growth, which in turn is due to a slowdown of net entry and a reduction in cost-reducing activity.[23]

As a result, a commodity price boom is welfare improving if and only if the windfall effect through $p\Omega$ is large enough to compensate for the cost of living effect through $c(p)$ and the curse effect through $\Delta < 0$. The closed-form solution (31) in Proposition 3 shows how model's parameters determine the relative weights of these effects.

6 The Prebisch-Singer Hypothesis Revisited

The Prebisch-Singer hypothesis posits that in the long-run commodity prices fall relative to the prices of the manufactures that the exporting country imports (see Prebisch, 1959; Singer, 1950).

In this section we study the long-run (steady-state) growth effects of the Prebisch-Singer hypothesis. Specifically, we make no attempt to explain why commodity prices would fall relative to the prices of imported goods. In contrast, we take the downward trend in the commodity/imports relative price as given, and derive the implications for steady-state growth.

Consider the case of a commodity exporting economy. The balanced trade condition in (6) suggests that an economy exporting part of its commodity endowement, i.e., $O < \Omega$, is, in fact, exchanging the commodity for the foreign consumption good. As a result, the relative price p/P_F is the one relevant for the Prebisch-Singer hypothesis. In the model, the price for the foreign comsumption good, P_F, is an exogenous constant. As such, a downward trend in the commodity price, p, results in the same trend in the relevant price ratio, p/P_F.

The following proposition characterizes the main result of the section.

Proposition 6. *Let:*

$$p(t) = p_0 e^{-g_p t},$$

[23] Note that as $t \to \infty$, the slope of the three transition paths depicted in Figure 5 converge to the same constant, ϕg, see equation (30) in Proposition 3. This happens because, as discussed in Section 4.3.1, the steady-state growth rate of aggregate TFP, g, is independent of the commodity price.

where $p_0 \equiv p(0)$ *is the initial price at* $t = 0$*, and* $g_p > 0$ *is the downward trend in the commodity price,* $p(t)$*. The steady-state growth rate of aggregate total factor productivity (TFP),* g*, is independent of the downward trend,* g_p*, i.e.,*

$$g(g_p') = g(g_p) = g \quad \textit{for all pairs} \left(g_p', g_p\right).$$

Proof. The result follows directly from the "commodity price super-neutrality" result in Propositions 2 and 3.

The result in Proposition 6 is an extension of the strong super-neutrality result discussed at length in Section 4.3: as the commodity price, $p(t)$, decreases, the value of manufacturing production, $Y\big(p(t)\big)$, instantaneously adjusts accordingly (the sign of the change depends on overall substitution/complementarity, see Proposition 4), this in turn induces an endogenous change in the mass of firms, via net entry/exit, which leaves unchanged the steady-state firms' market size and gross profitability.

7 Numerical Analysis

To further illustrate the dynamic properties of the model, we conduct a numerical exercise. We calibrate the model economy in the region $x(t) > x_Z$ of the phase diagram in Figure 3.

7.1 Calibration

One period is one year. Table 1 contains the baseline parameter values.

Table 1: Baseline Parameters

Parameter	Interpretation	Value
$\epsilon/(\epsilon - 1)$	Mfg price markup	1.3
θ	Mfg prod. function: $X_i = Z_i^{\theta} F(L_{X_i} - \phi, M_i)$	0.15
ρ	Discount rate	0.02
δ	Death rate	0.035
β	Mfg entry cost: $V_i = \beta \cdot \frac{WY}{N}$	1

We set $\epsilon = 4.33$ to match a price markup of 30 percent. Overall, the available evidence for the U.S. provides estimates of markups in value added data that range from 1.2 to 1.4.[24] Hence, we target a markup in the manufactuting sector of $\mu = \epsilon/(\epsilon - 1) = 1.3$ that is at the middle of the available range of estimates. The condition for a symmetric equilibrium, $\theta(\epsilon - 1) < 1$, imposes a restriction on the calibration of θ, i.e., $\theta \in \left[0, 1/(\epsilon - 1)\right]$. As a result, the calibrated value of $\epsilon = 4.33$ provides an upper bound on θ, i.e., $\theta \in (0, 0.3)$. Since we have no reference value guiding our choice, we set $\theta = 0.15$ at the middle of the possible range. The death rate is set to $\delta = 0.035$ to match the average closing rate of establishments in the U.S. manufacturing sector for 1992-2012. Data for closing establishments are from the Business Employment Dynamics (BED) survey of the Bureau of Labor Statistics (BLS).[25]

The requirement of positive eigenvalues over all the state space provides a restriction on the calibration of the entry cost's parameter, β. Specifically, $\nu > 0$ implies $\beta \in \left[0, \frac{1-\theta(\epsilon-1)}{\epsilon(\rho+\delta)}\right]$.[26] We set $\beta = 1$, which is within the set identified by the restriction above. Finally, we set the time discount rate to $\rho = 2\%$, which implies a 2 percent interest rate.

7.2 Dynamic Response to a "Profit Rate Shock"

In this section we compute the dynamic response of the gross profit rate, $x \equiv Y/\epsilon N$, to a "shock" that temporarily displaces x from its steady-state value, $x^* > x_Z$ (see Figure 3): we force the model to be in the neighborhood of the steady state (i.e., in transition dynamics) and illustrate how the model economy reverts back to the original steady state, x^*.[27]

Figure 6 plots the time path of

$$\frac{x(t)}{x^*} - 1 = \Delta e^{-\nu t},$$

where $x(t) = x_0 e^{-\nu t} + x^* \left(1 - e^{-\nu t}\right)$, the eigenvalue of the differential equation for x is

$$\nu = \frac{1 - \theta(\epsilon - 1)}{\beta\epsilon} - (\rho + \delta) \quad \text{for} \quad x(t) > x_Z,$$

[24]See Hall (1988), Morrison (1992), Norrbin (1993), Roeger (1995), and Basu and Fernald (1997, 2001).

[25]Survey homepage: `http://www.bls.gov/bdm/`.

[26]Let ν_N and ν_Z denote the eigenvalues of the dynamical system in the region $x_N < x(t) \leq x_Z$ and $x(t) > x_Z$ of the phase diagram in Figure 3, respectively. The two eigenvalues are in the following relationship: $\nu_Z = \nu_N - \theta(\epsilon - 1)/\beta\epsilon < \nu_N$.

[27]Recall that the gross profit rate x is the key variable of the model regulating the incentives to innovate and hence driving the relevant equilibrium dynamics of the model (see Proposition 2). Without loss of generality, we consider a shock to the variable x since this allows us to circumvent calibration of the function $\xi(p)$ in (20). Since there is a one-to-one mapping between the function $\xi(p)$ and the commodity price p, a shock to x can be interpreted as a transformation of the shock to the commodity price, p.

and the initial percentage displacement from the steady state is $\Delta = \frac{x_0}{x^*} - 1$— *"profit rate shock."* In Figure 6, we consider a profit rate shock of $\Delta = 10\%$. The parameter values in Table 1 result in an eigenvalue of $\nu = 0.06$, which implies an half-life of $t_{1/2} \approx 11.5$ years.[28]

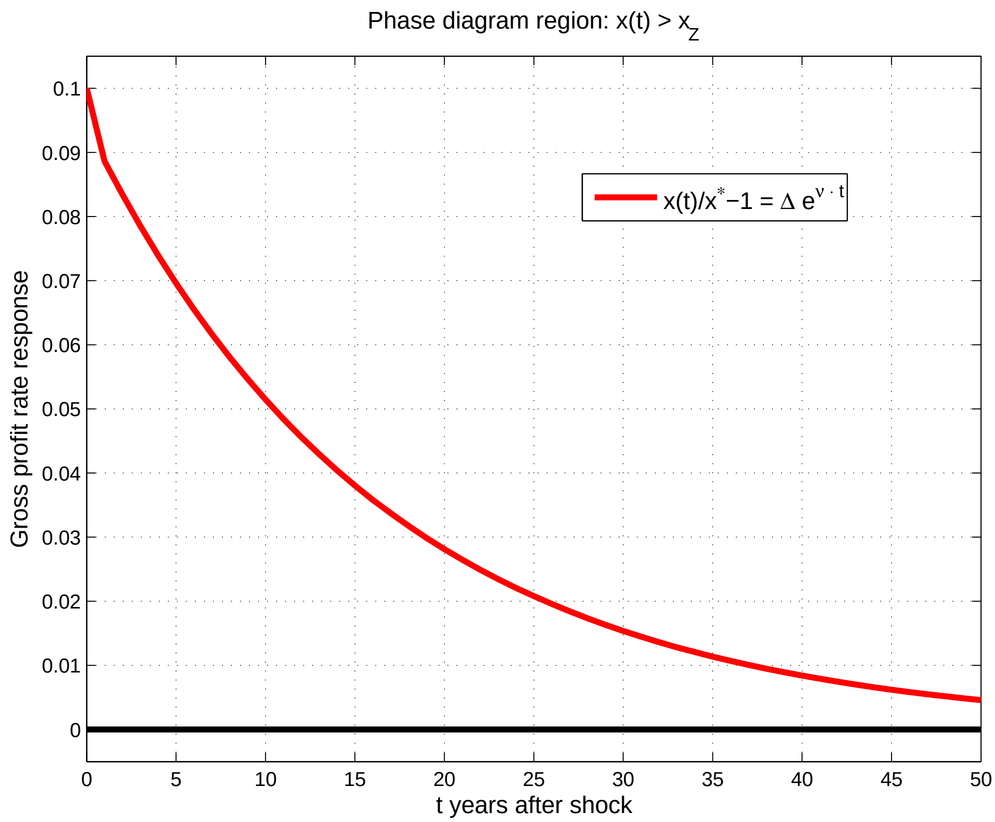

Figure 6: Dynamic Response to "Profit Rate Shock"

Notes: The figure plots the time path of the gross profit rate, x, as percentage deviation from the steady state, $x^* > x_Z$, in the region $x(t) > x_Z$ of the phase diagram in Figure 3: $x(t)/x^* - 1 = \Delta e^{-\nu t}$, where $x(t) = x_0 e^{-\nu t} + x^* \left(1 - e^{-\nu t}\right)$, $\Delta = 10\%$, and $\nu = \frac{1-\theta(\epsilon-1)}{\beta\epsilon} - (\rho + \delta) = 0.0604$. See Table 1 for parameter values.

Commodity price shock.—To explain the mapping between a commodity price shock and what we named "profit rate shock," we now consider the scenario of a permanent fall in the commodity price—from p to p' with $p' < p$—and an economy operating under global substitution, i.e., $\Delta_Y \equiv Y(p') - Y(p) > 0$ for all $p' < p$.[29] The "long-run commodity price super-neutrality" result in Proposition 2 implies that $x^*(p') = x(p)$ for all price pairs (p', p).

[28]Since the equilibrium gross profit flow x follows a linear differential equation, the speed of reversion to the steady state is fully determined by the magnitude of the eigenvalue, ν.

[29]See Proposition 4 for a formal definition of global substitution.

So, after an unexpected (permanent) fall in the commodity price, the value of manufacturing production jumps from $Y(p)$ to the new steady-state level $Y(p')$. The initial impact response, Δ, is then followed by transitional dynamics driven by net firm entry, eventually the mass of firms endogenously adjusts—from $N(p)$ to $N(p')$ with $N(p') > N(p)$—such that in steady state the initial jump in $Y(p')$ is fully neutralized, i.e., $x^*(p') = x^*(p)$.

8 Conclusions

We study the relationship between commodity prices and growth within an endogenous growth model of commodity-rich economies. In the model, long-run (steady-state) growth is endogenous and yet independent of commodity prices. However, commodity prices affect short-run growth through transitional dynamics in aggregate TFP. We argue that these predictions are consistent with historical data from the 19th to the 21st century: commodity prices exhibit large and persistent long-run movements whereas growth rates of real GDP per capita in the Western Offshoots (i.e., U.S., Australia, Canada, New Zealand) have been approximately constant since the 19th century. This argument, which parallels that in Jones (1995), draws an analogy between the effects of commodity price on growth and the literature on the (lack of) long-run growth effects of taxation (see Easterly and Rebelo, 1993; Easterly et al., 1993; Stokey and Rebelo, 1995; Mendoza et al., 1997).

Appendix

A.1 Firm's Behavior and the Free-Entry Equilibrium

To characterize the typical firm's behavior, consider the Current Value Hamiltonian (CVH, henceforth):

$$CVH_i = \left[P_i - C_X(W, P_M)Z_i^{-\theta}\right]X_i - W\phi - WL_{Z_i} + z_i\alpha KL_{Z_i},$$

where the co-state variable, z_i, is the value of the marginal unit of knowledge. The firm's knowledge stock, Z_i, is the state variable of the problem whose law of motion is equation (11); labor services allocated to cost reduction, L_{Z_i}, and the product's price, P_i, are control variables. Firms take the public knowledge stock, K, as given. Since the Hamiltonian is linear in L_{Z_i}, there are three cases: (1) $W > z_i\alpha K$ implies that the value of the marginal unit of knowledge is lower than its cost. As result, the firm does not allocate labor to cost-reducing activities; (2) $W < z_i\alpha K$ implies that the value of the marginal unit of knowledge is higher than its cost. This case violates general equilibrium conditions and, as such, it is ruled out since the firm would demand an infinite amount of labor to employ in cost reduction; and (3) $W = z_i\alpha K$, which is the first order condition for an interior solution given by the equality between marginal revenue and marginal cost of knowledge accumulation.

The problem of the firm also consists of the terminal condition,

$$\lim_{s\to\infty} e^{-\int_t^s [r(v)+\delta]dv} z_i(s)Z_i(s) = 0,$$

and a differential equation for the costate variable,

$$r + \delta = \frac{\dot{z}_i}{z_i} + \theta C_X(W, P_M)Z_i^{-\theta-1}\left(\frac{X_i}{z_i}\right),$$

that defines the rate of return to cost reduction as the ratio between revenues from the knowledge stock and its shadow price plus (minus) the appreciation (depreciation) in the value of knowledge. The revenue from the marginal unit of knowledge is given by the cost reduction it yields times the scale of production to which it applies.

The optimal pricing rule is

$$P_i = \left(\frac{\epsilon}{\epsilon - 1}\right) C_X(W, P_M)Z_i^{-\theta}. \tag{A.1}$$

Peretto (1998, Proposition 1) shows that under the restriction $1 > \theta(\epsilon - 1)$ the firm is always at the interior solution, where $W = z_i \alpha K$ holds, and the equilibrium is symmetric.

The cost function (10) produces the following conditional factor demands:

$$L_{X_i} = \frac{\partial C_X(W, P_M)}{\partial W} Z_i^{-\theta} X_i + \phi;$$

$$M_i = \frac{\partial C_X(W, P_M)}{\partial P_M} Z_i^{-\theta} X_i.$$

The price strategy (A.1), symmetry and aggregation across firms yield (13) and (14). In the symmetric equilibrium, $K = Z = Z_i$ yields $\dot{K}/K = \alpha L_Z/N$, where L_Z is aggregate labor in cost reduction. By taking logs and time-derivative of $W = z_i \alpha K$, using the demand curve (8), the cost-reduction technology (11), and the price strategy (A.1), one reduces the first-order conditions to (15).

Taking logs and time-derivative of V_i yields

$$r = \frac{\Pi_i}{V_i} + \frac{\dot{V}_i}{V_i} - \delta.$$

The sunk entry cost is $\beta WY/N$. Labor allocated to entry is L_N. The case $V > \beta WY/N$ yields an unbounded demand for labor in entry, $L_N = +\infty$, and, as such, it is ruled out since it would violate general equilibrium conditions. The case $V < \beta WY/N$ yields $L_N = -\infty$, which means that the non-negativity constraint on L_N binds as such $L_N = 0$. A free-entry equilibrium requires $V = \beta WY/N$. Using the price strategy (A.1), the rate of return to entry becomes (16).

A.2 Proof of Proposition 1

Since consumption goods and materials sectors are competitive, $\Pi_H = \Pi_M = 0$. The consumption expenditure allocation rule (4) and the choice of numeraire yield

$$\dot{A} = rA + L + p\Omega - \frac{1}{\varphi} Y_H.$$

By rewriting the domestic commodity demand (19) as

$$pO = Y \cdot \xi(p), \quad \xi(p) \equiv \frac{\epsilon - 1}{\epsilon} S_X^M(p) S_M^O(p),$$

allows us to rewrite the balanced trade condition as

$$\frac{1}{\varphi}Y_H - p\Omega = Y\left(1-\xi\left(p\right)\right).$$

Substituting the expressions for financial wealth, $A = \beta Y$, and the balanced trade condition in the household's budget constraint (3), and using the rate of return to saving in (5), yields

$$\begin{aligned}\frac{\dot{Y}}{Y} &= \rho + \frac{\dot{Y}_H}{Y_H} + \frac{L + p\Omega - \frac{1}{\varphi}Y_H}{\beta Y} \\ &= \rho + \frac{\dot{Y}_H}{Y_H} + \frac{L - Y\left(1-\xi\left(p\right)\right)}{\beta Y}.\end{aligned}$$

Differentiating the balanced trade condition yields

$$\frac{1}{\varphi}\dot{Y}_H = \dot{Y}\left(1-\xi\left(p\right)\right) \Rightarrow \frac{\dot{Y}_H}{Y_H} = \frac{\dot{Y}}{Y}\frac{Y}{Y_H}\varphi\left(1-\xi\left(p\right)\right) = \frac{\dot{Y}}{Y}\frac{Y\left(1-\xi\left(p\right)\right)}{Y\left(1-\xi\left(p\right)\right)+p\Omega}.$$

Substituting back in the budget constraint and rearranging terms yields

$$\frac{\dot{Y}}{Y} = \frac{Y\left(1-\xi\left(p\right)\right)+p\Omega}{p\Omega}\left[\rho + \frac{L - Y\left(1-\xi\left(p\right)\right)}{\beta Y}\right].$$

This differential equation has a unique positive steady-state value of manufacturing production:

$$Y\left(p\right) = \frac{L}{1-\xi\left(p\right)-\rho\beta}.$$

We ignore, for simplicity the issue of potential indeterminacy, assuming that Y jumps to this steady-state value. The associated expenditures on the home and foreign goods, respectively, are

$$\begin{aligned}Y_H\left(p\right) &= \varphi\left[\frac{L\left(1-\xi\left(p\right)\right)}{1-\xi\left(p\right)-\rho\beta} + p\Omega\right]; \\ Y_F\left(p\right) &= \left(1-\varphi\right)\left[\frac{L\left(1-\xi\left(p\right)\right)}{1-\xi\left(p\right)-\rho\beta} + p\Omega\right].\end{aligned}$$

Since $Y_H\left(p\right)$ and $Y_F\left(p\right)$ are constant, the saving rule (5) yields that the interest rate is $r = \rho$ at all times.

A.3 Proof of Proposition 2

The return to entry (16) and the entry technology $\dot{N} = (N/\beta Y) \cdot L_N - \delta N$ yield

$$L_N = \frac{Y}{\epsilon x}\left[x - \left(\phi + \frac{L_Z}{N}\right)\right] - \rho\beta Y.$$

Taking into account the non-negativity constraint on L_Z, we solve (11) and (15) for

$$\frac{L_Z}{N} = \begin{cases} \theta(\epsilon - 1)x - (\rho+\delta)/\alpha & x > x_Z \equiv \frac{\rho+\delta}{\alpha\theta(\epsilon-1)} \\ 0 & x \leq x_Z \end{cases}. \tag{A.2}$$

Therefore,

$$L_N = \begin{cases} \frac{Y}{\epsilon}\left[1 - \theta(\epsilon-1) - \frac{\phi - (\rho+\delta)/\alpha}{x}\right] - \rho\beta Y & x > x_Z \\ \frac{Y}{\epsilon}\left(1 - \frac{\phi}{x}\right) - \rho\beta Y & x \leq x_Z \end{cases}.$$

So we have

$$L_N > 0 \text{ for } \begin{cases} x > \frac{\phi - (\rho+\delta)/\alpha}{1 - \theta(\epsilon-1) - \epsilon\rho\beta} & x > x_Z \\ x > \frac{\phi}{1-\epsilon\rho\beta} & x \leq x_Z \end{cases}.$$

We look at the case

$$\frac{\phi}{1-\epsilon\rho\beta} \equiv x_N < \frac{\rho+\delta}{\alpha\theta(\epsilon-1)} \equiv x_Z,$$

which yields that the threshold for gross entry, x_N, is smaller than the threshold for cost reduction, x_Z.[30]

To obtain the value of Y when $L_N = 0$, first note that

$$L_N = 0 \quad \text{for} \quad \frac{1}{\epsilon}\left(1 - \frac{\phi}{x}\right) \leq \rho\beta.$$

The household budget constraint yields

$$0 = N\left(\frac{Y}{\epsilon N} - \phi\right) + L + \Omega p - \frac{1}{\varphi}Y_H.$$

Using the balanced trade condition and rearranging yields

[30] The global dynamics are well defined also when this condition fails and $x_N > x_Z$. We consider only the case $x_N < x_Z$ to streamline the presentation since the qualitative results and the insight about the role of the commodity price remain essentially the same.

$$Y = \frac{L - \phi N}{1 - \xi(p) - \frac{1}{\epsilon}}.$$

This equation holds for

$$x \leq x_N \equiv \frac{\phi}{1 - \epsilon\rho\beta} \Leftrightarrow N \geq N_N \equiv \frac{\phi}{1 - \epsilon\rho\beta}\frac{\epsilon}{Y}.$$

The interpretation is that with no labor allocated to entry, there is net exit and thus saving of fixed costs. This manifests itself as aggregate efficiency gains as intermediate firms move down their average cost curves. Note that in this region,

$$Y(t) = \frac{L - \phi N_0 e^{-\delta t}}{1 - \xi(p) - \frac{1}{\epsilon}},$$

which shows that intermediate production grows in value as a result of net exit. The consolidation of the market results in growing profitability, that is,

$$\frac{\dot{x}}{x} = \frac{\delta L}{L - \phi N_0 e^{-\delta t}} \Rightarrow \dot{x} = \frac{\delta L/\epsilon N_0}{1 - \xi(p) - \frac{1}{\epsilon}}.$$

This says that with the exit shock, the economy must enter the region where entry is positive because the very definition of steady state requires replacing firms that leave the market. Therefore, the only condition that we need to ensure convergence to the steady state with active cost reduction is $x^* > x_Z$.

A.4 Proof of Proposition 3

Taking logs of (26) yields

$$\log T(t) = \theta \log Z_0 + \theta \int_0^t \hat{Z}(s)\, ds + \chi \log N_0 + \chi \log\left(\frac{N(t)}{N_0}\right).$$

Using the expression for g in (27), and adding and subtracting $\hat{Z}^*$ from $\hat{Z}(t)$, we obtain

$$\log T(t) = \log\left(Z_0^{\theta} N_0^{\chi}\right) + gt + \theta \int_0^t \left[\hat{Z}(s) - \hat{Z}^*\right] ds + \chi \log\left(\frac{N(t)}{N_0}\right).$$

Using (A.2) and (28) we rewrite the third term as

$$\begin{aligned}\theta\int_0^t \left(\hat{Z}(s)-\hat{Z}^*\right)ds &= \alpha\theta^2(\epsilon-1)\int_0^t (x(s)-x^*)\,ds \\ &= \gamma\left(\frac{x_0}{x^*}-1\right)\int_0^t e^{-\nu s}ds \\ &= \frac{\gamma}{\nu}\left(\frac{x_0}{x^*}-1\right)\left(1-e^{-\nu t}\right),\end{aligned}$$

where

$$\gamma \equiv \alpha\theta^2(\epsilon-1)\,x^*.$$

Observing that $N(t) = Y(p)/\epsilon x(t)$ yields $\dot{N}/N = -\dot{x}/x$, we use (28) to obtain

$$\frac{N(t)}{N_0} = \frac{1+\left(\frac{N^*}{N_0}-1\right)}{1+\left(\frac{N^*}{N_0}-1\right)e^{-\nu t}}.$$

We then rewrite the last term as

$$\begin{aligned}\chi\log\left(\frac{N(t)}{N_0}\right) &= \chi\log\frac{1+\left(\frac{N^*}{N_0}-1\right)}{1+\left(\frac{N^*}{N_0}-1\right)e^{-\nu t}} \\ &= \chi\log\left(1+\left(\frac{N^*}{N_0}-1\right)\right)-\chi\log\left(1+\left(\frac{N^*}{N_0}-1\right)e^{-\nu t}\right).\end{aligned}$$

Approximating the log terms, we can write

$$\begin{aligned}\chi\log\left(\frac{N(t)}{N_0}\right) &= \chi\left(\frac{N^*}{N_0}-1\right)-\chi\left(\frac{N^*}{N_0}-1\right)e^{-\nu t} \\ &= \chi\left(\frac{N^*}{N_0}-1\right)\left(1-e^{-\nu t}\right).\end{aligned}$$

Observing that

$$\frac{N^*}{N_0}-1 = \frac{x_0}{x^*}-1,$$

these results yield (29).

Now consider

$$\begin{aligned}\log u &= \varphi \log\left(\frac{Y_H}{P_H L}\right) + (1-\varphi)\log\left(\frac{Y_F}{P_F L}\right) \\ &= \varphi \log\left(\frac{Y_H}{P_H L}\right) + (1-\varphi)\log\left(\frac{\frac{1-\varphi}{\varphi} Y_H}{P_F L}\right) \\ &= \log\left(\frac{Y_H}{L}\right) - \varphi \log P_H + (1-\varphi)\log\left(\frac{1-\varphi}{\varphi P_F}\right) \\ &= \log\left(\frac{Y_H}{L}\right) - \varphi \log c(p) + \varphi \log T - \varphi \log\left(\frac{\epsilon}{\epsilon-1}\right) + (1-\varphi)\log\left(\frac{1-\varphi}{\varphi P_F}\right).\end{aligned}$$

To simplify the notation, and without loss of generality, we set

$$(1-\varphi)\log\left(\frac{1-\varphi}{\varphi P_F}\right) + \varphi \log\left(N_0^{\chi} Z_0^{\theta}\right) - \varphi \log\left(\frac{\epsilon}{\epsilon-1}\right) \equiv 0.$$

This is just a normalization that does not affect the results. We then substitute the expression derived above into (1) and write

$$\begin{aligned}U(p) &= \int_0^\infty e^{-\rho t}\left[\log \varphi\left(\frac{1-\xi(p)}{1-\xi(p)-\rho\beta} + \frac{p\Omega}{L}\right) - \varphi \log\left(c(p)\right) + \varphi g t\right] dt \\ &\quad + \varphi\left(\frac{\gamma}{\nu} + \chi\right)\Delta \int_0^\infty e^{-\rho t}\left(1-e^{-\nu t}\right) dt.\end{aligned}$$

Integrating, we obtain (31).

A.5 Proof of Lemma 1

Observe that

$$\epsilon_X^M \equiv -\frac{\partial \log M}{\partial \log P_M} = 1 - \frac{\partial \log S_X^M}{\partial \log P_M} = 1 - \frac{\partial S_X^M}{\partial P_M}\frac{P_M}{S_X^M}$$

so that $\epsilon_X^M \leq 1$ if

$$\frac{\partial S_X^M}{\partial P_M} = \frac{\partial}{\partial P_M}\left(\frac{P_M M}{P_M M + L_X}\right) \geq 0.$$

This in turn is true if

$$\left(1 - S_X^M\right)\frac{\partial (P_M M)}{\partial P_M} - S_X^M \frac{\partial L_X}{\partial P_M} \geq 0.$$

Recall now that total cost is increasing in P_M so that

$$\frac{\partial (P_M M)}{\partial P_M} + \frac{\partial L_X}{\partial P_M} > 0 \Rightarrow \frac{\partial (P_M M)}{\partial P_M} > -\frac{\partial L_X}{\partial P_M}.$$

It follows that

$$\frac{\partial L_X}{\partial P_M} \leq 0$$

is a sufficient condition for $\epsilon_X^M \leq 1$ since it implies that both terms in the inequality above are positive. The proof for $\epsilon_M^O \leq 1$ is analogous.

A.6 Proof of Proposition 4

Differentiating (20) yields

$$\frac{d \log Y(p)}{dp} = -\frac{d \log (1 - \xi(p) - \beta\rho)}{dp} = \frac{\xi'(p)}{1 - \xi(p) - \beta\rho}.$$

It is useful to write $\xi'(p)$ as

$$\xi'(p) = \frac{\xi(p)}{p}\left[\left(1 - \epsilon_X^M(p)\right) S_M^O(p) + 1 - \epsilon_M^O(p)\right],$$

which shows that the sign of $\xi'(p)$ depends on the upstream and downstream price elasticities of demand, and on the overall contribution of the commodity to manufacturing costs. Assume for example that $1 - \epsilon_X^M(p) < 0$ (i.e., labor-materials substitution) and $1 - \epsilon_M^O(p) > 0$ (i.e., labor-commodity complementarity), then there exists a threshold price p^v such that:

$$\xi'(p^v) = \frac{\xi(p^v)}{p^v}\left[\left(1 - \epsilon_X^M(p^v)\right) S_M^O(p^v) + 1 - \epsilon_M^O(p^v)\right] = 0,$$

i.e.,

$$\left(\epsilon_X^M(p^v) - 1\right) S_M^O(p^v) = 1 - \epsilon_M^O(p^v).$$

A.7 Proof of Proposition 5

Equations (19) and (20) yield

$$\Omega \gtrless O \Leftrightarrow \frac{\Omega}{L} \gtrless \frac{\xi(p)}{p\left[1 - \xi(p) - \beta\rho\right]}.$$

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