Risk Aversion and the Financial Accelerator *

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Abstract†

We extend the Bernanke, Gertler and Gilchrist (1999) framework with risk-neutral entrepreneurs to entrepreneurs with CRRA preferences. The optimal contract in this case is identical to debt when there is no default, while when there is default entrepreneurs retain part of their assets. The partial equilibrium analysis suggests that, when entrepreneurs are risk-averse, leverage becomes more sensitive to fluctuations in expected returns and to shocks to the variance of unobserved idiosyncratic productivity —"risk shocks". In general equilibrium, the higher responsiveness of leverage to asset prices, makes asset prices more stable and acts to reduce business cycle fluctuations relative to a case where entrepreneurs are risk neutral. With risk aversion, technology and monetary shocks have quantitatively similar effects on key macro variables, although about 20 percent smaller, relative to the ones in the model with risk neutrality. However, the response of output to "risk shocks" is 60 percent smaller when entrepreneurs are risk averse than when they are risk neutral.

Keywords: Financial accelerator; financial frictions; risk; optimal contract; agency costs.

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1 Introduction

With the recent financial crisis, the analysis of the links between the financial sector and the real economy has attracted a lot of attention among macroeconomists. A standard modeling technique that allows for meaningful feedback between the financial sector and the real economy in a DSGE framework is based on the contribution by Bernanke, Gertler and Gilchrist (BGG, 1999). In their model the relationship between lenders and borrowers is characterized by a credit friction, which introduces a wedge between expected capital returns and the safe rate of return.[1] This wedge, which is also interpretable as an external finance premium, is countercyclical and amplifies the business cycle. For example, a favorable shock to the economy that increases asset prices also raises net worth and reduces the external finance premium. The decline in the external finance premium leads to an increase in borrowing, and investment and output rise beyond the effect of the initial shock. One important characteristic of this model is that borrowers are risk neutral, therefore, they always want to borrow as much as possible, as long as the expected capital returns are higher than the borrowing cost.

In practice entrepreneurs face uninsurable risks with respect to which they may not be indifferent. Among other studies, the empirical analysis of Heaton and Lucas (2000) shows that for a subset of households that receive income from entrepreneurial ventures, their variable business income represents a large source of undiversified risk. The behavior of risk averse entrepreneurs in the face of this risk can be very different from that of risk neutral agents. In particular, entrepreneurs with higher risk aversion may borrow less and they become more sensitive to changes in undiversified risk. This different behavior may have important implications for the role of financial frictions in the amplification of the business cycle.

In this paper we explore this line of thought by extending the BGG framework to the case of entrepreneurs with constant relative risk-aversion (CRRA) preferences, yet maintaining an analytically tractable, log-liner framework. This framework allows us to investigate the role of financial frictions in the presence of uninsurable risk. We have four main results. First, risk-averse borrowers choose a lower leverage in steady state than their risk-neutral counterpart, *ceteris paribus*. Second, in partial equilibrium, when entrepreneurs are risk-averse, leverage becomes more sensitive to fluctuations in expected returns and to shocks to the variance of unobserved idiosyncratic productivity — so called "risk shocks". This finding is consistent with the results of Chen *et al.* (2010), who study investment and financing decisions for entrepreneurial firms in a dynamic capital structure model with incomplete markets. Third, in general equilibrium the response of key macro variables to aggregate shocks, such as technology and monetary shocks is very similar for risk-averse and risk-neutral borrowers, although about 20 percent smaller in the former case.[2] Finally, the response of output to "risk shocks" is 60

[1] Throughout the text we will use the expressions "credit friction" and "financial friction" interchangeably.

[2] We find that the response of key macro variables to government spending shocks is very similar for risk-averse and risk-neutral borrowers, although about 15% smaller in the risk-averse case. We do not report these

percent smaller when entrepreneurs are risk-averse than when they are risk-neutral.

On the methodological side, we are the first to incorporate risk-aversion in a model of idiosyncratic, uninsurable risk such as BGG, while keeping the analytical tractability of a log-linear framework. Modeling costly state verification problems with risk-averse borrowers has several difficulties which we need to address. To begin with, the optimal contract is no longer a debt contract, as for the case of risk neutrality (Townsend, 1979). Under a standard debt contract, in case of default the lender confiscates all the net worth of the borrower. Such an arrangement is no longer optimal for the risk-averse borrower because it would imply a zero-consumption scenario. We use Tamayo's (2013) results in the costly state verification literature who shows that, in a static, partial-equilibrium setting, risk-averse entrepreneurs would offer a different optimal contract to the lender. This contract ensures that the borrower retains some of his net worth even in the case of default. We extend Tamayo's (2013) financial contract to a general equilibrium framework that features optimal history-independent loans with predetermined returns for lenders.[3]

The second difficulty lies in the aggregation of individual histories in the presence of uninsurable idiosyncratic risk and non-linear preferences, whose combination implies that every entrepreneur chooses a different leverage. This form of heterogeneity normally requires giving up the traditional frameworks with a limited number of agents in favor of a more computational approach, e.g. Krussel and Smith (1998) . We instead allow entrepreneurs to be risk-averse and make two assumptions that lead to identical leverage choices for potentially different entrepreneurs. Specifically, we allow only newborn entrepreneurs to work, so that labor income does not affect the financial decision of entrepreneurs. Moreover, we assume that all net worth is reinvested in every period and entrepreneurs consume only in the case of death, which occurs with an exogenous probability. These two assumptions keep the aggregation of individual histories simple and ensure, as in BGG, that only aggregate net worth matters for the economy dynamics.

We contribute to the theoretical literature on financial friction by demonstrating that the financial accelerator mechanism is robust to the presence of risk-averse entrepreneurs in response to technology and monetary shocks. A partial equilibrium analysis would suggest that, for a given movement in excess capital returns, the response of leverage should be larger with risk-averse borrowers. However, in general equilibrium the endogenous movement in excess capital returns is such that key macro variables respond in a very similar way to aggregate shocks under different degrees of risk aversion, although the effects are somewhat smaller with higher risk-aversion.

results in the simulations.

[3] Precisely, we derive the optimal one-period contract with deterministic monitoring. An excellent list of references for partial equilibrium multi-period contracts includes Monnett and Quintin (2005) for stochastic monitoring, Wang (2005) for deterministic monitoring, and Cole (2013) for self-enforcing stochastic monitoring.

For risk shocks, instead, the presence of risk-averse borrowers significantly reduces amplification. The increase in the volatility of idiosyncratic productivity, increases expected monitoring costs, and reduces net worth and the price of capital temporarily. However, the price of capital is expected to rise. Other things the same, this higher future return induces entrepreneurs to borrow more and invest in the remunerative capital. In the risk-neutral case, however, the risk shock increases the cost of external finance so much that entrepreneurs are not able to borrow enough to buy the temporarily cheap capital. In the risk-averse case, the smaller fluctuations in the external finance premium and the more moderate decline in net worth result in a smaller decline in borrowing. As a result, the decline in the price of capital, investment and output is more than two times smaller than in the risk-neutral scenario.

The paper proceeds as follows. Section 2 derives the static optimal contract in partial equilibrium. Section 3 introduces aggregate risk and dynamics. Section 4 incorporates the resulting contract into the general equilibrium framework. Section 5 contains our quantitative analysis and results. Section 6 concludes.

2 Static Optimal Contract in Partial Equilibrium

In this section we study the optimal contract between a risk-averse borrower (the entrepreneur) and a risk-neutral lender. In the financial frictions literature popularized by BGG, borrowers are assumed to be risk neutral hence indifferent to aggregate or idiosyncratic risk. In the present context instead, the borrower should be thought of as a risk-averse agent who is subject to uninsurable risk. Lenders are risk-neutral with respect to the idiosyncratic (i.e. entrepreneur specific) risk because, as will be true in the general equilibrium model developed below, they can diversify their lending activity across a large number of projects.

The static contract between the lender and borrower follows the traditional CSV framework and resembles the optimal contract developed by Tamayo (2013). Entrepreneurs invest in a risky asset (capital) in the amount of QK, where K denotes the quantity of capital purchased and Q its relative price. The return on the investment is $QKR^k\omega$, where R^k indicates aggregate returns to capital and $\log(\omega) \sim \mathcal{N}(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2)$ the idiosyncratic return component that is specific to the entrepreneur with pdf $\phi(\omega)$. We assume that the lender cannot observe the realization of the idiosyncratic shock to the entrepreneurs unless he pays monitoring costs μ which are in fixed percentage of total assets. In each state of the world $\omega \in \Omega$, the risk-averse entrepreneur chooses to report $s(\omega)$ and the report is verified in the verification set $\Omega^V \subset \Omega$. Following the literature, we assume that reports are always truthful so that $s(\omega) = \omega$ for all $\omega \in \Omega$, which implies that the repayment function depends only on ω.[4]

Definition 1 *A contract under CSV is an amount of borrowed money B, a repayment function $R(\omega)$ in the state of nature ω and a verification set Ω^V, where the lender chooses to verify the state of the world.*

[4]See Tamayo (2013) for details.

The static problem in the presence of only idiosyncratic risk ω can be formulated as

$$\max_{K,R()} \frac{\int_0^\infty [QKR^k(\omega - R(\omega))]^{1-\rho} \phi(\omega) d\omega}{1-\rho} \tag{1}$$

$$BR \leq QKR^k \int_0^\infty R(\omega)\phi(\omega)d\omega - \mu QKR^k \int_{\omega \in \Omega^V} \omega\phi(\omega)d\omega \tag{2}$$

$$QK = B + N \tag{3}$$

$$0 \leq R(\omega) \leq \omega \quad \forall\omega \tag{4}$$

The first equation is the expected utility of the entrepreneur from the investment return. The second equation is a participation constraint for the lender; it says that he should be paid on average the gross safe rate of return, R. The third equation just says that the entrepreneur uses the loan (B) and his own net worth (N) for acquiring capital. The final inequality constraint states that repayments should be non-negative and cannot exceed the total amount of assets. The following Proposition is a special case of Tamayo's (2013) Theorem 1 case iii).

Proposition 1 *Under the optimal contract that solves the problem (1) subject to (2), (3), (4), the repayment function $R(\omega)$ can be written as that*

- *$\exists\ \bar{\omega}$ and $\underline{\omega}$, such that*

$$R(\omega) = \begin{cases} 0 & if \quad \omega < \underline{\omega} \\ \omega - \underline{\omega} & if \quad \underline{\omega} \leq \omega \leq \bar{\omega} \\ \bar{R} & if \quad \omega > \bar{\omega}, \text{ where } \bar{\omega} \geq \bar{R} \geq \bar{\omega} - \underline{\omega} \end{cases}$$

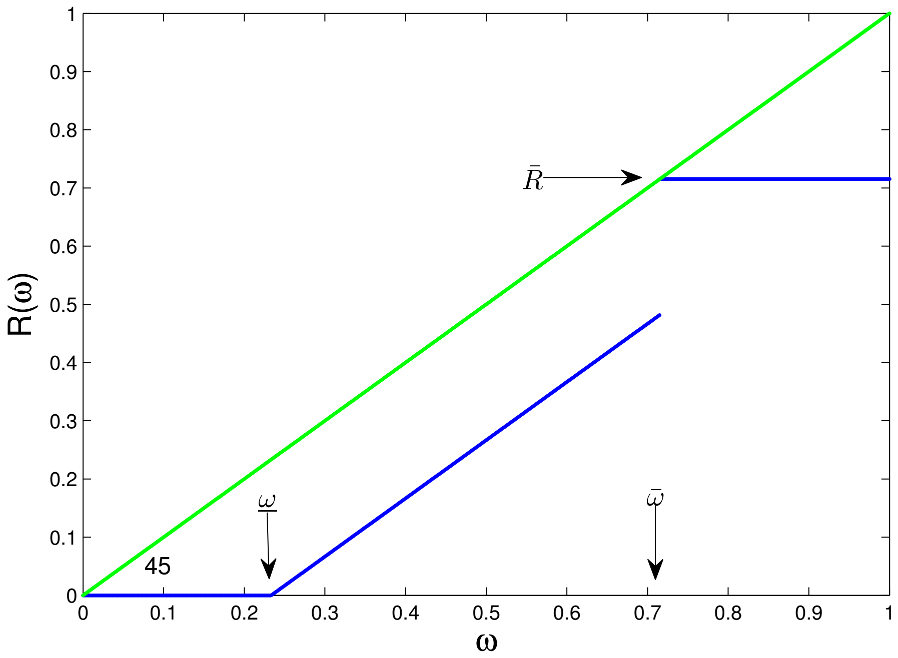
$$\Omega^V = [0, \bar{\omega})$$

Proof *See the Appendix.*

The optimal contract is illustrated in Figure 1. Proposition 1 highlights the difference between the optimal contract with risk-neutral borrowers and the case with risk-averse borrowers. When the lender monitors the borrower ($\omega \leq \bar{\omega}$), he does not seize all assets. If the borrower's returns are very small ($\omega < \underline{\omega}$), the lender receives no repayment; if the borrower does a little better ($\underline{\omega} < \omega < \bar{\omega}$), he keeps a fixed amount $\underline{\omega}$ of resources, while the lender seizes the rest. As in Townsend's debt contract, when the borrower is not monitored, the lender receives a flat payoff. The structure of the optimal contract in the defaulting region is the result of the borrower's attempt to smooth his return across different states of the world.[5] Therefore, optimal risk sharing requires that the borrower is initially prioritized in the repayment. At the same time the lender is indifferent to the structure of the repayment function, as long as his net payment covers the opportunity cost of his funds on average.

[5]Effectively, in the region $\omega \in (\underline{\omega}, \bar{\omega})$ the borrower always receives $\underline{\omega}$.

Figure 1: Optimal contract with risk-averse entrepreneurs



Corollary 1 *When $\rho \to 0$, $\underline{\omega} \to 0$, $\bar{R} \to \bar{\omega}$, so that the optimal contract replicates the original BGG contract.*

Corollary 1 states that when the borrower becomes risk-neutral the optimal contract converges to the debt contract of BGG. In this case the repayment function is completely characterized by $\bar{\omega}$, as $\bar{R}$ becomes equal to $\bar{\omega}$ and $\underline{\omega}$ goes to zero. In other words, the debt contract of BGG is a special case of the richer risk-sharing agreement described in Proposition 1.

An interesting implication of Proposition 1 is that, notwithstanding the complexity of the problem under risk-aversion, the repayment function $R(\omega)$ is completely characterized by the thresholds $(\underline{\omega}, \bar{\omega})$ and by the non-default repayment $\bar{R}$. This allows us to reformulate the contracting problem as follows:

$$\mathcal{L} = \max_{\bar{\omega}, \underline{\omega}, \bar{R}, \kappa, \lambda} \frac{(\kappa R^k)^{1-\rho} g(\bar{\omega}, \underline{\omega}, \bar{R})}{1-\rho} + \lambda\Big(\kappa R^k h(\bar{\omega}, \underline{\omega}, \bar{R}) - (\kappa - 1)R\Big)$$

where $\kappa \equiv \frac{QK}{N}$, $g(\bar{\omega}, \underline{\omega}, \bar{R})$ and $h(\bar{\omega}, \underline{\omega}, \bar{R})$ are correspondingly:

$$g(\bar{\omega}, \underline{\omega}, \bar{R}) = \int_0^{\underline{\omega}} \omega^{1-\rho}\phi(\omega)d\omega + \underline{\omega}^{1-\rho} \int_{\underline{\omega}}^{\bar{\omega}} \phi(\omega)d\omega + \int_{\bar{\omega}}^{\infty} (\omega - \bar{R})^{1-\rho}\phi(\omega)d\omega \quad (5)$$

$$h(\bar{\omega}, \underline{\omega}, \bar{R}) = (1-\mu) \int_{\underline{\omega}}^{\bar{\omega}} \omega\phi(\omega)d\omega - \underline{\omega} \int_{\underline{\omega}}^{\bar{\omega}} \phi(\omega)d\omega + \bar{R} \int_{\bar{\omega}}^{\infty} \phi(\omega)d\omega - \mu \int_0^{\underline{\omega}} \omega\phi(\omega)d\omega \quad (6)$$

The optimal $\kappa, \bar{\omega}, \underline{\omega}, \bar{R}$ are only functions of exogenous variables R^k, R and parameters σ_ω, μ. The first-order conditions for this problem are reported in the Appendix.

Figure 2: Optimal leverage

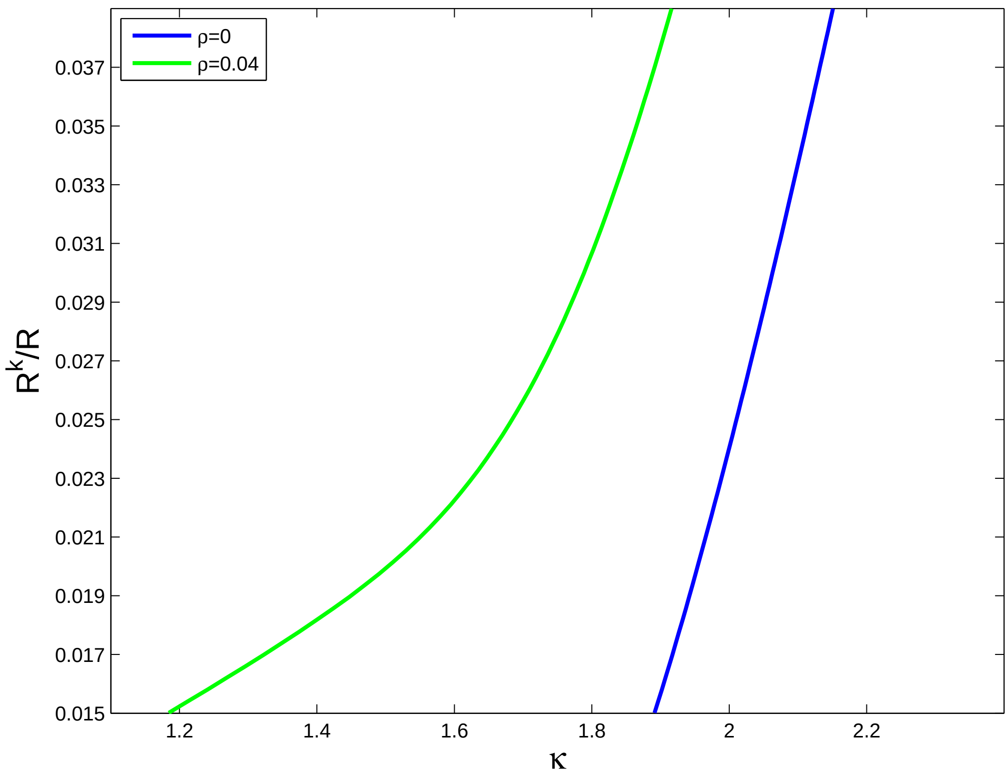


Figure 2 shows the relationship between the (annualized) discounted returns to capital (R^k/R) and leverage κ. The relationship is positive as higher returns to capital lower expected defaults, thereby reducing agency costs and allowing entrepreneurs to borrow more. From the Figure we also see that for any given premium, as risk-aversion increases, leverage decreases. This is what we should expect as, when risk aversion rises, entrepreneurs will try to reduce the volatility of their returns by cutting leverage.

3 Dynamic Optimal Contract in Partial Equilibrium With Aggregate Risk

In this section we extend the contract to a dynamic setting where entrepreneurs maximize their expected consumption path and returns to capital are subject to aggregate risk. For the moment, aggregate returns to capital and the risk-free rate are still exogenous. We largely use notation from Dmitriev Hoddenbagh (2013).

At time t, the entrepreneur j purchases capital $K_t(j)$ at a unit price of Q_t, which he will rent to wholesale goods producers in the next period. The entrepreneur uses his net worth $N_t(j)$ and a loan $B_t(j)$ from the representative lender to purchase capital:

$$Q_t K_t(j) = N_t(j) + B_t(j). \tag{7}$$

In period $t+1$, entrepreneur j is hit with an idiosyncratic shock $\omega_{t+1}(j)$ and an aggregate shock R^k_{t+1}, so that he is able to deliver $Q_t K_t(j) R^k_{t+1} \omega_{t+1}(j)$ units of assets. The idiosyncratic shock $\omega_{t+1}(j)$ is a log-normal random variable with distribution $\log(\omega_{t+1}(j)) \sim \mathcal{N}(-\frac{1}{2}\sigma^2_{\omega,t}, \sigma^2_{\omega,t})$ so that the mean of ω is equal to 1.[6] When the realization of $\omega_{t+1}(j)$ exceeds $\bar{\omega}_{t+1}$ the entrepreneur is able to repay the loan at the contractual rate Z_{t+1}. That is,

$$B_t Z_{t+1} = Q_t K_t R^k_{t+1} \bar{R}_{t+1} \tag{8}$$

Following BGG, we assume that entrepreneurs die with constant probability $1-\gamma$. It is well known, for instance from the work of Krussell and Smith (1999), that if agents are risk-averse and subject to uninsurable idiosyncratic risk, there is no simple way of aggregating individual histories and one would need to keep track of the wealth distribution of all the entrepreneurs. Consider the case where entrepreneurs receive a wage income in every period. In this case, different entrepreneurs would choose different leverages, depending on their net worth. For example, entrepreneurs with a very low net worth would realize that, even in the case of very low idiosyncratic returns to capital, if they survive to the next period, they would be able to make up for their losses with their wages. Given their low net worth today the variance of their net worth tomorrow is still pretty low even for a high leverage, therefore it will be optimal to choose a high leverage. Consider instead an entrepreneur with a very high net worth today. In case of a low idiosyncratic realization tomorrow, he would lose almost all his wealth and end up consuming only his wage. This entrepreneur will choose a lower leverage than the low-net-worth entrepreneur. The issue of different leverages does not arise in BGG because entrepreneurs are risk-neutral and thus are indifferent to the variance of their future wealth.

To resolve the aggregation problem we assume that entrepreneurs work only in the first period of their lives and that they consume all their net worth only upon the event of death. If entrepreneurs survive they do not consume anything and reinvest all their proceeds. In order to keep aggregate dynamics of net worth the same of BGG, we assume that in the first period entrepreneurs provide $\frac{1}{1-\gamma}$ units of labor, so that total labor income is identical in both models. Entrepreneur j's value function is

$$V^e_t(j) = (1-\gamma) \sum_{s=1}^{\infty} \gamma^s \mathbb{E}_t \frac{(C^e_{t+s}(j))^{1-\rho}}{1-\rho} \tag{9}$$

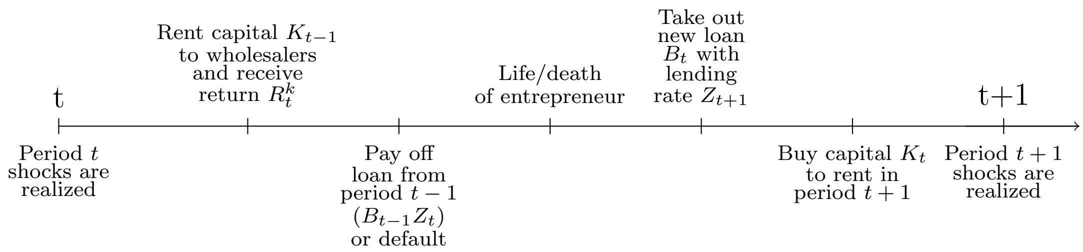
[6] The timing is meant to capture the fact that the variance of ω_{t+1} is known at the time of the financial arrangement, t.

where $C^e_{t+s}(j)$ is the entrepreneur j's consumption in case of his death,

$$C^e_t(j) = N_t(j) \tag{10}$$

defined as wealth accumulated from operating firms. The timeline for entrepreneurs is plotted in Figure 3.

Figure 3: Timeline for Entrepreneurs



The dynamic problem can be formulated recursively as follows:

$$\max_{K_t, \bar{R}_{t+1}, \bar{\omega}_{t+1}, \underline{\omega}_{t+1}} \mathbb{E}_t \left[\frac{(\kappa_t R_{k,t+1})^{1-\rho} g(\bar{\omega}_{t+1}, \underline{\omega}_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t}) \Psi_{t+1}}{1-\rho} \right] \tag{11}$$

$$s.t. \Psi_t = 1 + \gamma \mathbb{E}_t \left[(\kappa_t R_{k,t+1})^{1-\rho} g(\bar{\omega}_{t+1}, \underline{\omega}_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t}) \Psi_{t+1} \right] \tag{12}$$

$$s.t. \beta \kappa_t R_{k,t+1} h(\bar{\omega}_{t+1}, \underline{\omega}_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t}) = (\kappa_t - 1) R_t \tag{13}$$

As in BGG, R_t is the safe rate known at time t. Lenders require to be paid R_t on average, which implies that the contract must specify a triplet $\{\underline{\omega}_{t+1}, \bar{\omega}_{t+1}, \bar{R}_{t+1}\}$ contingent on R^k_{t+1}.[7] This assumption about the repayment to the lenders makes entrepreneurs effectively bear the aggregate risk. The following Proposition summarizes the solution to the dynamic contracting problem.

Proposition 2 *Solving problem (11)-(13) and log-linearizing the solution gives the following relationship between leverage and the expected discounted return to capital*

$$\hat{\kappa}_t = \nu_p (\mathbb{E}_t \hat{R}^k_{t+1} - R_t) \tag{14}$$

Moreover, when the standard deviation of idiosyncratic productivity varies over time, the rela-

[7]Later in the general equilibrium model R_t will be equal to the inverse of the household's stochastic factor.

tionship becomes

$$\hat{\kappa}_t = \nu_p(\mathbb{E}_t\hat{R}^k_{t+1} - R_t) + \nu_\sigma\hat{\sigma}_{\omega,t} \tag{15}$$

Proof *Equations (14) and (15) are obtained in the Appendix.*

Following our assumptions about entrepreneurial wage and consumption, all entrepreneurs choose the same leverage regardless of their net worth, so that aggregate leverage κ_t will simply be equal to the leverage chosen by each entrepreneur. Moreover, to a first-order approximation the complex financial agreement between borrowers and lenders boils down to the single equation (14) that links leverage to the expected discounted return to capital, which in equilibrium is the external finance premium. Note that equation (14) is identical in form to the one in BGG (equation (4.17) in their paper). The presence of risk-aversion only changes the elasticity of leverage to the external finance premium ν_p or to the volatility of idiosyncratic productivity ν_σ, if σ_ω is allowed to change over time. In this sense, our framework fully nests the BGG framework and this is what allows us to compare the two models in a meaningful way.

When borrowers are risk averse ($\rho > 0$) the values of the elasticities ν_p and ν_σ will be different from the risk neutral case. For all the calibrations that we considered we have that

$$\frac{\partial \nu_p}{\partial \rho} > 0 \qquad\qquad \left|\frac{\partial \nu_\sigma}{\partial \rho}\right| > 0$$

To understand this result it is useful to to think about how ρ affects steady-state leverage and marginal monitoring costs. Marginal monitoring costs represent the marginal cost of increasing leverage and, importantly, they are a convex function of leverage itself. Therefore, when leverage is lower, marginal monitoring costs are also lower and less sensitive to leverage.

An increase in risk aversion reduces steady state leverage, as explained in Section 2. Lower leverage means that the steady state is in a region where marginal monitoring costs are flatter relative to the risk neutral case. Hence, the response of κ_t to a given change in excess returns to capital (ν_p) will be larger when steady state leverage is lower because marginal monitoring costs are less sensitive to changes in κ_t.

Proposition 2 indicates that, for a given change in prices, leverage is more volatile when entrepreneurs are risk averse. If leverage varies more also in general equilibrium we might expect investment and output to be more volatile, so that risk aversion would constitute an additional channel of amplification of shocks through the financial accelerator. However, in general equilibrium, excess returns to capital adjust endogenously to changes in the economic environment and it might well be that this adjustment acts as a stabilizer rather than as an amplifier of shocks. Hence, we proceed with the analysis by embedding the optimal contract just derived in the BGG general equilibrium framework. This allows us to study the effect of the financial accelerator with risk-averse entrepreneurs when expected discounted returns to capital are determined endogenously.

4 The Model in General Equilibrium

We now embed our partial equilibrium framework in a standard dynamic New Keynesian model, where returns to capital and returns to lenders are determined endogenously. There are six agents in our model: households, entrepreneurs, financial intermediaries, capital producers, wholesalers and retailers. A graphical overview of the model is provided in Figure 4. The dotted lines denote financial flows, while the solid lines denote real flows (goods, labor, and capital).

4.1 Households

The representative household maximizes its utility by choosing the optimal path of consumption, labor and money

$$\max \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \chi \frac{H_{t+s}^{1+\eta}}{1+\eta} \right] \right\}, \tag{16}$$

where C_t is household consumption, and H_t is household labor effort. The budget constraint of the representative household is

$$C_t = W_t H_t - T_t + \Pi_t + R_{t-1} D_t - D_{t+1} + R_{t-1}^n \frac{B_t}{P_t} - \frac{B_{t+1}}{P_t} \tag{17}$$

where W_t is the real wage, T_t is lump-sum taxes, Π_t is lump-sum profits received from final goods firms owed by the household, D_t are deposits in financial intermediaries (banks) that pay a real non-contingent gross interest rate R_{t-1} and B_t are nominal bonds that pay a gross non-contingent interest rate R_{t-1}^n.

Households maximize their utility (16) subject to the budget constraint (17) with respect to consumption, labor, bonds and deposits yielding the following first order conditions:

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left\{ C_{t+1}^{-\sigma} \right\} R_t, \tag{18}$$

$$C_t^{-\sigma} = \beta R_t^n \mathbb{E}_t \left\{ \frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right\} \tag{19}$$

$$W_t C_t^{-\sigma} = \chi H_t^{\eta}. \tag{20}$$

We define the gross rate of inflation as $\pi_{t+1} = P_{t+1}/P_t$.

4.2 Retailers

The final consumption good consists of a basket of intermediate retail goods which are aggregated together in a CES fashion by the representative household:

$$C_t = \left(\int_0^1 c_{it}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{21}$$

The demand for retailer i's unique variety is

$$c_{it} = \left(\frac{p_{it}}{P_t} \right)^{-\varepsilon} C_t, \tag{22}$$

where p_{it} is the price charged by retail firm i. The aggregate price index is defined as

$$P_t = \left(\int_0^1 p_{it}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \tag{23}$$

Retailers costlessly differentiate the wholesale goods and sell them to households at a markup over marginal cost. They have price-setting power and are subject to Calvo (1979) price rigidities. With probability $1-\theta$ each retailer is able to change its price in a particular period t. Retailer i maximizes the following stream of real profits:

$$\max_{p_{it}^*} \sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t,s} \frac{p_{it}^* - P_{t+s}^w}{P_{t+s}} \left(\frac{p_{it}^*}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} \right\}, \tag{24}$$

where P_t^w is the wholesale goods price and $\Lambda_{t,s} \equiv \beta \frac{U_{C,t+s}}{U_{C,t}}$ is the household's (i.e. shareholder's) stochastic discount factor. The first order condition with respect to the retailer's price p_{it}^* is

$$\sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t,s} \left(\frac{p_{it}^*}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} \left[\frac{p_{it}^*}{P_{t+s}} - \frac{\varepsilon}{\varepsilon-1} \frac{P_{t+s}^w}{P_{t+s}} \right] \right\} = 0. \tag{25}$$

From this condition it is clear that all retailers that are able to reset their prices in period t will choose the same price $p_{it}^* = P_t^* \ \ \forall i$. The price level will evolve according to

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{26}$$

Dividing the left and right hand side of (26) by the price level gives

$$1 = \left[\theta \pi_{t-1}^{\varepsilon-1} + (1-\theta)(p_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \tag{27}$$

where $p_t^* = P_t^*/P_t$. Using the same logic, we can normalize (25) and obtain:

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{s=0}^{\infty} \theta^s \mathbb{E}_{t-1} \left\{ \Lambda_{t,s} (1/p_{t+s})^{-\varepsilon} Y_{t+s} p_{t+s}^w \right\}}{\sum_{s=0}^{\infty} \theta^s \mathbb{E}_{t-1} \left\{ \Lambda_{t,s} (1/p_{t+s})^{-\varepsilon} Y_{t+s} \right\}}, \tag{28}$$

where $p_{t+s}^w = \frac{P_{t+s}^w}{P_t}$ and $p_{t+s} = P_{t+s}/P_t$.

4.3 Wholesalers

Wholesale goods are produced by perfectly competitive firms and then sold to monopolistically competitive retailers who costlessly differentiate them. Wholesalers hire labor from households and entrepreneurs in a competitive labor market at real wage W_t and W_t^e and rent capital from entrepreneurs at rental rate R_t^r. Note that capital purchased in period t is used in period $t+1$. Following BGG, the production function of the representative wholesaler is given by

$$Y_t = A_t K_{t-1}^{\alpha} (H_t)^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)}, \tag{29}$$

where A_t denotes aggregate technology, K_t is capital, H_t is household labor, H_t^e is entrepreneurial labor, and Ω defines the relative importance of household labor and entrepreneurial labor in the production process. Entrepreneurs inelastically supply one unit of labor, so that the production function simplifies to

$$Y_t = A_t K_{t-1}^{\alpha} H_t^{(1-\alpha)\Omega}. \tag{30}$$

One can express the price of the wholesale good in terms of the price of the final good. In this case, the price of the wholesale good will be

$$\frac{P_t^w}{P_t} = p_t^w = \frac{1}{\mathcal{X}_t}, \tag{31}$$

where $\mathcal{X}_t$ is the variable markup charged by final goods producers. The objective function for wholesalers is then given by

$$\max_{H_t, H_t^e, K_{t-1}} \frac{1}{\mathcal{X}_t} A_t K_{t-1}^{\alpha} (H_t)^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)} - W_t H_t - W_t^e H_t^e - R_t^r K_{t-1}. \tag{32}$$

Here wages and the rental price of capital are in real terms. The first order conditions with respect to capital, household labor and entrepreneurial labor are

$$\frac{1}{\mathcal{X}_t} \alpha \frac{Y_t}{K_{t-1}} = R_t^r, \tag{33}$$

$$\frac{\Omega}{\mathcal{X}_t} (1-\alpha) \frac{Y_t}{H_t} = W_t, \tag{34}$$

$$\frac{\Omega}{\mathcal{X}_t} (1-\alpha) \frac{Y_t}{H_t^e} = W_t^e. \tag{35}$$

Given that equilibrium entreprenerial labor in equilibrium is 1, we have

$$\frac{\Omega}{\mathcal{X}_t}(1-\alpha)Y_t = W_t^e. \tag{36}$$

4.4 Capital Producers

While entrepreneurs hold capital between periods, perfectly competitive capital producers hold capital within a given period, and use available capital and final goods to produce new capital. Capital production is subject to adjustment costs, according to

$$K_t = I_t + (1-\delta)K_{t-1} - \frac{\phi_K}{2}\left(\frac{I_t}{K_{t-1}} - \delta\right)^2 K_{t-1}, \tag{37}$$

where I_t is investment in period t, δ is the rate of depreciation and ϕ_K is a parameter that governs the magnitude of the adjustment cost. The capital producer's objective function is

$$\max_{I_t} K_t Q_t - I_t, \tag{38}$$

where Q_t denotes the price of capital. The first order condition of the capital producer's optimization problem is

$$\frac{1}{Q_t} = 1 - \phi_K\left(\frac{I_t}{K_{t-1}} - \delta\right). \tag{39}$$

4.5 Lenders

One can think of the representative lender in the model as a perfectly competitive bank which costlessly intermediates between households and borrowers. The role of the lender is to diversify the household's funds among various entrepreneurs. The bank takes nominal household deposits D_t and lends out the nominal amount B_t to entrepreneurs. In equilibrium, deposits will equal loanable funds ($D_t = B_t$). Households receive a predetermined real rate of return R_t on their deposits.

4.6 Entrepreneurs

We have already described the entrepreneur's problem and timing in detail in Section 3. At the beginning of each period entrepreneurs rent out the capital they bought at the end of the previous period to perfectly competitive wholesalers. Later wholesalers return to the entrepreneurs depreciated capital and pay them the rental rate. After that entrepreneurs sell their capital and settle their position with the banks, either by repaying their loans or by defaulting. Following the arrangements with the banks, nature decides which entrepreneurs are going to survive, and which entrepreneurs are going to die and consume all of their net worth. Subsequently, new entrepreneurs are born with zero net worth and supply inelastically one unit of labor in the aggregate. Then newborn and surviving old entrepreneurs borrow money from banks and buy capital from capital producers.

Wholesale firms rent capital at rate $R^r_{t+1} = \frac{\alpha Y_t}{\mathcal{X}_t K_{t-1}}$ from entrepreneurs. After production takes place entrepreneurs sell the undepreciated capital back to capital goods producers for the unit price Q_{t+1}. Aggregate returns to capital are then given by

$$R^k_{t+1} = \frac{\frac{1}{\mathcal{X}_t}\frac{\alpha Y_{t+1}}{K_t} + Q_{t+1}(1-\delta)}{Q_t}. \tag{40}$$

Consistent with the partial equilibrium specification, entrepreneurs die with probability $1-\gamma$, which implies the following dynamics for aggregate net worth:

$$N_{t+1} = \gamma\big(Q_t K_t R^k_{t+1} - (Q_t K_t - N_t)R_t - \mu Q_t K_t R^k_{t+1} \int_0^{\bar{\omega}_{t+1}} \omega\phi(\omega)d\omega\big) + W^e_{t+1}. \tag{41}$$

The terms inside the brackets reflect the aggregate returns to capital to entrepreneurs net of loan repayments and monitoring costs. Aggregate entrepreneurial consumption is given by

$$C^e_t = (1-\gamma)(N^e_t - W^e_t) \tag{42}$$

Given that each entrepreneur chooses the same leverage, we can define leverage as the ratio of aggregate capital expenditure to aggregate net worth

$$\kappa_t = Q_t K_t / N_t. \tag{43}$$

4.7 Goods market clearing

The goods market clearing condition is

$$Y_t = C_t + I_t + G_t + C^e_t + \mu Q_{t-1} K_{t-1} R^k_t \int_0^{\bar{\omega}_t} \omega\phi(\omega)d\omega \tag{44}$$

where the last term reflects aggregate monitoring costs.

4.8 Monetary Policy

As in BGG, we assume that there is a central bank which conducts monetary policy by choosing the nominal interest rate R^n_t according to the following rule

$$\log(R^n_t) - \log(R^n) = \rho^{R^n}\Big(\log(R^n_{t-1}) - \log(R)\Big) + \xi\pi_{t-1} + \epsilon^{R^n}_t \tag{45}$$

where ρ^{R^n} and ξ determine the relative importance of the past interest rate and past inflation in the central bank's interest rate rule. Shocks to the nominal interest rate are given by ϵ^{R^n}. It should be noted that the interest rule in BGG differs from the conventional Taylor rule, where current inflation rather than past inflation is targeted.

4.9 Shocks

The shocks in the model follow a standard AR(1) process. The AR(1) processes for technology, government spending and idiosyncratic volatility are given by

$$\log(A_t) = \rho^A \log(A_{t-1}) + \epsilon_t^A, \tag{46}$$

$$\log(G_t/Y_t) = (1-\rho^G)\log(G_{ss}/Y_{ss}) + \rho^G \log(G_{t-1}/Y_{t-1}) + \epsilon_t^G, \tag{47}$$

$$\log(\sigma_{\omega,t}) = (1-\rho^{\sigma_\omega})\log(\sigma_{\omega,ss}) + \rho^{\sigma_\omega}\log(\sigma_{\omega,t-1}) + \epsilon_t^{\sigma_\omega} \tag{48}$$

where ϵ^A, ϵ^G and ϵ^{σ_ω} denote exogenous shocks to technology, government spending and idiosyncratic volatility, and $(G_{ss}/Y_s s)$ and $\sigma_{\omega,ss}$ denote the steady state values for government spending relative to output and idiosyncratic volatility respectively. Recall that σ_ω^2 is the variance of idiosyncratic productivity, so that σ_ω is the standard deviation of idiosyncratic productivity. Nominal interest rate shocks are defined by the BGG Rule in (45).

4.10 Equilibrium

The nonlinear model has 26 endogenous variables and 26 equations. The endogenous variables are: R, R^n, H, C, π, p^*, p^w, $\mathcal{X}$, Y, W, W^e, I, Q, K, R^k, N, C^e, k, $\bar{\omega}$, $\underline{\omega}$, $\bar{R}$, Ψ, λ, G, A, σ_ω, where the new variable λ corresponds to the Lagrange multiplier for the optimality conditions used in the Appendix . The equations defining these endogenous variables are: (18), (19), (20), (27), (28), (30), (31), (34), (36), (37), (39), (40), (41), (42), (43), (44), and financial contract participation (13), discounting condition (12) and optimality conditions (84), (85), (86), (87). The exogenous processes for technology, government spending and idiosyncratic volatility follow (46), (47) and (48) respectively. Nominal interest rate shocks are defined by the Taylor rule in (45).

4.11 Log-linear Model

The log-linear model has 19 equations and 19 variables, because algebraic manipulations with the Calvo model allow to replace (27), (28) and (31) with (52), and drop p^* and p^w, while simplifying the financial contract allows to replace (12), (13), (84), (85), (86), (87) with (63) and drop $\bar{\omega}, \underline{\omega}, \bar{R}, \Psi$. The equations are

$$-\sigma\Big(\mathbb{E}_t\hat{C}_{t+1}-\hat{C}_t\Big)+\hat{R}_t=0, \tag{49}$$

$$\hat{R}^n_t=\hat{R}_t+\mathbb{E}_t\hat{\pi}_{t+1}, \tag{50}$$

$$\hat{Y}_t-\hat{H}_t-\hat{\mathcal{X}}_t-\sigma\hat{C}_t=\eta\hat{H}_t, \tag{51}$$

$$\hat{\pi}_t=-\frac{(1-\theta)(1-\theta\beta)}{\theta}\hat{\mathcal{X}}_t+\beta\mathbb{E}_t\hat{\pi}_{t+1}, \tag{52}$$

$$\hat{Y}_t=\hat{A}_t+\alpha\hat{K}_{t-1}+(1-\alpha)(1-\Omega)\hat{H}_t, \tag{53}$$

$$\hat{K}_t=\delta\hat{I}_t+(1-\delta)\hat{K}_{t-1}, \tag{54}$$

$$\hat{Q}_t=\delta\phi_K(\hat{I}_t-\hat{K}_{t-1}), \tag{55}$$

$$\hat{R}^k_{t+1}=(1-\epsilon)(\hat{Y}_{t+1}-\hat{K}_t-\hat{\mathcal{X}}_{t+1})+\epsilon\hat{Q}_{t+1}-\hat{Q}_t, \tag{56}$$

$$Y\hat{Y}_t=C\hat{C}_t+I\hat{I}_t+G\hat{G}_t+C^e\hat{C}^e_t+\phi N(\hat{\phi}_t+\hat{N}_{t-1}), \tag{57}$$

$$\hat{\phi}_t=\hat{Q}_{t-1}+\hat{K}_{t-1}-\hat{N}_{t-1}+\nu^m_\sigma\hat{\sigma}_{\omega,t-1}+\nu^m_p(\mathbb{E}_{t-1}R_{k,t}-\hat{R}_{t-1}), \tag{58}$$

$$\hat{N}_t=\gamma\big(\kappa R_k(\hat{\kappa}_{t-1}+\hat{R}_{k,t})-\kappa R\hat{\kappa}_{t-1}-(\kappa-1)R\hat{R}_{t-1}-\phi\hat{\phi}_t\big)+\frac{W^e}{N}(\hat{W}^e_t)+\frac{N-W^e}{N}\hat{N}_{t-1}, \tag{59}$$

$$\hat{\kappa}_t=\hat{K}_t+\hat{Q}_t-\hat{N}_t, \tag{60}$$

$$C^e\hat{C}^e_t=(1-\gamma)(N\hat{N}_t-W^e\hat{W}^e_t), \tag{61}$$

$$\hat{W}^e_t=\hat{Y}_t-\hat{\mathcal{X}}_t, \tag{62}$$

$$\hat{\kappa}_t=\nu_p(\mathbb{E}_t\hat{R}^k_{t+1}-\hat{R}_t)+\nu_\sigma\hat{\sigma}_{\omega,t}, \tag{63}$$

$$\hat{A}=\rho^A\hat{A}_{t-1}+\epsilon^A_t, \tag{64}$$

$$\hat{R}^n_t=\rho^{R^n}\hat{R}^n_{t-1}+\xi\hat{\pi}_t+\rho^Y\hat{Y}_t+\epsilon^{R^n}_t, \tag{65}$$

$$\hat{G}_t=\rho^G\hat{G}_{t-1}+\epsilon^G_t, \tag{66}$$

$$\hat{\sigma}_{\omega,t}=\rho^{\sigma_\omega}\hat{\sigma}_{\omega,t-1}+\epsilon^{\sigma_\omega}_t \tag{67}$$

5 Quantitative Analysis

In section 3 we discussed the role of risk aversion in determining the elasticities of leverage with respect to the expected discounted return to capital and to the standard deviation of idiosyncratic productivity. In particular, we have highlighted the fact that in partial equilibrium leverage becomes more responsive to the latter with higher risk aversion as marginal monitoring costs build up more slowly. While the partial equilibrium analysis suggests higher sensitivity of leverage and hence higher amplification under risk aversion, the general equilibrium effect depends on the endogenous adjustment of prices and returns. In this section we investigate quantitatively the general equilibrium effects of technology shocks, monetary shocks and idiosyncratic volatility shocks for different coefficients of risk aversion.

5.1 Calibration and Benchmarks

Our baseline calibration largely follows BGG. We set the discount factor $\beta = 0.99$, the risk aversion parameter $\sigma = 1$ so that the utility of households is logarithmic in consumption, and the elasticity of labor supply to 3 ($\eta = 1/3$). The share of capital in the Cobb-Douglas production function is $\alpha = 0.35$. Capital adjustment costs are $\phi_k = 10$ to generate an elasticity of the price of capital with respect to the investment capital ratio of 0.25. Quarterly capital depreciation is $\delta = .025$. Monitoring costs are $\mu = 0.12$. The death rate of entrepreneurs is $1 - \gamma = .0275$, yielding an annualized business failure rate of eleven percent. The weight of household labor relative to entrepreneurial labor in the production function is $\Omega = 0.99$.

For price-setting, we set the Calvo parameter $\theta = 0.75$, so that 25% of firms can reset their prices in each period, meaning the average length of time between price adjustments is four quarters. As our baseline, we follow the BGG monetary policy rule and set the autoregressive parameter on the nominal interest rate to $\rho^{R^n} = 0.9$ and the parameter on past inflation to $\xi = 0.11$. We set the persistence of the shocks to technology at $\rho^A = 0.99$, and keep the standard deviation at 1 percent. Following BGG, for monetary shocks we consider a 25 basis point shock (in annualized terms) to the nominal interest rate with persistence $\rho^{R_n} = 0.9$.

For our purposes, the most important part of the calibration regards the volatility to idiosyncratic productivity and the risk-aversion parameter. We want to compare the impulse responses of the model with risk-averse entrepreneurs to those of the benchmark model with risk-neutral ones. To do so we set ρ to zero to recover the BGG model and a to 0.1 for the risk-averse model. Following Christiano, Motto and Rostagno (2013), we set the persistence of idiosyncratic volatility at $\rho^{\sigma_\omega} = 0.9706$. As to the standard deviations of idiosyncratic volatility shocks σ_ω, we choose two different values for each coefficient of risk-aversion. If we set σ_ω to be the same for the different coefficients of risk-aversion, the model with the smaller ρ would imply a higher steady-state leverage. It follows that a shock of a given size would have a stronger effect on impact, since similar movements in prices and returns to capital would induce larger fluctuations in net worth when leverage is higher. Thus, when we increase risk-aversion, we decrease the idiosyncratic volatility to numerically align the steady-state leverage and risk premium in two models.[8]

Following BGG, when entrepreneurs are risk-neutral we set σ_ω to 0.28, which implies a steady-state leverage 2.1 and a value of R^K/R of 1.0084, corresponding to an annualized excess return of 3.3 percent. In the case of risk-averse entrepreneurs we set $\rho = 0.5$ and $\sigma_\omega = 0.085$, which generate leverage of 2.1 and R^K/R of 1.0076, corresponding to annualized excess returns of 3 percent. Why this particular coefficient of risk aversion and level of idiosyncratic volatility? If we look at the literature on cross-sectional volatility of sales growth, Castro, Clementi and

[8]We do not report the results for the two models with different risk-aversion and other identical parameters. In the model with higher risk-aversion and lower leverage the effect on the endogenous variables on impact is smaller for all shocks.

Lee (2010) obtain a value firm specific volatility of TFP between 0.04 and 0.12. Comin and Mulani (2006), Davis, Haltiwanger, Jarmin and Miranda (2006) and a more recent study by Veirman and Levin (2014) report the volatility for the annual growth of sales to be between 0.24 and 0.3, however that volatility corresponds to a much smaller standard deviation of quarterly idiosyncratic productivity. We simulate our model in the steady state, where aggregate shocks are absent, but idiosyncratic shocks still affect firms and find that $\sigma_\omega = 0.08$ and $\sigma_\omega = 0.1$ imply a value of volatility of annual sales of 0.24 and 0.3, which is the range observed in the data. We settle for a value of σ_ω of 0.085 and subsequently choose a value for ρ that delivers a leverage of two. The results reported in our simulations are robust to the choice of ρ and σ_ω as long as we select these two parameters to match the leverage and the average excess returns observed in the data.

5.2 Leverage, Premium and Amplification

Our calibration implies that the two cases we consider - risk-averse and risk-neutral entrepreneurs - have very similar steady states in terms of leverage and external finance premium. The first two columns of Table 1 show that in the risk-neutral calibration, the steady-state leverage and R^k are 2.1 and 1.0186, respectively. The risk-averse calibration delivers similar values - leverage of 2.1 and R^k equal to 1.0176 - using a higher risk aversion and a lower volatility of idiosyncratic productivity. We do not report the other steady-state variables but they are very similar across the two models.[9] The table also shows an intermediate case where we increase risk-aversion but leave σ_ω to 0.28. Clearly in this case leverage is smaller than in the other two cases.

Table 1: Steady-state comparison

	κ	R^k	ν_p	ν_σ
Risk-neutral case (σ_ω=0.28, ρ= 0.0)	2.098	1.0186	18.74	-0.71
Risk-averse case (σ_ω=0.085, ρ= 0.5)	2.084	1.0176	125.36	-1.94

$$\hat{\kappa}_t = \nu_p(\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}_{t+1}) + \nu_\sigma\hat{\sigma}_{\omega,t}$$

Despite the fact that steady states are similar, entrepreneurial risk-aversion still affects the way in which the economy reacts to shocks. This different sensitivity is captured by the different values of the two elasticities ν_p and ν_σ in equation (15) for the two calibrations. Table 1 shows that these elasticities are higher in absolute value for the risk-averse case. As we discussed in section 3, an increase in ρ increases both elasticities in absolute value. The decrease in σ_ω further increases ν_p and brings down the absolute value of ν_σ although the intermediate case

[9]From the model equations one can see that if leverage, external finance premium and defaults are identical, then the two steady states will coincide. Although with higher risk aversion defaults are smaller, they are in both cases very small compared to GDP so that in practice the steady states are almost identical.

shows that most of the change in the elasticities between our two preferred scenarios is really driven by the increase in ρ. Notably, in our risk-averse calibration the elasticity to the financial premium grows by about seven times whereas the elasticity to "risk" grows only by about three times relative to our risk-neutral calibration.

How would higher sensitivity of leverage to the financial premium and to the volatility of idiosyncratic productivity affect business cycles? In partial equilibrium, for a given change in prices or idiosyncratic volatility, the larger fluctuations in leverage should strengthen amplification. However, in general equilibrium the impact of ν_p and ν_σ is less obvious because the movement of prices is endogenous and it differs with and without risk-aversion.

To predict the outcome it is helpful to think about the elasticity ν_p in two extreme cases: the frictionless case and the risk-neutral case. In a world without financial frictions $\nu_p \to \infty$. Even the smallest increase in expected capital returns makes entrepreneurs be willing to hold an infinite amount of capital, owing to constant returns, so that the external finance premium is always equal to zero in equilibrium. At the opposite end of the spectrum, when entrepreneurs are risk-neutral, ν_p is small, reflecting the fact that even if capital returns rise, borrowing cannot increase much because marginal borrowing costs increase very quickly with leverage. Here large swings in the premium are required to generate movements in leverage. Given that ν_p in the risk-averse case is larger than in the risk-neutral case, we should expect the premium in the risk-averse case to still react to shocks (because financial frictions are still present), but more mildly than in the risk-neutral case. With smaller movements in the returns to capital and, therefore, the price of capital, we expect smaller fluctuations in net worth and less volatile business cycles. Our simulations in the following section confirm our intuition.

5.3 Simulations

Figure 5 plots the impulse responses of the two models under risk-neutrality and risk-aversion to a technology shock. In both cases the direction of the responses is the same and follows the intuition of BGG. In particular, the productivity shock immediately stimulates the demand for capital leading to an investment boom. The increase in investment raises asset prices, which raises net worth thus reducing the external finance premium. The decline in the premium further stimulates investment and the financial accelerator mechanism arises: an initial increase in investment increases asset prices and net worth, which further stimulates investment. The financial accelerator model also deliver more persistence than standard New Keynesian models because net worth reverts to steady state very slowly, as can be seen from the Figure. As usual for all models with sticky prices, a one percent increase in total factor productivity leads to less than one percent response of GDP for both models, since marginal costs go down, while prices do not adjust completely on impact, and as a result markups in the economy go up.

The responses of output, investment, consumption and other macroeconomic variables is similar across the two scenarios. The output response is almost identical because consumption and investment behave very similarly in the two cases. As we expected, the financial premium response is much milder in the risk-averse case, about one fifth of the response of the risk-neutral case. Movements in net worth and leverage are somewhat larger in the risk-averse case but the price of capital increases in a very similar fashion across the two scenarios, which leads to similar responses in investment.

One of the appealing feature of general equilibrium models with costly state verification and risk-neutral borrowers is that they amplify monetary shocks and they make the responses of macro variables more persistent, thanks to the endogenous dynamics in net worth. Figure 6 shows the impulse responses of the two models with varying degrees of risk-aversion with respect to 25 basis shock to the interest rate. The model with higher risk aversion and more precautionary behavior displays responses to monetary shocks that are about twenty percent smaller on impact vis-a-vis the risk-neutral case. As for the technology shock, the financial premium goes down much less in the case of risk-aversion. Nevertheless, because of the higher sensitivity of leverage to the premium, the response of leverage and net worth is quantitatively similar - only about 20% smaller in the risk-averse scenario. The price of capital and investment go up to a smaller extent in the risk-aversion case, therefore, we observe a somewhat smaller reaction of output to the same shock. Nevertheless, the responses are very similar in the two cases. The endogenous adjustment of the external finance premium is such that the financial accelerator mechanism is fundamentally robust to the presence of risk-averse entrepreneurs.

In case of risk shocks the quantitative responses differ more markedly across the models, as shown in Figure 7. The risk shock increases expected monitoring costs, and reduces net worth and the price of capital temporarily. However, the price of capital is expected to rise. Other things the same, this higher future return induces entrepreneurs to borrow more and invest in the remunerative capital. In the risk-neutral case, however, the risk shock increases the cost of external finance so much that entrepreneurs are not able to borrow enough to buy the temporarily cheap capital. In the risk-averse case, the smaller fluctuations in the external finance premium and the more moderate decline in net worth put these entrepreneurs in a better financial position to buy the inexpensive capital. In fact the impulse responses show that, conditional on the risk shock, borrowing declines much less in the risk-averse case relative to the risk neutral case. As a result, the decline in the price of capital, investment and output is more than two times smaller than in the risk-neutral scenario.

6 Conclusion and Future Research

In this paper we extend the BGG framework to allow borrowers to have constant relative risk-aversion preferences instead of risk-neutral. We find that the model with risk-averse borrowers compared to the model with risk-neutral borrowers demonstrates similar responses for technology and monetary shocks, but significantly weaker response for shocks to the volatility of idiosyncratic productivity or "risk-shocks". These results are closely related to the conclusions of Christiano, Motto and Rostagno (2013), who demonstrated the importance of risk shocks for business cycles with risk-neutral entrepreneurs.

For subsequent research our framework can be extended in several directions. It is possible to have several types of entrepreneurs with different preferences and leverage, while maintaining analytical tractability. Such specification would allow the average risk-aversion to be time-varying, since positive shocks would redistribute resources towards agents with higher leverage and lower risk-aversion. In this case, a sequence of good shocks would decrease average risk-aversion and increase leverage, which might make economy more fragile to negative shocks.

Our framework also allows for contracts with optimal risk-sharing of aggregate risk between lenders and borrowers. In the current framework returns to lenders are predetermined, and entrepreneurs effectively carry all aggregate risk, so it would be interesting to investigate, whether the amplification of monetary and technology shocks is robust to the trade of state-contingent claims on the aggregate state of the world. From Dmitriev and Hoddenbagh (2013) and Carlstrom, Fuerst and Paustian (2013) and Carlstrom, Fuerst, Ortiz and Paustian (2013) we know that the financial accelerator is not robust to the presence of state-contingent contracts for risk-neutral entrepreneurs. Dmitriev and Hoddenbagh (2014) demonstrate that amplification is also not robust to state-contingent contracts in costly enforcement environment, developed by Kiyotaki and Moore (1997) and extended by Iacoviello (2005) to risk-averse agents environment. The robustness of the accelerator to state-contingent contracts in costly state verification framework with risk-averse agents remains an important question for future research.

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7 Appendix

7.1 Proof of Proposition 1

The proof follows Tamayo (2013). First, note that when the report is not verified ($\omega \notin \Omega^V$) the repayment function must only depend on the report $\tilde{\omega}$, i.e. we have $R(\tilde{\omega})$. Therefore, the entrepreneur will choose $\omega^* = \arg\min_{\tilde{\omega}} R(\tilde{\omega})$ so the contract may as well set $R(\tilde{\omega}) = \bar{R}$. Second, under the optimal contract, in the verification region $R(\omega) \leq \bar{R}$ because otherwise the contract would not be incentive compatible. Specifically, the entrepreneur would prefer to misreport $\omega \notin \Omega^V$ and pay $\bar{R}$. Finally, it can also be shown that Ω^V must be a lower interval (for the proof see Lemma 3 in Tamayo (2013)). These findings can be summarized by saying that the optimal repayment function follows:

$$R(\omega) = \begin{cases} R(\omega) \leq \bar{R}, & \text{if } \omega \leq \bar{\omega} \\ \bar{R}, & \text{if } \omega > \bar{\omega} \end{cases} \tag{68}$$

Now let us rewrite the contracting problem using the above results as

$$\max \int_0^{\bar{\omega}} \left(\kappa \frac{R^k}{R}\right)^{1-\rho} [\omega - R(\omega)]^{1-\rho} d\Phi(\omega) + \int_{\bar{\omega}}^{\infty} \left(\kappa \frac{R^k}{R}\right)^{1-\rho} [\omega - \bar{R}]^{1-\rho} d\Phi(\omega) \tag{69}$$

$$\text{s.t. } \kappa \frac{R^k}{R} \left(\int_0^{\bar{\omega}} R(\omega) d\Phi(\omega) + R[1 - \Phi(\bar{\omega})] - \mu\Phi(\bar{\omega}) \right) \geq (\kappa - 1) \tag{70}$$

$$\bar{R} \leq \bar{\omega} \tag{71}$$

$$R(\omega) \leq \omega \ \forall \omega \leq \bar{\omega} \tag{72}$$

$$R(\omega) \geq 0 \ \forall \omega \leq \bar{\omega} \tag{73}$$

where we have plugged in the constraint (3), used the definition of leverage $\kappa = \frac{QK}{N}$ and rescaled the objective function and constraints by the exogenous parameters N and R. Assign the multipliers $\lambda, \xi, \gamma_1(\omega)$ and $\gamma_2(\omega)$ to the constraints. The Lagrangian reads:

$$\max \int_0^{\bar{\omega}} \left(\kappa \frac{R^k}{R}\right)^{1-\rho} [\omega - R(\omega)]^{1-\rho} d\Phi(\omega) + \int_{\bar{\omega}}^{\infty} \left(\kappa \frac{R^k}{R}\right)^{1-\rho} [\omega - \bar{R}]^{1-\rho} d\Phi(\omega) + \tag{74}$$

$$\lambda \left[\kappa \frac{R^k}{R} \left(\int_0^{\bar{\omega}} R(\omega) d\Phi(\omega) + R[1 - \Phi(\bar{\omega})] - \mu\Phi(\bar{\omega}) \right) - (\kappa - 1) \right] + \tag{75}$$

$$\xi(\bar{\omega} - \bar{R}) + \int_0^{\bar{\omega}} \gamma_1(\omega)(\omega - R(\omega))\phi(\omega) d\omega + \int_0^{\bar{\omega}} \gamma_2(\omega)(R(\omega))\phi(\omega) d\omega \tag{76}$$

The first order necessary conditions with respect to $R(\omega), \bar{R}, \bar{\omega}$ after appropriate rescaling of the multipliers can be written as[10]:

$$-\gamma_1(\omega)\phi(\omega) - \left(\kappa\frac{R^K}{R}\right)^{1-\rho}\{[\omega - R(\omega)]^{-\rho}\phi(\omega) + \lambda\left(\kappa\frac{R^K}{R}\right)\phi(\omega) + \gamma_2(\omega)\phi(\omega) = 0 \text{ for every } \omega \leq \bar{\omega} \tag{77}$$

$$-\xi - \left(\kappa\frac{R^K}{R}\right)^{1-\rho}\int_{\bar{\omega}}^{\infty}[\omega - \bar{R}]^{-\rho}d\Phi(\omega) + \lambda\left(\kappa\frac{R^K}{R}\right)[1 - \Phi(\bar{\omega})] = 0 \tag{78}$$

$$-\frac{\xi}{\phi(\bar{\omega})} - \left(\kappa\frac{R^K}{R}\right)^{1-\rho}[\bar{\omega} - R(\bar{\omega})]^{1-\rho} + \left(\kappa\frac{R^K}{R}\right)^{1-\rho}[\bar{\omega} - \bar{R}]^{1-\rho} - \lambda\left(\kappa\frac{R^K}{R}\right)[R(\bar{\omega}) - \bar{R} - \mu] = 0 \tag{79}$$

and the complementary slackness conditions:

$$0 = \lambda\left\{\kappa\frac{R^k}{R}\left(\int_0^{\bar{\omega}}\bar{R}(\omega)d\Phi(\omega) + R[1 - \Phi(\bar{\omega})] - \mu\Phi(\bar{\omega})\right) - (\kappa - 1)\right\} \tag{80}$$

$$0 = \xi[\bar{\omega} - \bar{R}] \tag{81}$$

$$0 = \gamma_1(\omega)[\omega - R(\omega)] \tag{82}$$

$$0 = \gamma_2(\omega)R(\omega) \tag{83}$$

Suppose that $\gamma_1(\omega) > 0$ for all $\omega < \bar{\omega}$. Then it must be that $\gamma_2(\omega) = 0$, from the complementary slackness conditions. Then equation (75) would imply that $\lambda > \left(\kappa\frac{R^K}{R}\right)^{-\rho}(0)^{-\rho}$ which is not possible. Hence it must be true that $\gamma_1(\omega) = 0$ for all $\omega \leq \bar{\omega}$ and a standard debt contract is not optimal. We know from (75) that $\gamma_1(\omega) = 0 \iff (\omega - R(\omega))^{-\rho} \geq \lambda$. Now there are two possible cases. Suppose $\gamma_2(\omega) = 0$ for all $\omega \leq \bar{\omega}$. Then the contract specifies that $R(\omega) = \omega - \lambda^{-1/\rho}\left(\frac{R}{R^K\kappa}\right)$. By complementary slackness it should be the case that $R(\omega) > 0$ for all ω, which is not possible because if $\omega = 0$, $R(\omega) > 0$ would not be feasible. Then it must be the case that $\gamma_2(\omega) > 0$ for some ω which implies $R(\omega) = 0$ and $\omega \leq \lambda^{-1/\rho}\left(\frac{R}{R^K\kappa}\right)$ for the same ω. Hence there is a lower interval where $R(\omega) = 0$. Call the upper bound of this interval $\underline{\omega} \equiv \lambda^{-1/\rho}\left(\frac{R}{R^K\kappa}\right)$. Therefore $R(\omega) = 0$ if $\omega \leq \underline{\omega}$ and $R(\omega) = \omega - \underline{\omega}$ if $\underline{\omega} \leq \omega \leq \bar{\omega}$.■

7.2 FOCs for the dynamic contract and proof of Proposition 2

The Lagrangian is

$$\mathcal{L} = \mathbb{E}_t\left\{\frac{(\kappa_t R_{t+1}^k)^{1-\rho}g(\bar{\omega}_{t+1}, \underline{\omega}_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t})\Psi_{t+1}}{1-\rho} + \lambda_{t+1}\left(\kappa_t R_{t+1}^k h(\bar{\omega}_{t+1}, \underline{\omega}_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t}) - (\kappa_t - 1)R_t\right)\right\}$$

[10]We do not need the first-order condition with respect to κ to prove the proposition.

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial k_t} = \mathbb{E}_t \Big\{ (\kappa_t R^k_{t+1})^{1-\rho} g_{t+1} \Psi_{t+1} - \lambda_{t+1} R_t \Big\} = 0 \tag{84}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\omega}} = \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\bar{\omega},t+1} \Psi_{t+1}}{1-\rho} + \lambda_{t+1} \kappa_t R^k_{t+1} h_{\bar{\omega},t+1} = 0 \tag{85}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{\omega}} = \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\underline{\omega},t+1} \Psi_{t+1}}{1-\rho} + \lambda_{t+1} \kappa_t R^k_{t+1} h_{\underline{\omega},t+1} = 0 \tag{86}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{R}} = \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\bar{R},t+1} \Psi_{t+1}}{1-\rho} + \lambda_{t+1} \kappa_t R^k_{t+1} h_{\bar{R},t+1} = 0 \tag{87}$$

Now we can express λ_{t+1} from $\frac{\partial \mathcal{L}}{\partial \bar{\omega}} = 0$

$$\lambda_{t+1} = -\frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\bar{\omega},t+1} \Psi_{t+1}}{1-\rho} \frac{1}{\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} \tag{88}$$

Now we plug this condition into the three other equations and obtain

$$\frac{\partial \mathcal{L}}{\partial k_t} = \mathbb{E}_t \Big\{ (\kappa_t R^k_{t+1})^{1-\rho} g_{t+1} \Psi_{t+1} + \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\bar{\omega},t+1} \Psi_{t+1}}{1-\rho} \frac{1}{\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} R_t \Big\} = 0 \tag{89}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{\omega}} = \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\underline{\omega},t+1} \Psi_{t+1}}{1-\rho} - \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\bar{\omega},t+1} \Psi_{t+1}}{1-\rho} \frac{1}{\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} \kappa_t R^k_{t+1} h_{\underline{\omega},t+1} = 0 \tag{90}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{R}} = \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\bar{R},t+1} \Psi_{t+1}}{1-\rho} - \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\bar{\omega},t+1} \Psi_{t+1}}{1-\rho} \frac{1}{\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} \kappa_t R^k_{t+1} h_{\bar{R},t+1} = 0 \tag{91}$$

we can transform this system to

$$\frac{\partial \mathcal{L}}{\partial k_t} = \mathbb{E}_t \Big\{ (R^k_{t+1})^{1-\rho} \Psi_{t+1} \Big(g_{t+1} + \frac{g_{\bar{\omega},t+1}}{(1-\rho)\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} R_t \Big) \Big\} = 0 \tag{92}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{\omega}} = g_{\underline{\omega},t+1} - g_{\bar{\omega},t+1} \frac{h_{\underline{\omega},t+1}}{h_{\bar{\omega},t+1}} = 0 \tag{93}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{R}} = g_{\bar{R},t+1} - \frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} h_{\bar{R},t+1} = 0 \tag{94}$$

Since in the equation (92) Ψ_{t+1} and $\hat{R}_{k,t+1}$ enter as multiplicative terms and the term $g_{t+1} + \frac{g_{\bar{\omega},t+1}}{(1-\rho)\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} R_t$ is equal to zero in the steady state, Ψ_{t+1} and $\hat{R}_{k,t+1}$ have no effect in the first order approximation. Therefore, to find the approximate solution it is sufficient to consider the following system:

$$\kappa_t R^k_{t+1} h(\bar{\omega}_{t+1}, \underline{\omega}_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t}) = (\kappa_t - 1) R_t \tag{95}$$

$$\mathbb{E}_t \left\{ g_{t+1} + \frac{g_{\bar{\omega},t+1}}{(1-\rho)\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} R_t \right\} = 0 \tag{96}$$

$$\frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} = \frac{g_{\bar{R},t+1}}{h_{\bar{R},t+1}} \tag{97}$$

$$\frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} = \frac{g_{\underline{\omega},t+1}}{h_{\underline{\omega},t+1}} \tag{98}$$

We can substitute k_t and obtain

$$\frac{\mathbb{E}_t \left[\frac{g_{\bar{\omega}} R_{t+1}}{(1-\rho) R_{k,t+1} h_{\omega,t+1}} \right]}{E_t g_{t+1}} = \frac{1}{1 - \frac{R_{k,t+1}}{R_t} h_{t+1}} \tag{99}$$

$$\frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} = \frac{g_{\bar{R},t+1}}{h_{\bar{R},t+1}} \tag{100}$$

$$\frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} = \frac{g_{\underline{\omega},t+1}}{h_{\underline{\omega},t+1}} \tag{101}$$

Whenever the gradient of this system has full rank at the steady state, we will be able to find an approximate solution of $\hat{\bar{\omega}}_{t+1}, \hat{\underline{\omega}}_{t+1}, \hat{\bar{R}}_{t+1}$ as functions of $\mathbb{E}_t \hat{R}_{k,t+1} - \hat{R}_t$, $\hat{R}_{k,t+1} - \hat{R}_t$ and $\hat{\sigma}_{\omega,t}$. Using this fact and log-linearizing equation (95) will give us

$$\hat{k}_t = \nu_p (\mathbb{E}_t \hat{R}_{k,t+1} - \hat{R}_t) + \nu_\sigma \hat{\sigma}_{\omega,t} \tag{102}$$

Figure 4: Overview of the Model

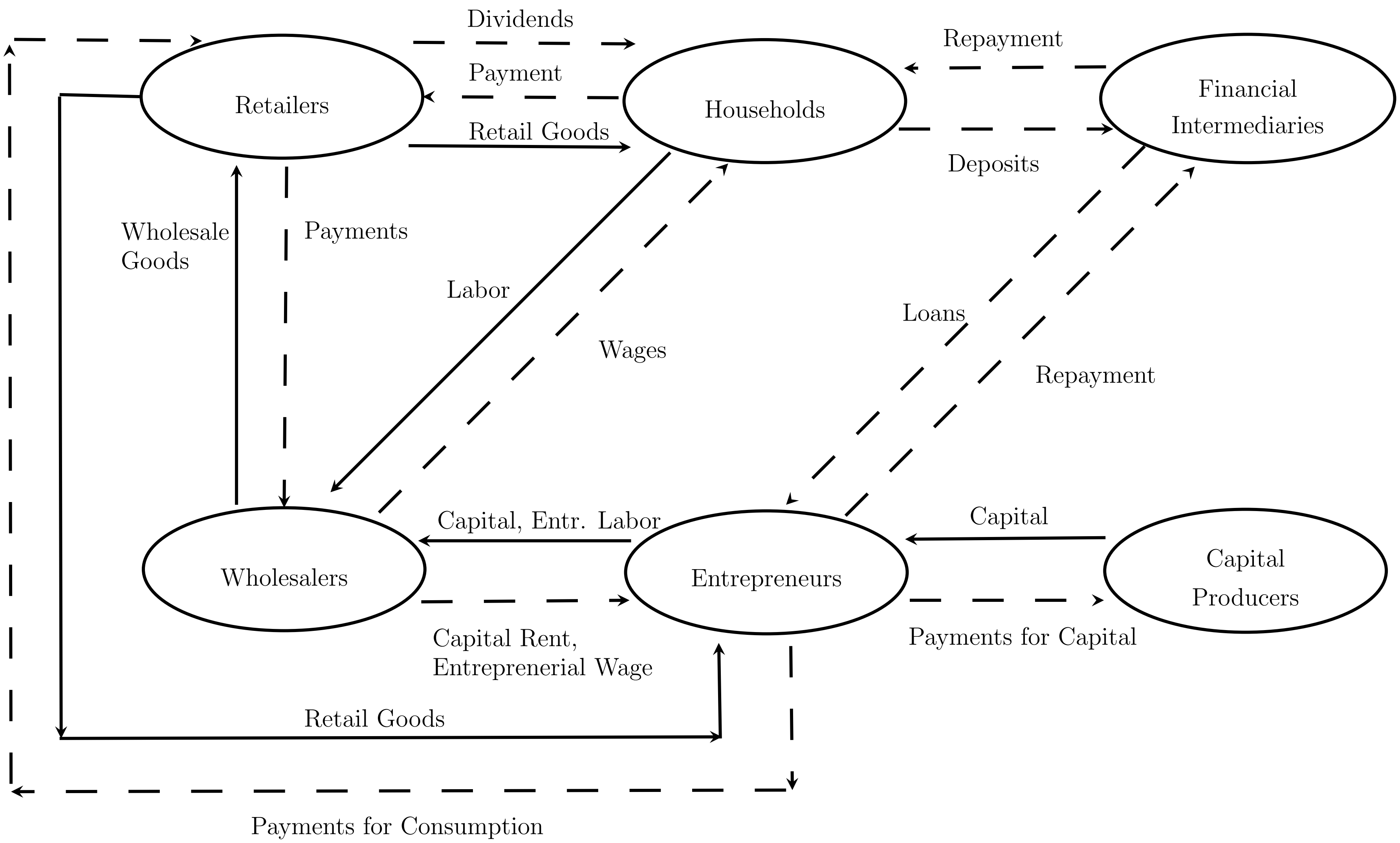

Figure 5: Impulse Response to Technology Shocks

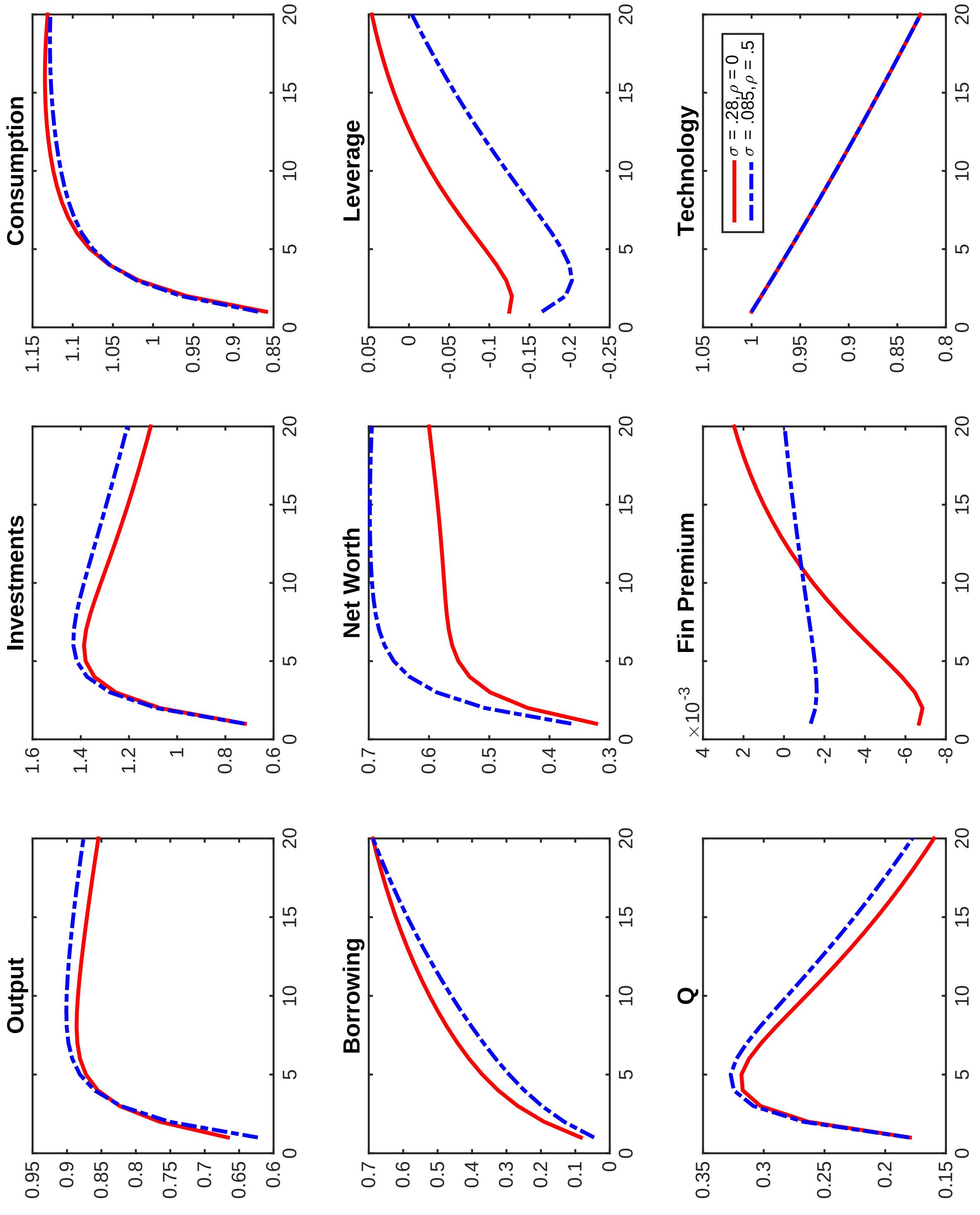

Figure 6: Impulse Response to Monetary Shocks

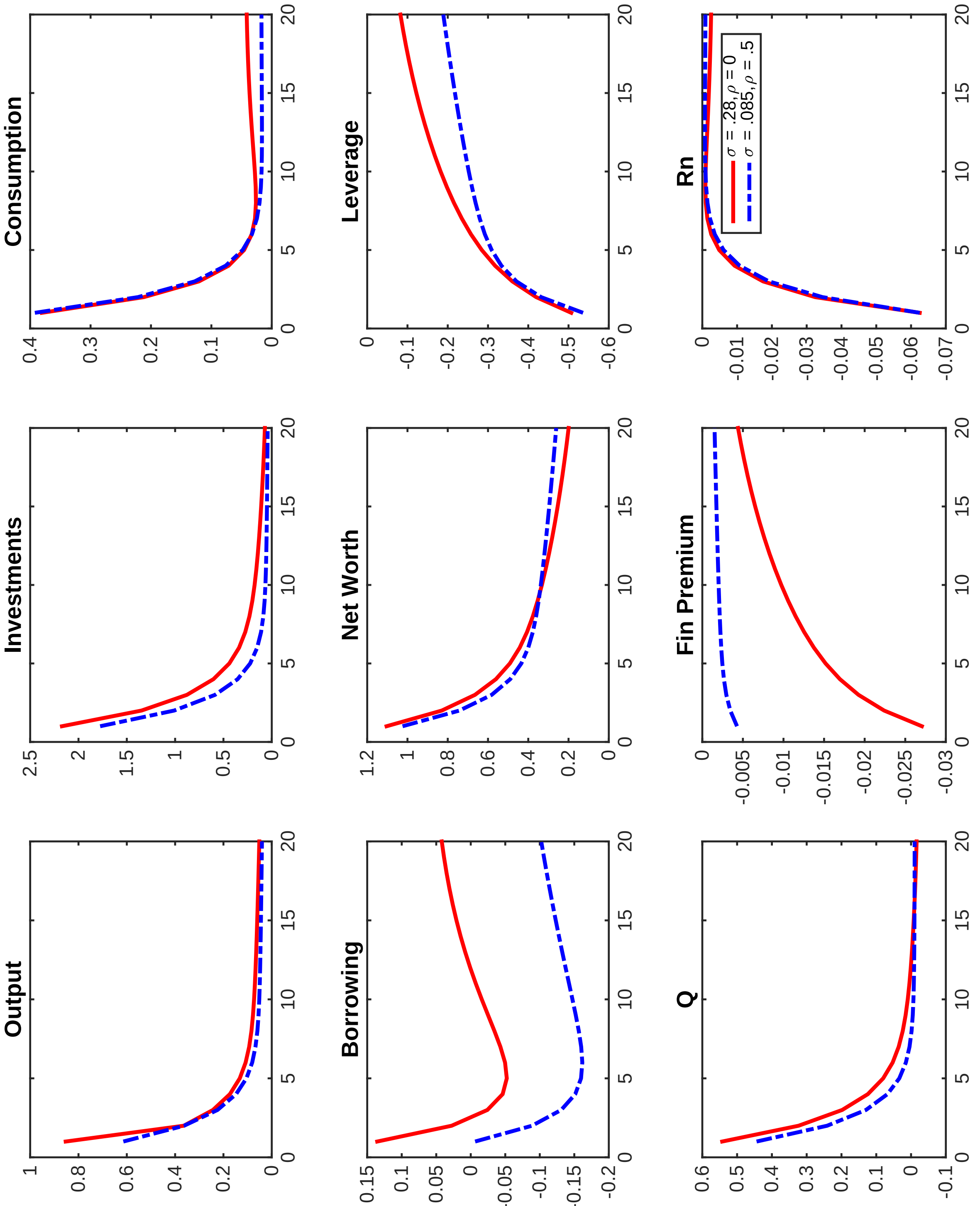

Figure 7: Impulse Response to Risk Shock